Feedback Control

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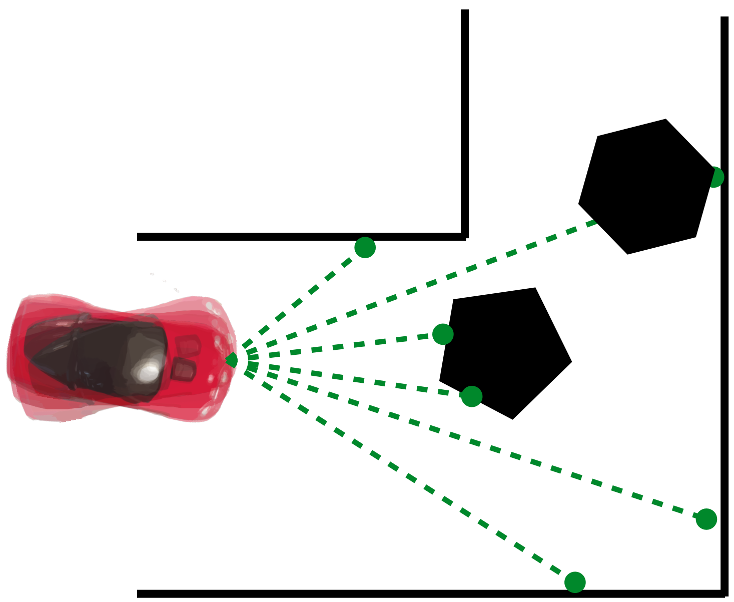
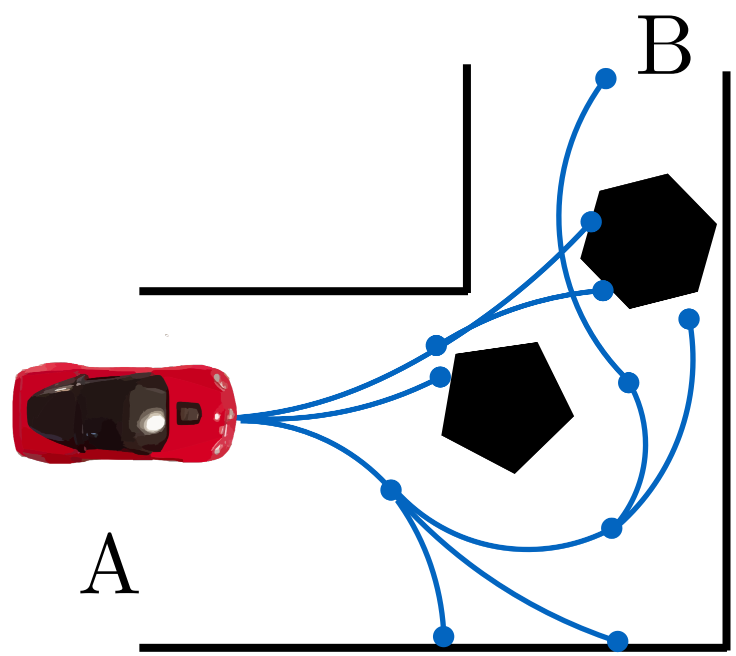
*Slides based on or adapted from Sanjiban Choudhury, Russ Tedrake

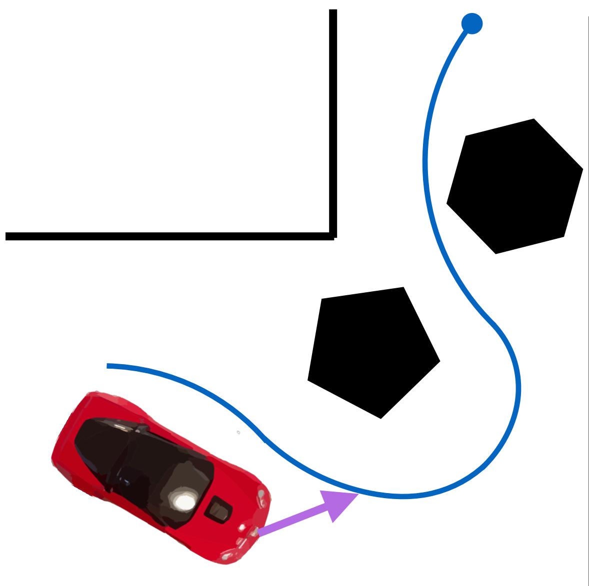
Estimate state



Plan a sequence of motions

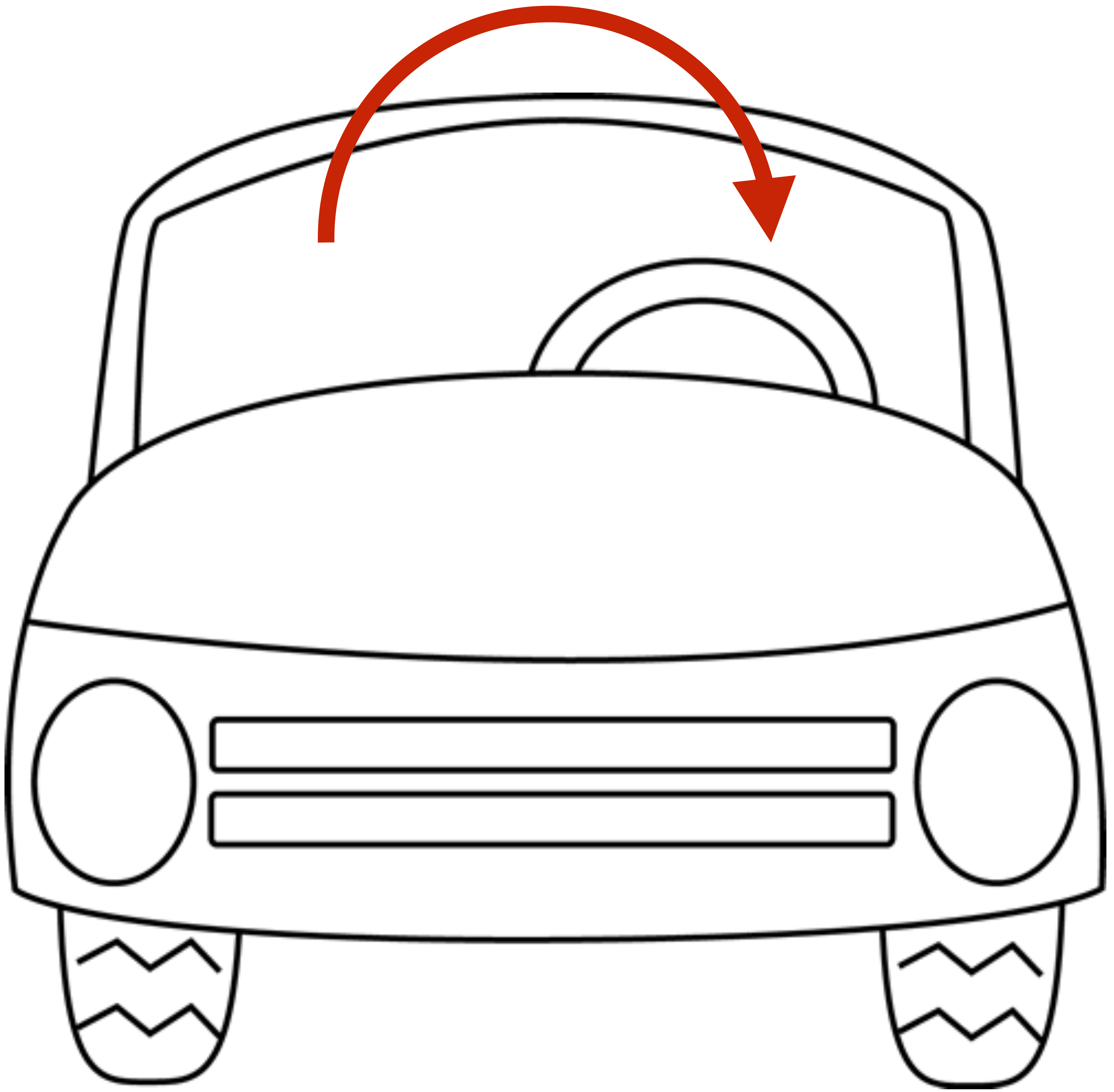


Control robot to follow path



- Robot pose known
- Path is given

From perception to control ...



