Introduction to State Estimation

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*Slides based on or adapted from Sanjiban Choudhury
Logistics

• Begin working on Assignment 0!
• Post questions, discuss any issues you are having on piazza.
• Students with no access to 022, e-mail cardkey@cs.washington.edu with your student ID and CC me.
• Students that have not been added to the class, come talk to me after the lecture.
• If you did not attend recitation yesterday, come talk to me after the lecture.
Robot interacts with environment

Sensors
- Laser
- Camera
- GPS

World Model

State Estimation

Global Planning

Local Planning

Safety Planning

Control

Actuators
- Stick
- Lever
- Pedals

Helicopter Models

Robot interacts with environment
Estimate state

Plan a sequence of motions

Control robot to follow plan
Today’s objective

1. Formulate state estimation as a Bayes filtering problem

2. Discuss various components of a Bayes filter

3. Intuition behind Bayes filtering
A robot interacting with the world
A robot interacting with the world

World Model (Belief)

Robot

Measurement

Environment, state

Action
Bayes filtering in action!
Example 1: Rich information

Measurement: LiDAR, camera, GPS

World Model (Belief): Location, cars, people...

Action: Steering, speed

Urmson “How a driverless car sees the road” TED (2015)
Example 2: Medium information

Measurement: Planar LiDAR

World Model (Belief): State of robot

Action: Pitch, Roll
Example 3: Very low information

Guilliard, et al. Autonomous thermalling as a partially observable Markov decision process. In RSS, 2018

Measurement: Pitot tube (airpseed), GPS

World Model (Belief): Location of wind thermal sensor

Action: Pitch, Roll
A robot interacting with the world

World Model (Belief) → Robot → Measurement → Environment, state → Action → Robot

World Model, belief, environment, time, robot, functioning, interacting, perception, action, data, measurement, control, system, state, belief, internal, state, static, or noisy, functioning, dynamical, robotic, systems, that, maintain, state, prediction, or, belief, error, which, predict, the, environment, variables, to, maintain, the, robot's, state, and, knowledge, of, its, location, in, the, world, model. Throughout, this, book, we, discuss, the, relationship, between, robot, and, environment, it's, sensors, that, represent, the, environment, in, the, robot's, state, model. This, model, includes, the, robot, location, in, the, environment, and, its, interaction, with, the, world, through, its, sensors. The, environment, variables, are, used, to, update, the, robot's, state, model, through, interaction, and, perception, of, its, environment.
Given data (stream of measurements and actions) \[\rightarrow\] ESTIMATE \[\rightarrow\] Belief (probability of states)

How do we formally define this problem?
State: A very abstract definition

State: statistic of history sufficient to predict the future
State ($x_t$)

Collection of variables sufficient to predict the future (future that we care about)

What are some examples of state?

1. Pose of a robot - Usually 6 dof (3 position, 3 for orientation)
   - 3 dof for planar mobile robot (x, y, heading)

2. Configuration of a manipulator - Collection of joint angles

3. Location of objects in environment

State can be static/dynamic, discrete/continuous/hybrid
Measurement ($z_t$)

Measurements are sensor values that provide information about the current state.

(Measurement does not always tell you state directly!!)

What are some examples of measurements?

1. GPS - absolute information about robot pose

2. Laser scan - relative geometric information between pose and environment

3. Camera image - information about color / texture (harder to model)
**Action** \( (u_t) \)

Actions affect how a state changes from one time to another.

What are some examples of actions?

1. Active forces applied by the robot - (measure motor currents, force torque sensors, odometers)

2. Passive actions that change environment - weather (can detect with sensors)

3. NOP actions - doing nothing is also an action. State does not change.
Fundamental problem: State is hidden

All the robot sees is a stream of actions and measurements

$$u_1, z_1, u_2, z_2, u_3, z_3, \ldots$$

But robot never sees the state

$$x_1, x_2, x_3, \ldots$$
Fundamental problem: State is hidden

But all decision making depends on knowing state

Solution: Estimate belief over state

\[ \text{bel}(x_t) = P(x_t | z_{1:t}, u_{1:t}) \]

Belief is a probability of each possible state given history

Also called Posterior / Information state / State of knowledge

Represent belief? Parametric (Gaussian), Non-parametric (Histogram)
Let’s think about causality of events

Assumptions:

1. Robot receives a stream of measurements / actions.

2. One measurement / action per time-step.
Problem: How do we estimate belief?

\[ P(\text{current state} | \text{all past information}) \]

\[ P(x_t | z_t, u_t, x_{t-1}, \ldots) \]
Problem: How do we estimate belief?

\[ P(\text{current state } | \text{all past information}) \]

\[ P(x_t | z_t, u_t, x_{t-1}, ...) \]
Solution

Andrey Andreyevich Markov (1856 - 1922)
Solution: Markov assumption

Markov assumption:
Future state conditionally independent of past actions, measurements given present state.

\[ P(x_t | u_t, x_{t-1}, z_{t-1}, u_{t-1}, \ldots ) = P(x_t | u_t, x_{t-1}) \]
\[ P(z_t | x_t, u_t, x_{t-1}, z_{t-1}, u_{t-1}, \ldots ) = P(z_t | x_t) \]
Reminder: Conditional Independence

\[ P(A|B, C) = P(A|C) \]

iff \( A, B \) conditionally independent given \( C \)
Probabilistic models

State transition probability / dynamics / motion model

\[ P(x_t | x_{t-1}, u_t) \]

Measurement probability / Observation model

\[ P(z_t | x_t) \]
When does Markov not hold?

\[ P(x_t | x_{t-1}, u_t) \quad P(z_t | x_t) \]

whenever state doesn’t capture all requisite information

- Camera images at different times of the day
- Unmodelled pedestrians in front of laser
- Steady gusts of wind
Central Question: How do we tractably calculate belief?

Input data

Measurement \( z_{1:t} \) \hspace{1cm} Actions \( u_{1:t} \)

Belief

\( bel(x_t) = P(x_t|z_{1:t}, u_{1:t}) \)

Ans: Bayes filter!
Bayes filter in a nutshell

Key Idea: Apply Markov to get a recursive update!
Bayes filter in a nutshell

Step 0. Start with the belief at time step $t-1$

$$bel(x_{t-1})$$
Bayes filter in a nutshell

Step 1: Prediction - push belief through dynamics given action

\[
\text{bel}(x_t) = \sum \frac{P(x_t | u_t, x_{t-1}) \text{bel}(x_{t-1})}{\text{bel}(x_{t-1})}
\]
Bayes filter in a nutshell

Step 2: Correction - apply Bayes rule given measurement

\[ bel(x_t) = \eta P(z_t|x_t) \overline{bel}(x_t) \]

\[ \eta = \frac{1}{\sum P(z_t|x_t) \overline{bel}(x_t)} \]

\[ \overline{bel}(x_t) = \frac{u_t}{b}(x_t) \text{ bel}(x_{t-1}) \]

\[ bel(x_t) \]

\[ \overline{bel}(x_t) \]

\[ z_t \]
Bayes filter in a nutshell

**Key Idea:** Apply Markov to get a recursive update!

Step 0. Start with the belief at time step t-1

\[ \text{bel}(x_{t-1}) \]

Step 1: Prediction - push belief through dynamics given *action*

\[ \overline{\text{bel}}(x_t) = \sum P(x_t | u_{t}, x_{t-1}) \text{bel}(x_{t-1}) \]

Step 2: Correction - apply Bayes rule given *measurement*

\[ \text{bel}(x_t) = \eta P(z_t | x_t) \overline{\text{bel}}(x_t) \]
Bayes filter is a powerful tool.