When I think about control ...

https://www.youtube.com/watch?v=WNR4MqG45pk

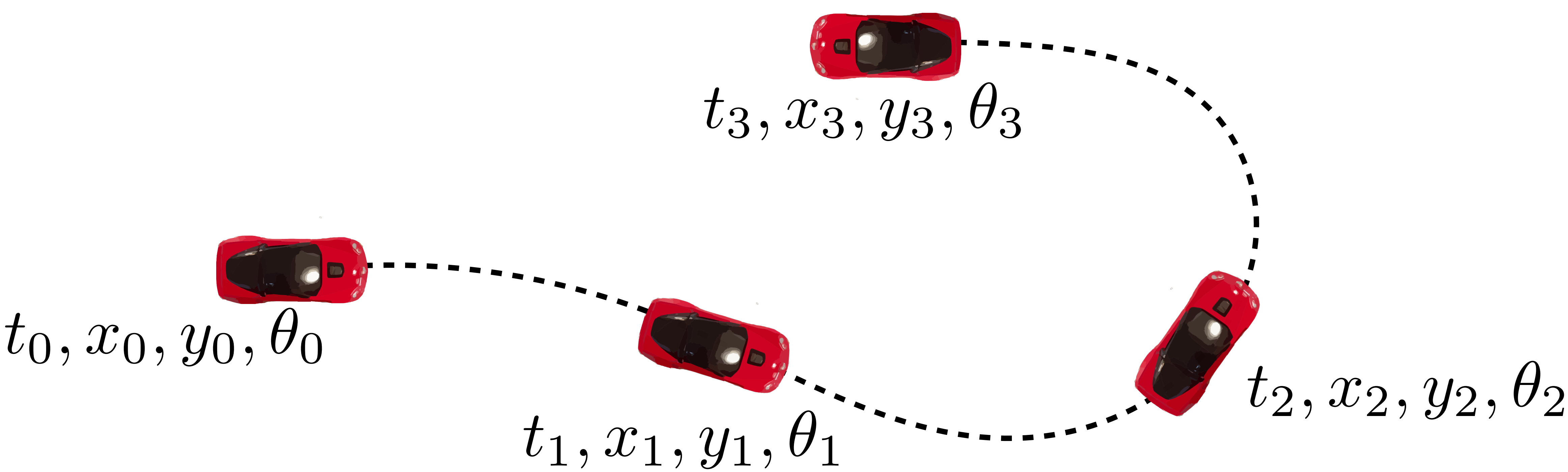
Today's objective

1. Introduce terms and definitions in feedback control

2. Go through challenges in current control research

The control framework

Say we want to get the car to hit a sequence of poses



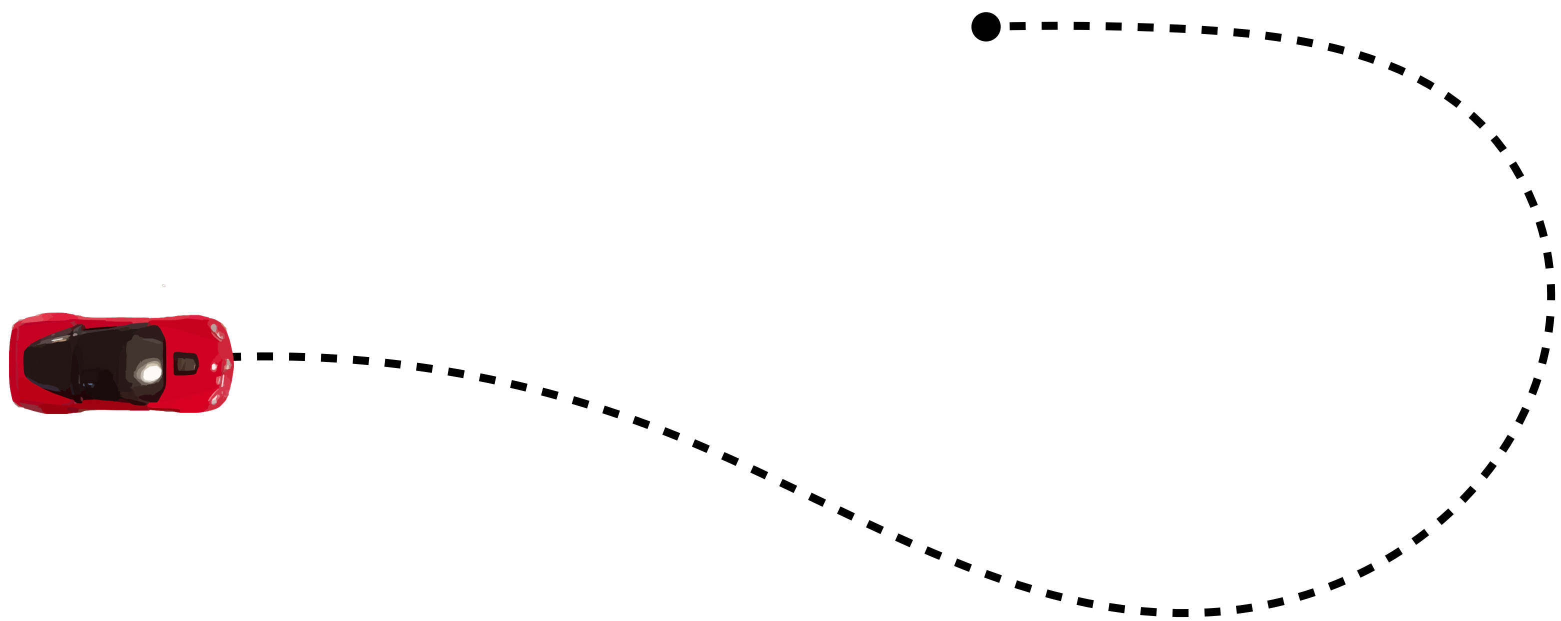
Can express this as wanting to track a reference *trajectory*

$$x(t), y(t), \theta(t)$$

The control framework

Let's say we want to track a reference trajectory

$$x(t), y(t), \theta(t)$$



Objective: Figure out a **control** trajectory $u(t)$ to achieve this

In our case, we will focus on steering angle $\delta(t)$ as control input

Why do we need feedback?

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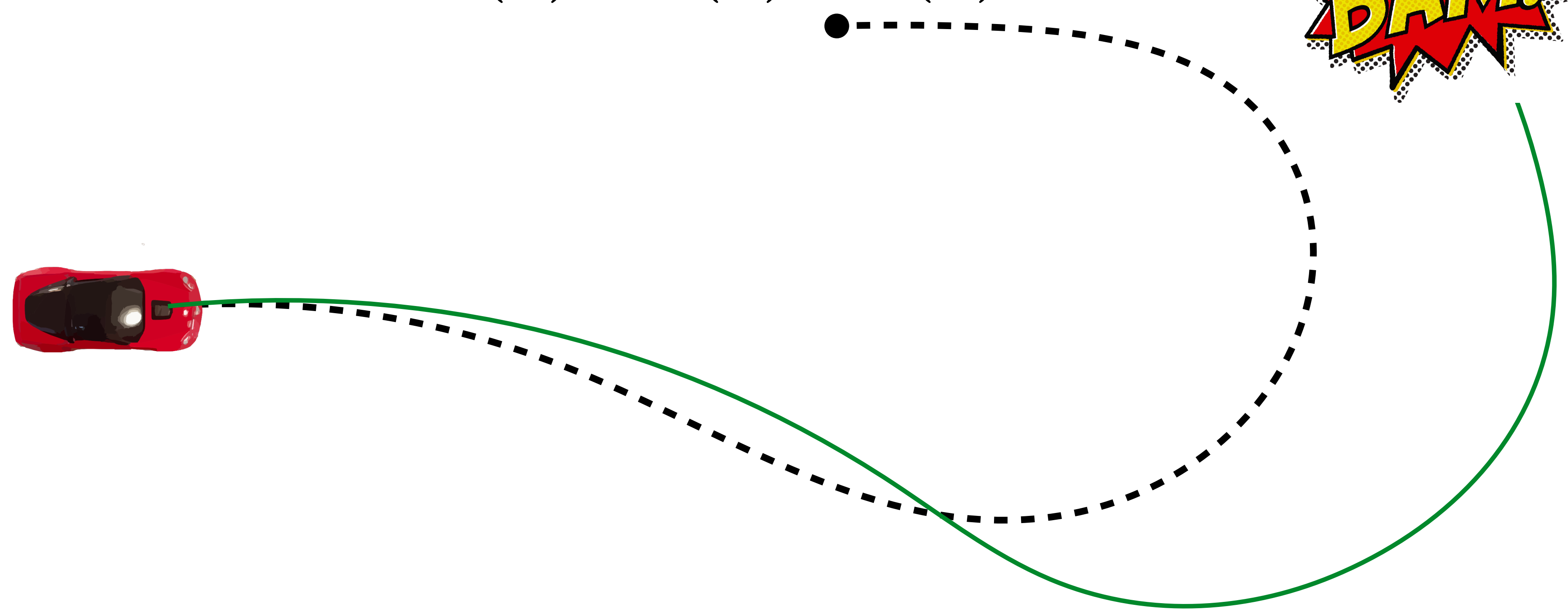
Let's say we want to track a reference trajectory

$$x(t), y(t), \theta(t)$$



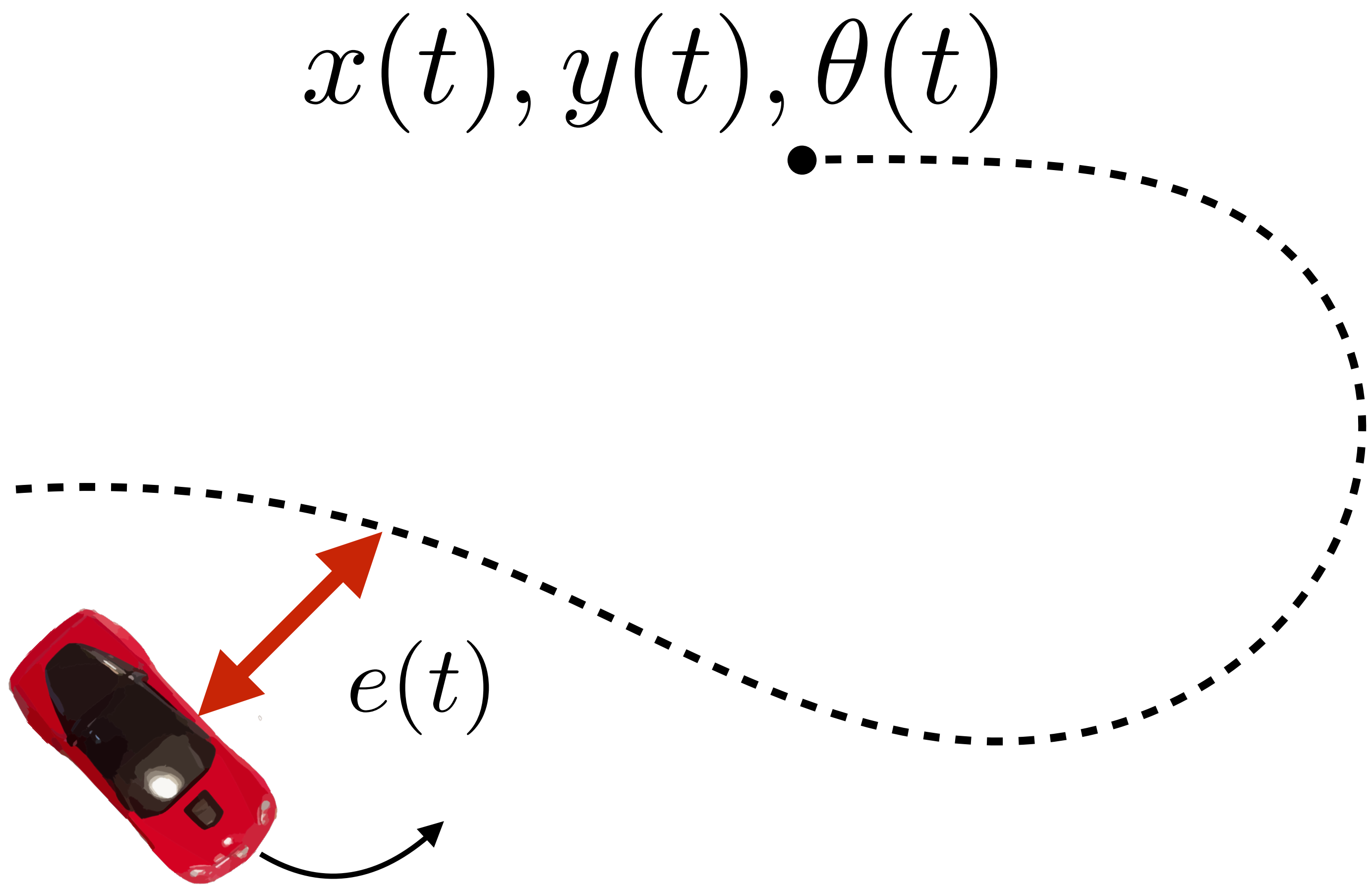
What if we send out steering angles $\delta(t)$ obtained from kinematic car model?

Open loop control leads to accumulating errors!

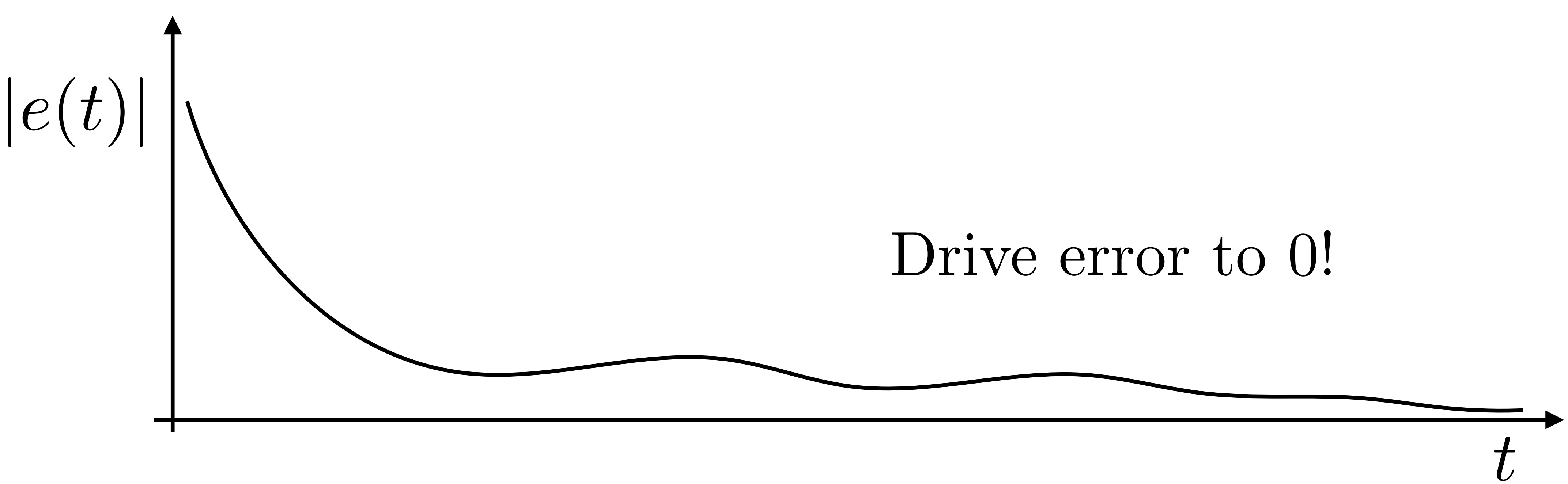
Overarching principle of feedback control:

Measure error between reference and state. Minimize this error.

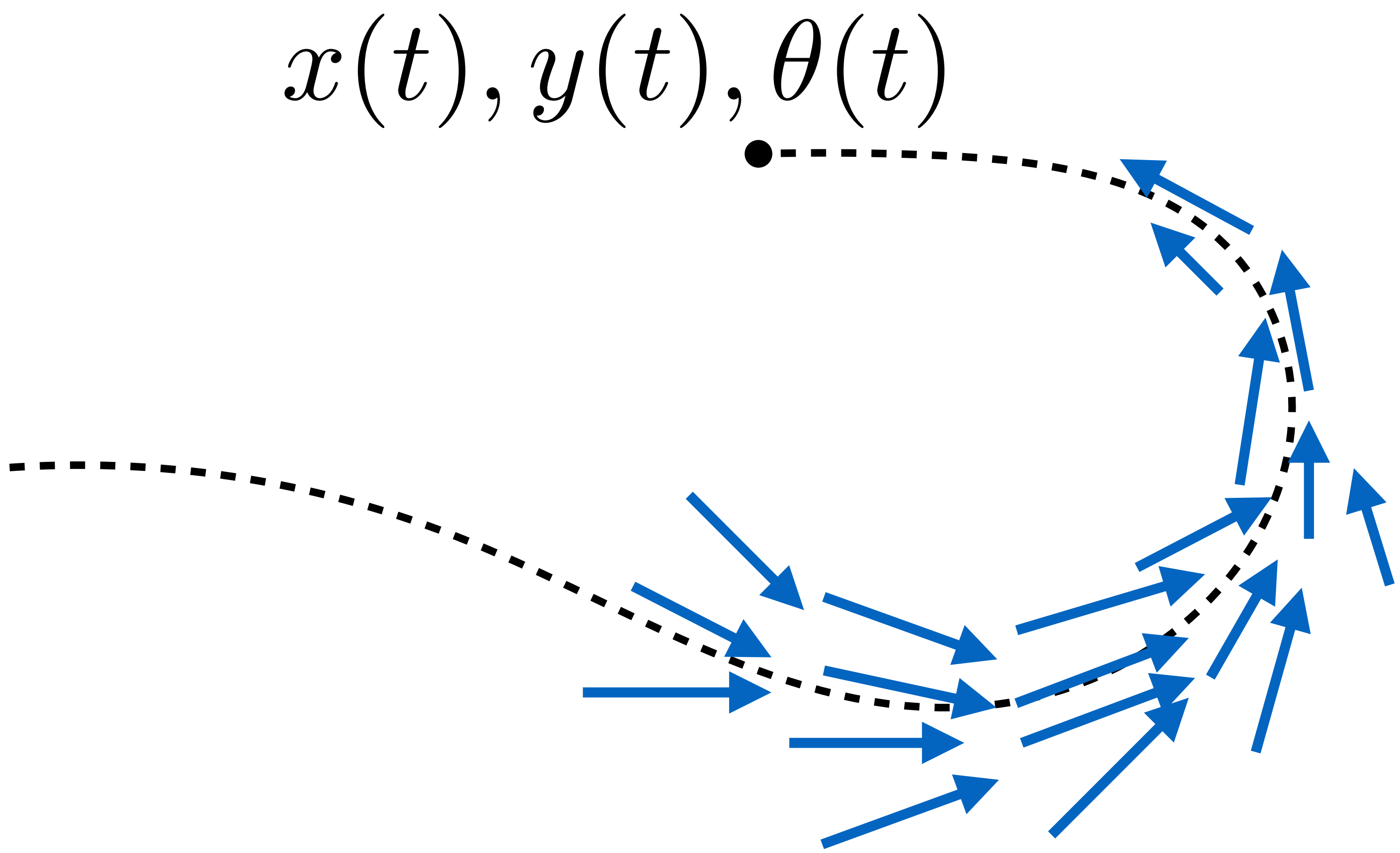
Measure error and minimize it



Error between state and reference

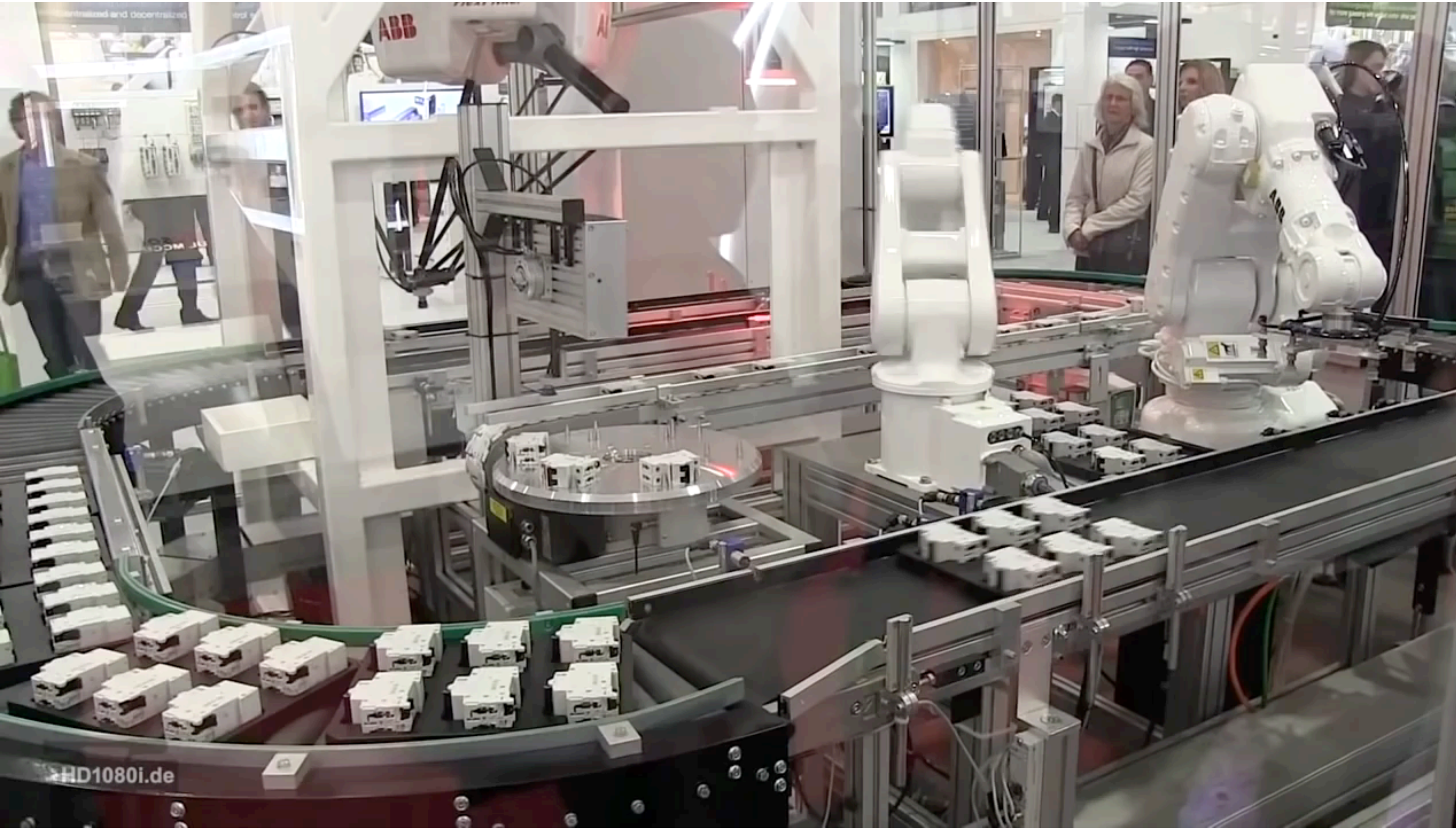


Useful to think of control laws as vector fields



Is this still a research problem?

Industrial robots hard at work



https://www.youtube.com/watch?v=J_8OnDsQVZE&t=315s

Assumptions made by such controllers

1. Fully actuated: There exists an inverse mapping from reference to control actions

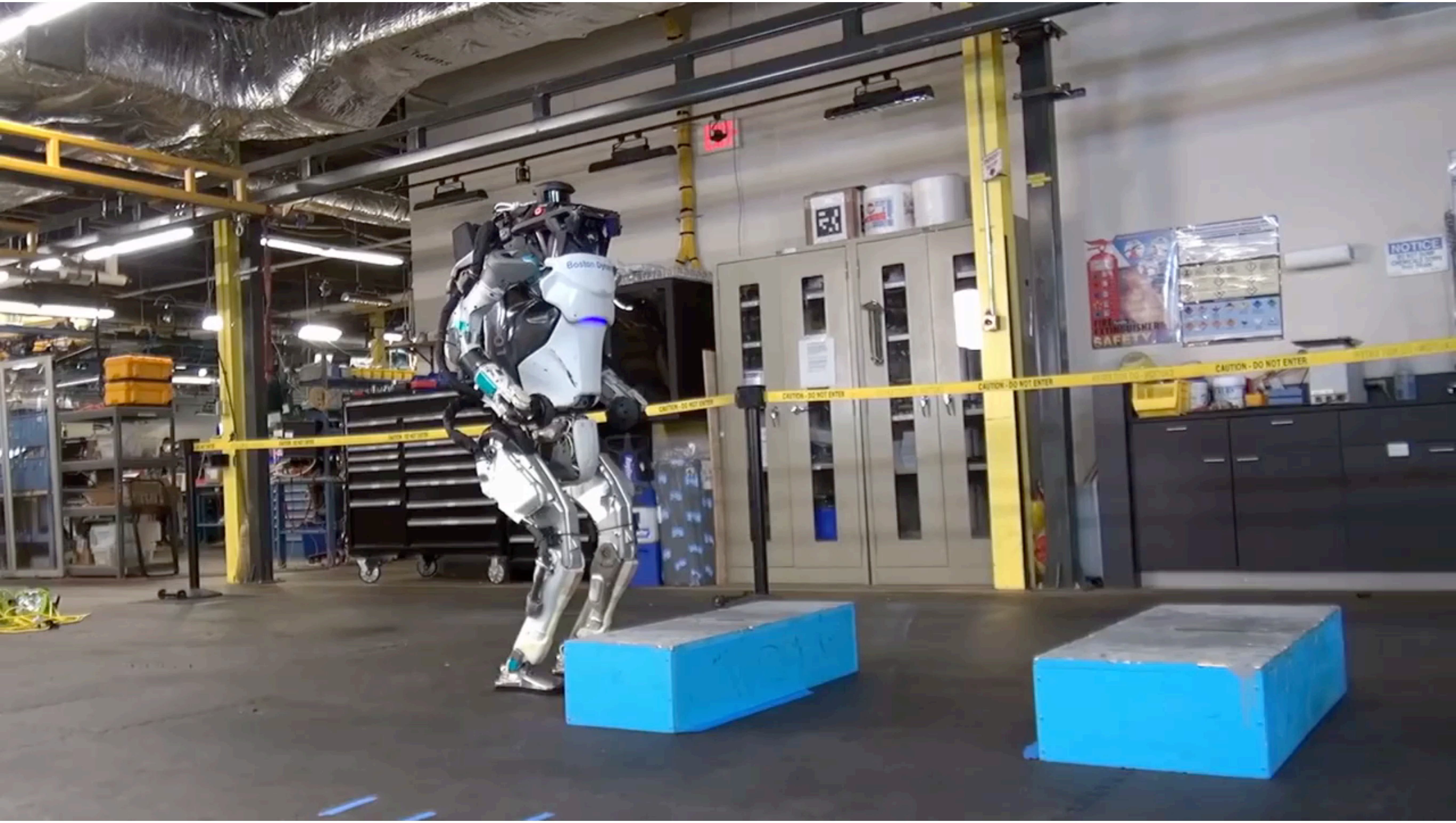
$$\sigma(t) \rightarrow u(t)$$

2. Almost no execution error or state estimation error

3. Enough control authority to clamp down errors / overcome disturbances

Instead, what are harder problems we want to think about?

The Atlas robot hard at … play?



https://www.youtube.com/watch?v=fRj34o4hN4I

Why is this a hard problem?

Challenge 1: Underactuated systems

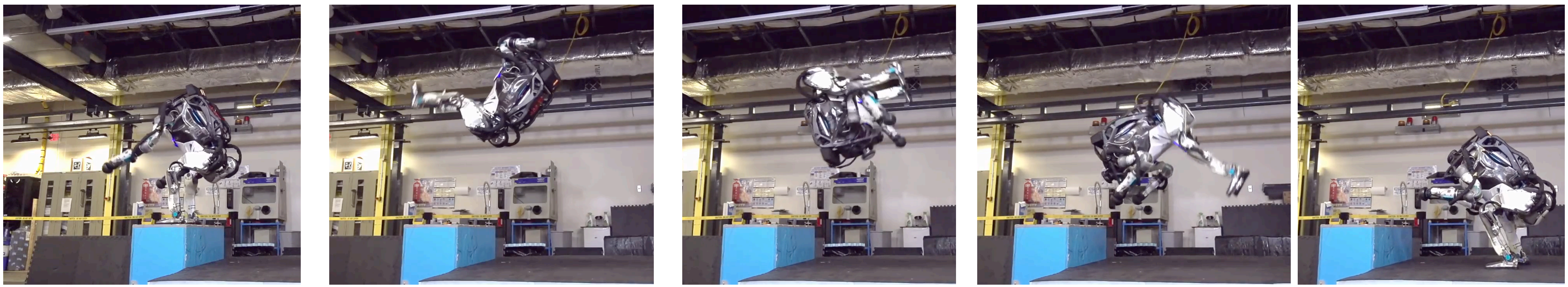
Fully actuated: There exists a[illegible] inver[illegible] mapping from reference to control actions

$$\sigma(t) \; [illegible] \; u(t)$$

We don’t have full authority to move the system along arbitrary trajectories

Challenge 1: Underactuated systems

What affects the error between robot state and reference?



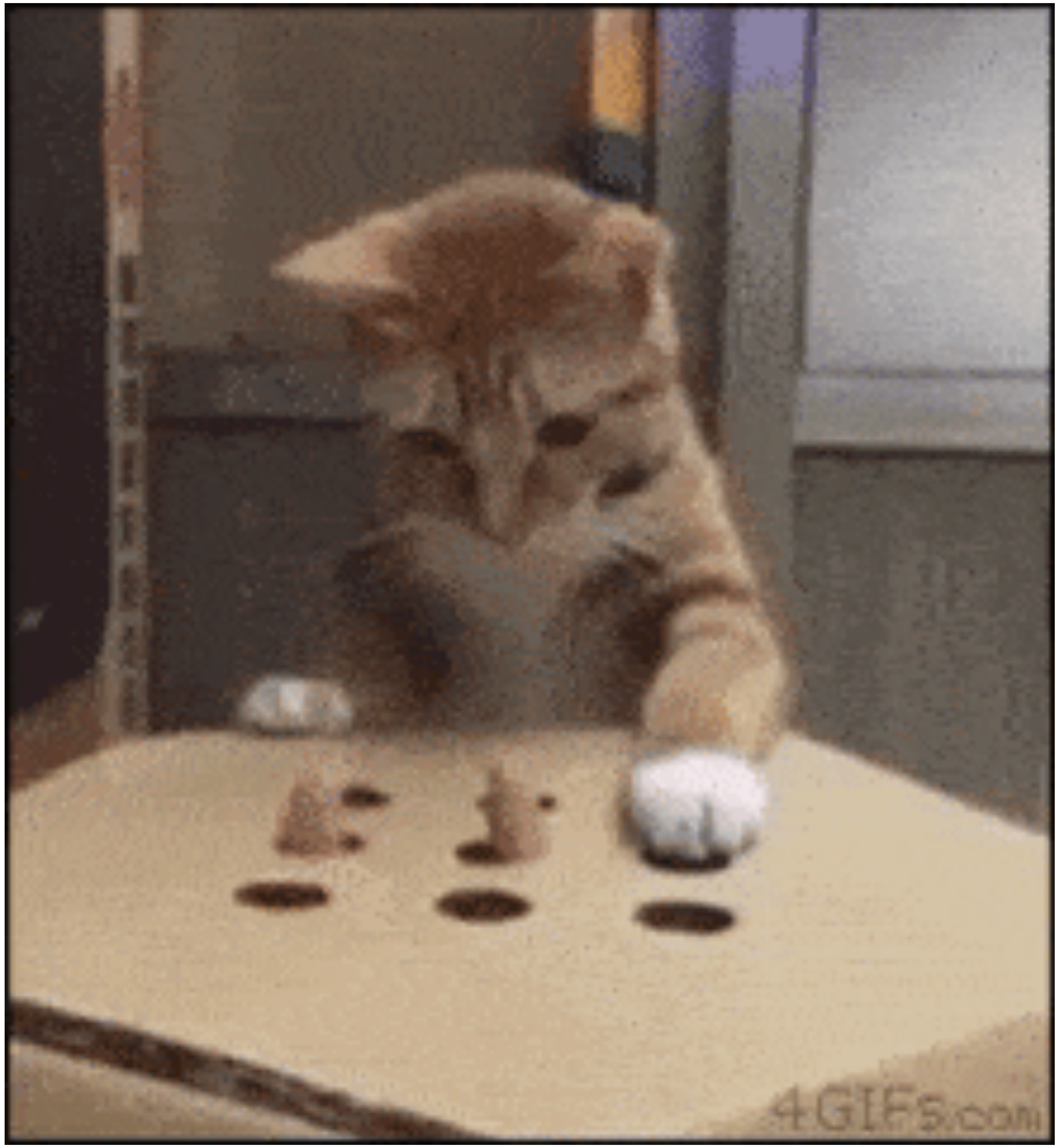
Some initial motor thrust ...

Whole lot of gravity!

Whole lot of momentum!

... some precise control adjustments

Myth: Control is a battle with nature!



Is control like playing whack-a-mole with error terms??

Embrace dynamics instead of fighting it

State-of-the-art humanoid
(at the time)

Honda, 1996

Passive walker
(no motors, powered by gravity!)

Steven H. Collins, Martijn Wisse, and Andy Ruina, 2001

Which one looks more natural? Which one consumes less energy?

Question:

If we know the model of our robot, can't we solve a ginormous optimization to figure out control ...?

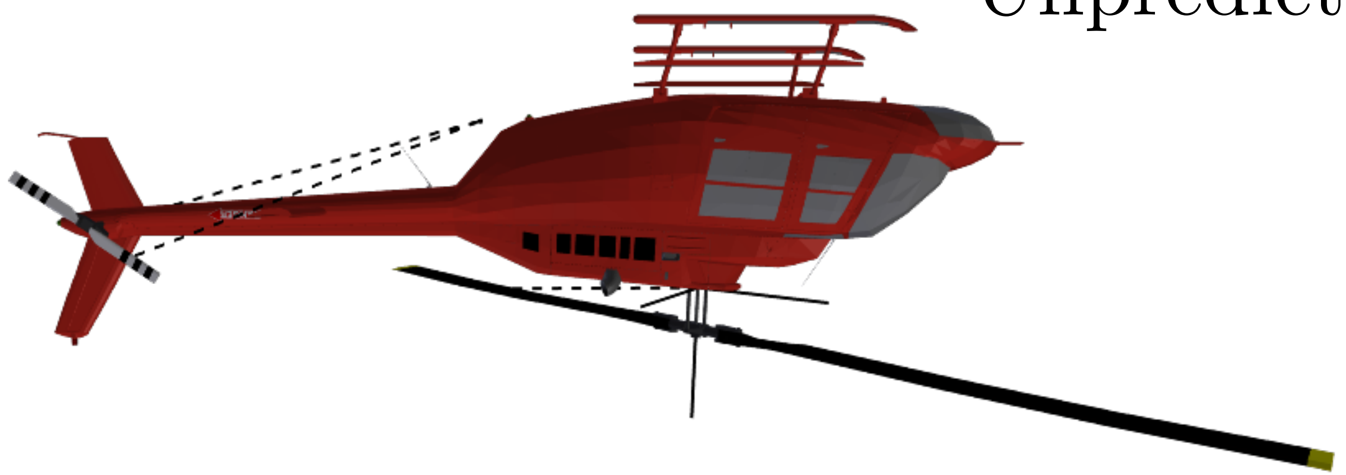
Doing backflips with a helicopter

Redbull Eurocopter BO-105 https://www.youtube.com/watch?v=RGu45s1_QPU

And just what is this model ?!?

Unpredictable drag forces!

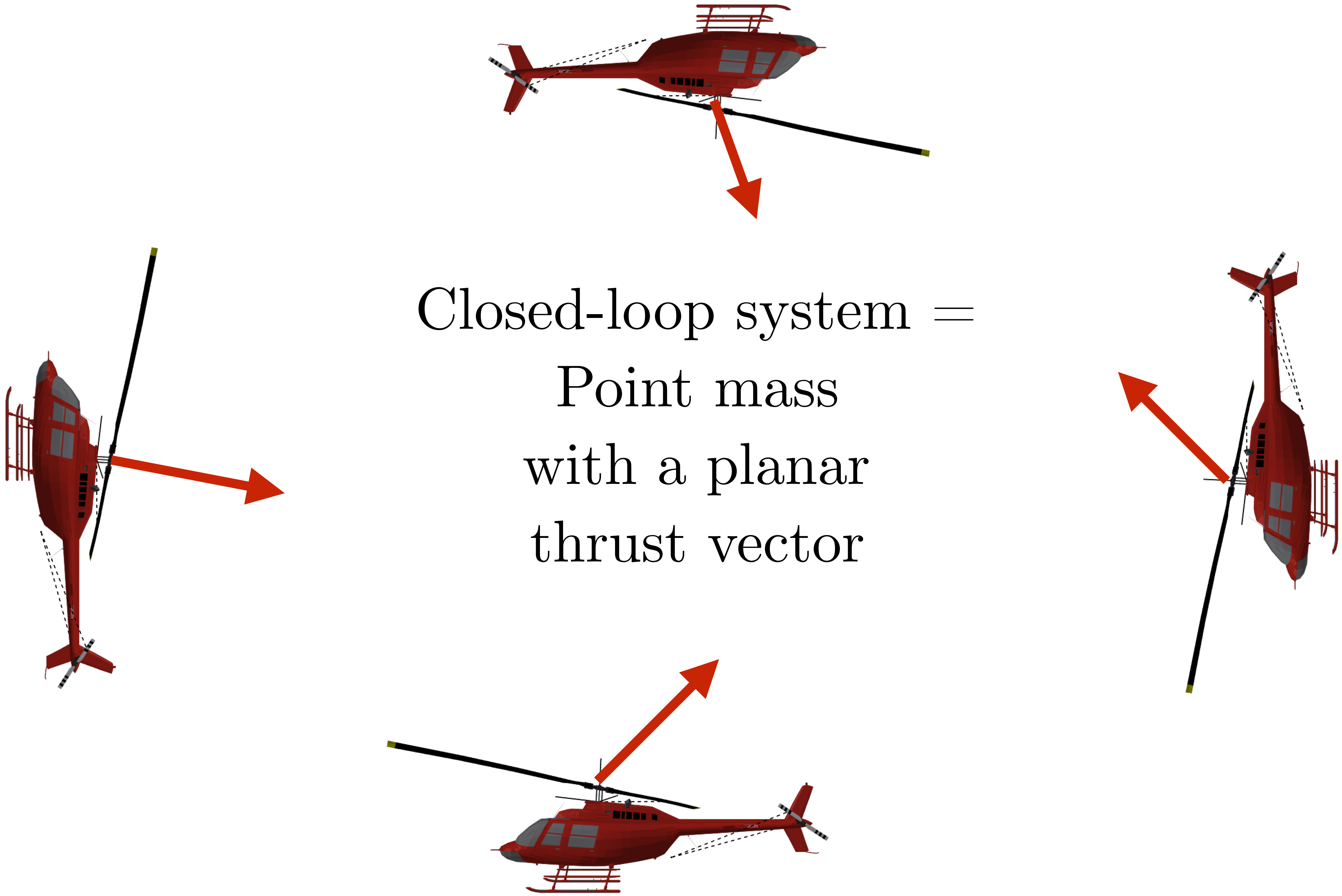
Nothing countering gravity!



Chaotic vortex around blades!

Hopeless to assume we know exactly how the helicopter will behave upside down...

Challenge 2: Choose good closed-loop models



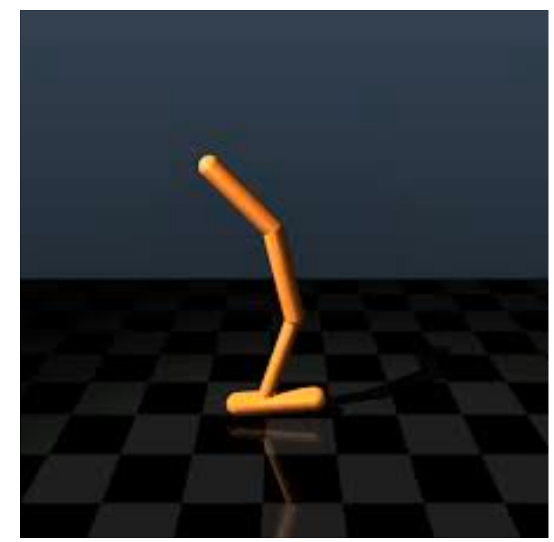
Closed-loop system =
Point mass
with a planar
thrust vector

Complex dynamics + Feedback control law = Well-behaved system

Trotting quadruped = biped = one-leg

Quadruped

Biped



One-leg

Marc Raibert showed how these closed loop systems are equivalent

Question:

Is this all offline? Can I pre-compute a bunch of controllers once and call it a day?

Figure 8s while drifting

Stanford's MARTY

https://www.youtube.com/watch?v=nTK56vPb8Zo

Challenge 3: Model changing on the fly!

Run real-time estimators for wheel characteristics

Need control laws for all possible model parameters

Other challenges for mobile robot control

1. Unexpected obstacle avoidance

2. Noisy state estimation

3. Controllers that guarantee safety

Enough with the motivation ...

Let's start writing some controllers

A generic template for a controller

1. Get a reference path / trajectory to track

2. Pick a point on the reference

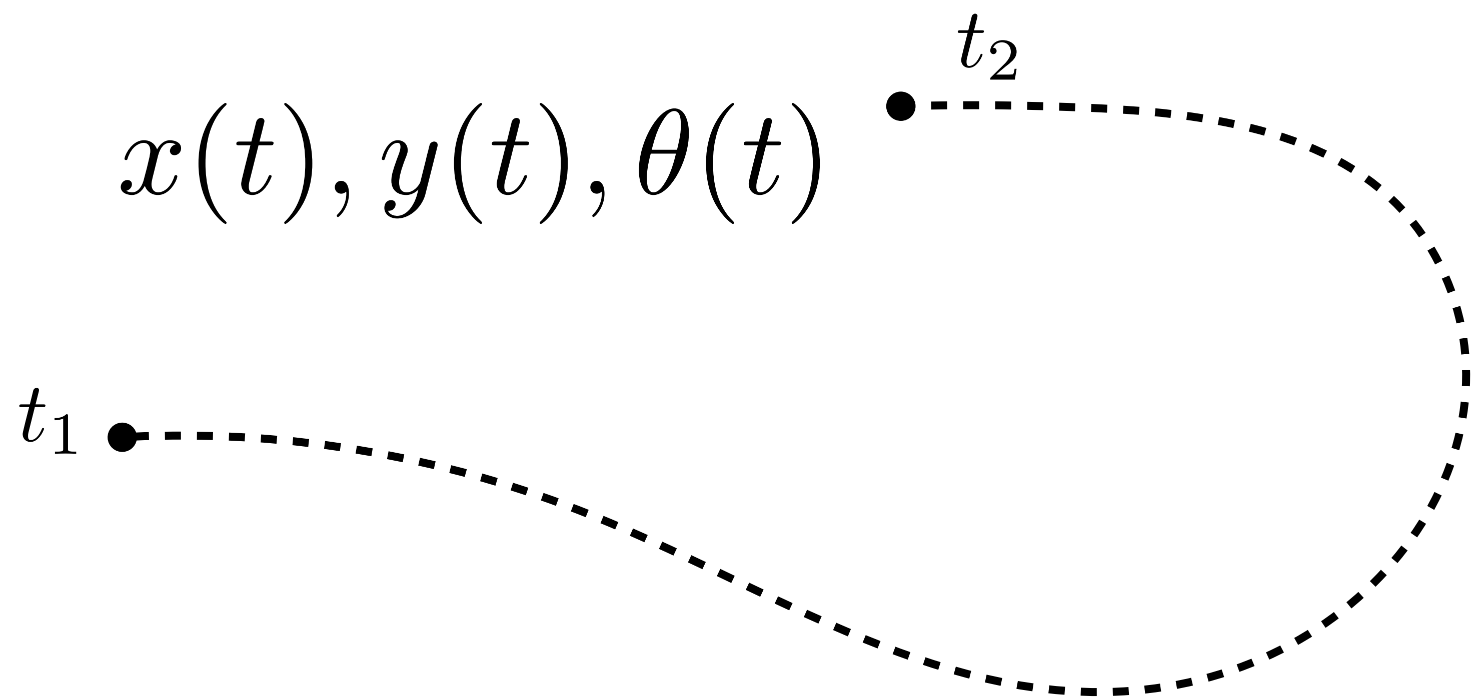
3. Compute error to reference point

4. Compute control law to minimize error

Step 1: Get a reference

Reference can be a time-parameterized trajectory

$$x(t), y(t), \theta(t)$$



The time index refers to a desired pose we want the system to achieve at a given time

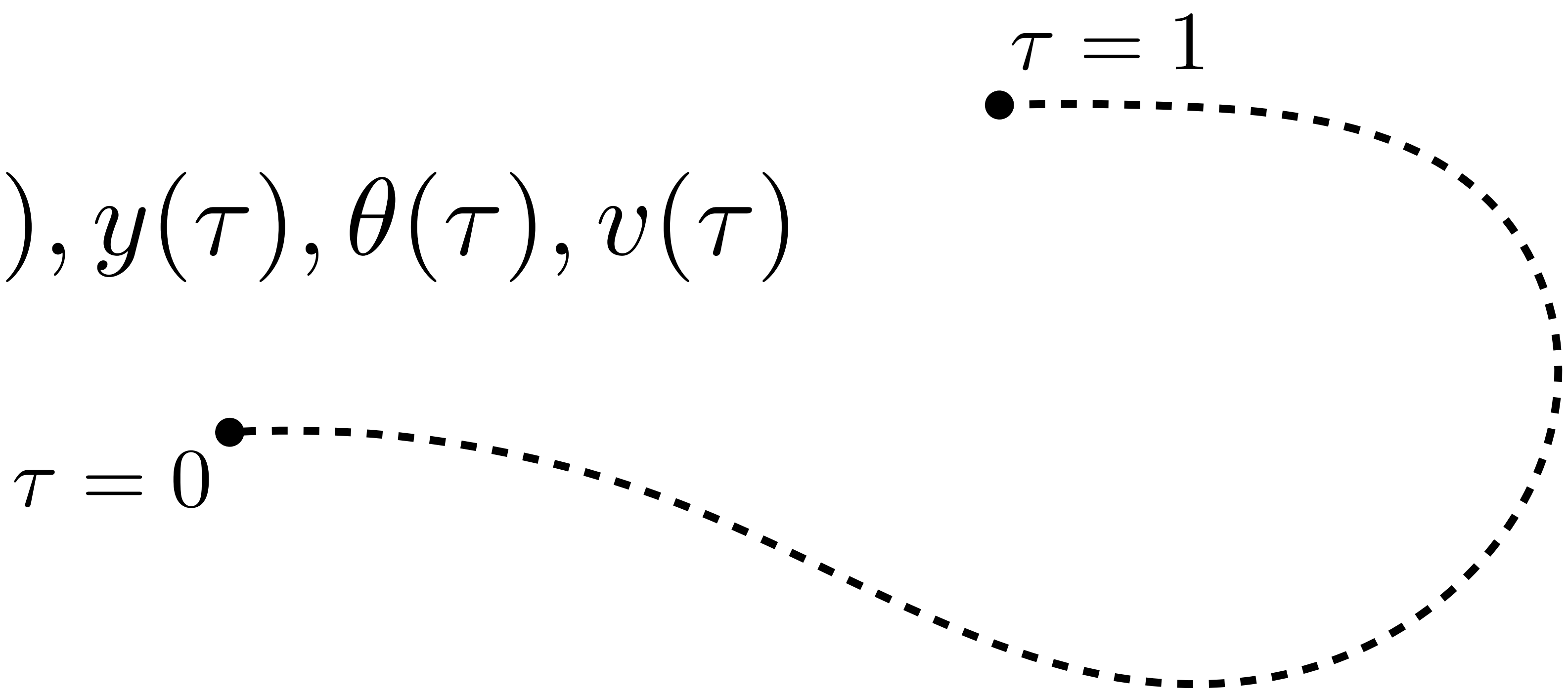
Pro: Useful if we want the robot to respect time constraints

Con: Sometimes we care only about deviation from reference

Step 1: Get a reference

Reference can be a index-parameterized path

$$x(\tau), y(\tau), \theta(\tau), v(\tau)$$



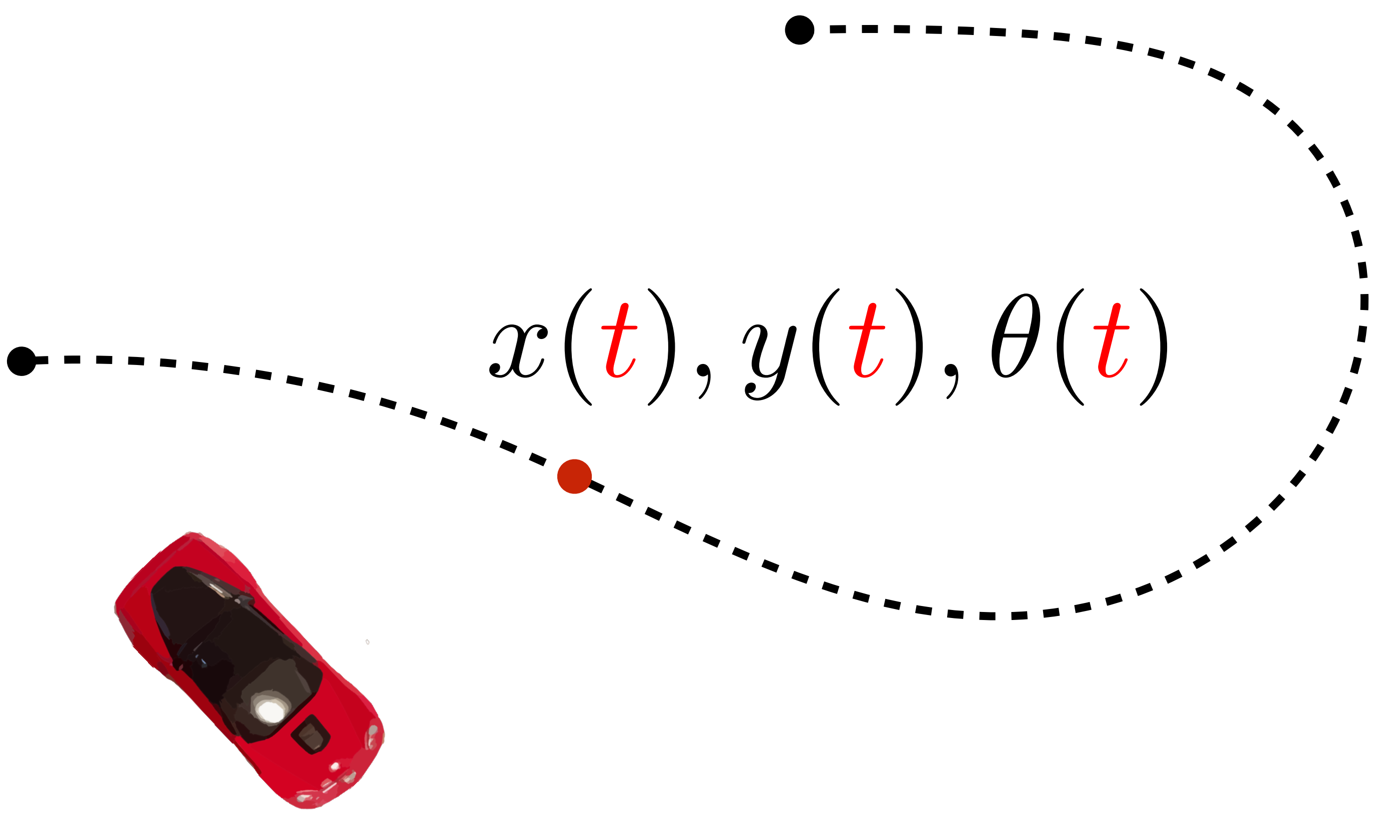
The index is simply a way to access the path; there is no notion of time

Pro: Useful for conveying the shape you want the robot to follow

Con: Can't control when robot will reach a point

Step 2: Pick a point on the reference

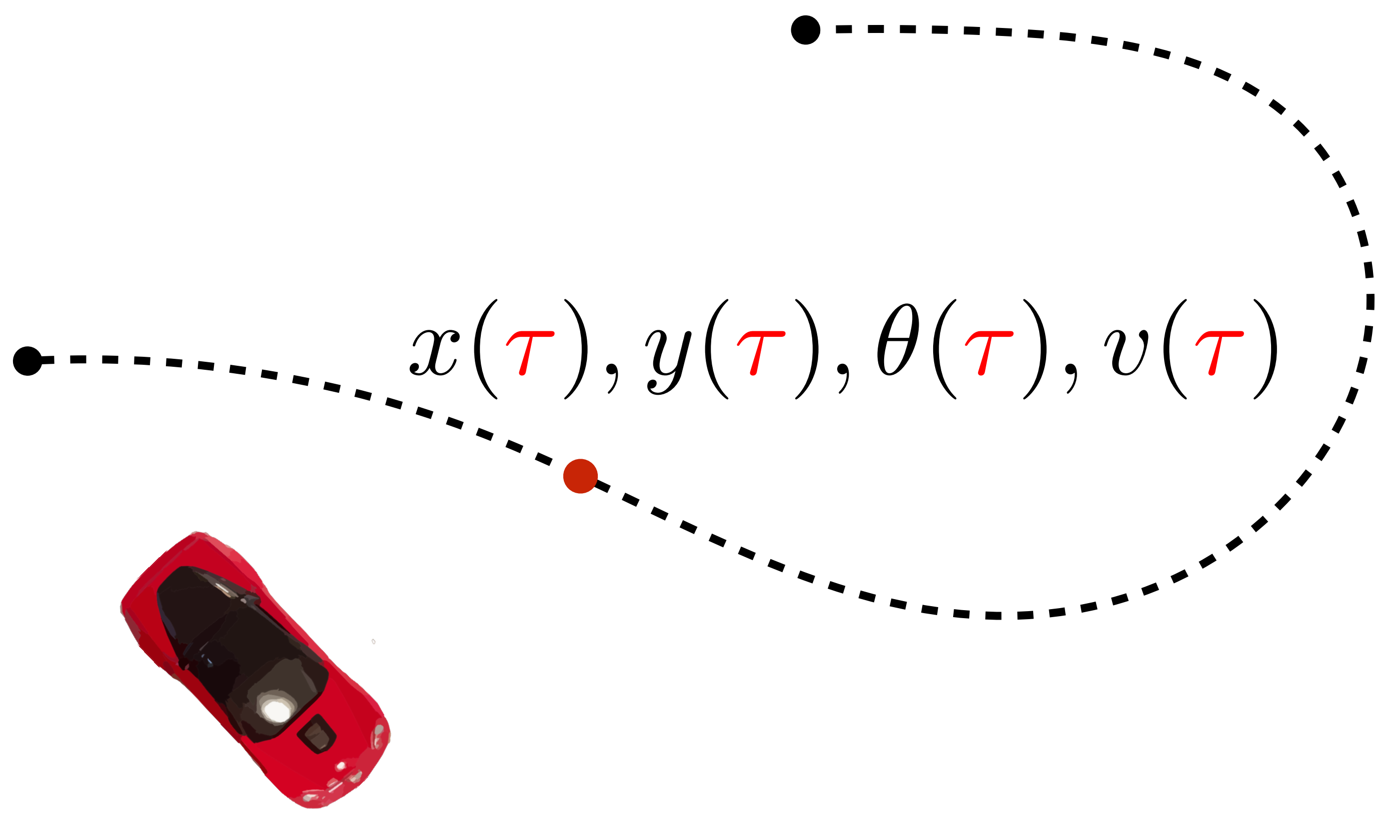
Pick a reference point on the trajectory / path



If we are using a time-parameterized trajectory, the current time is the natural reference

Step 2: Pick a point on the reference

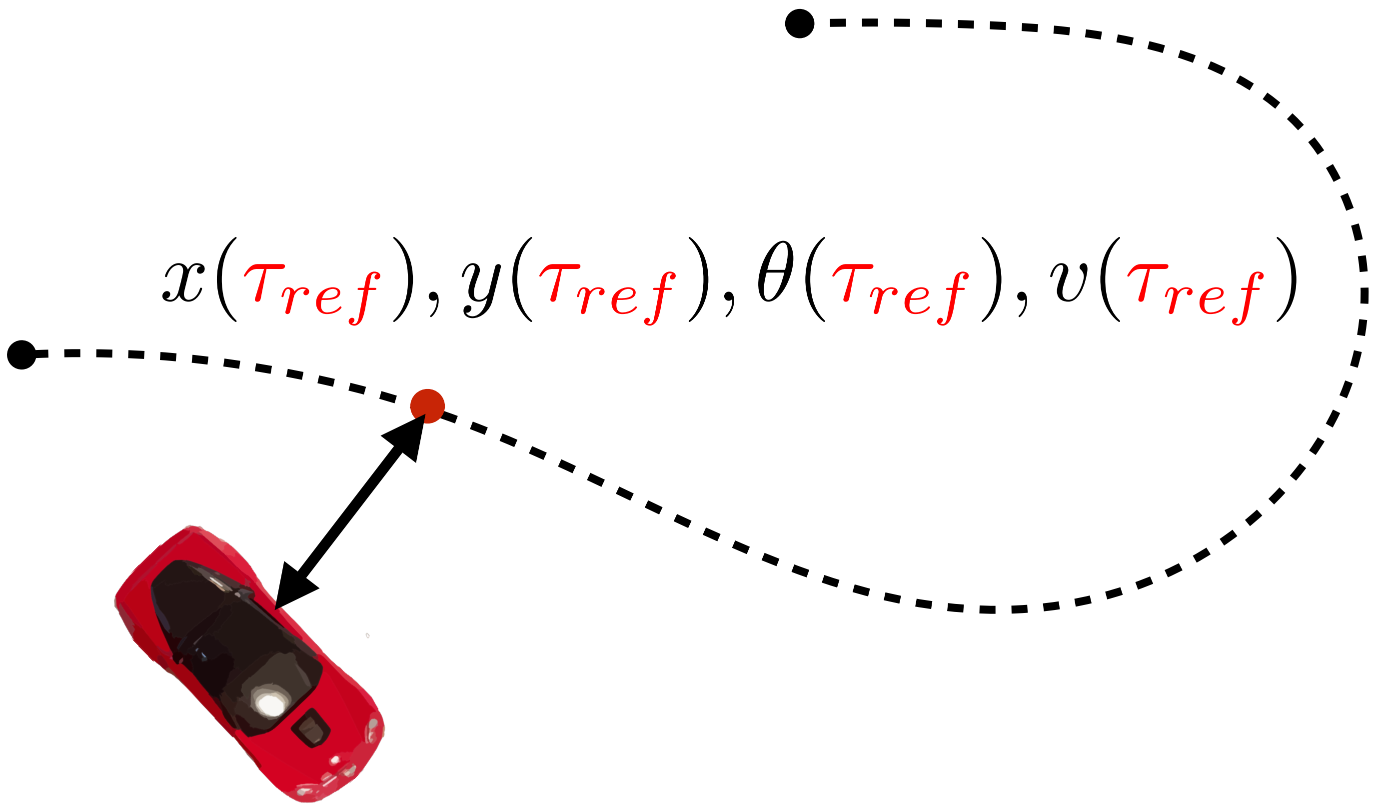
Pick a reference point on the trajectory / path



If we are using a index-parameterized trajectory,
there are multiple options

Step 2: Pick a point on the reference

Option 1: Pick the closest point on the path

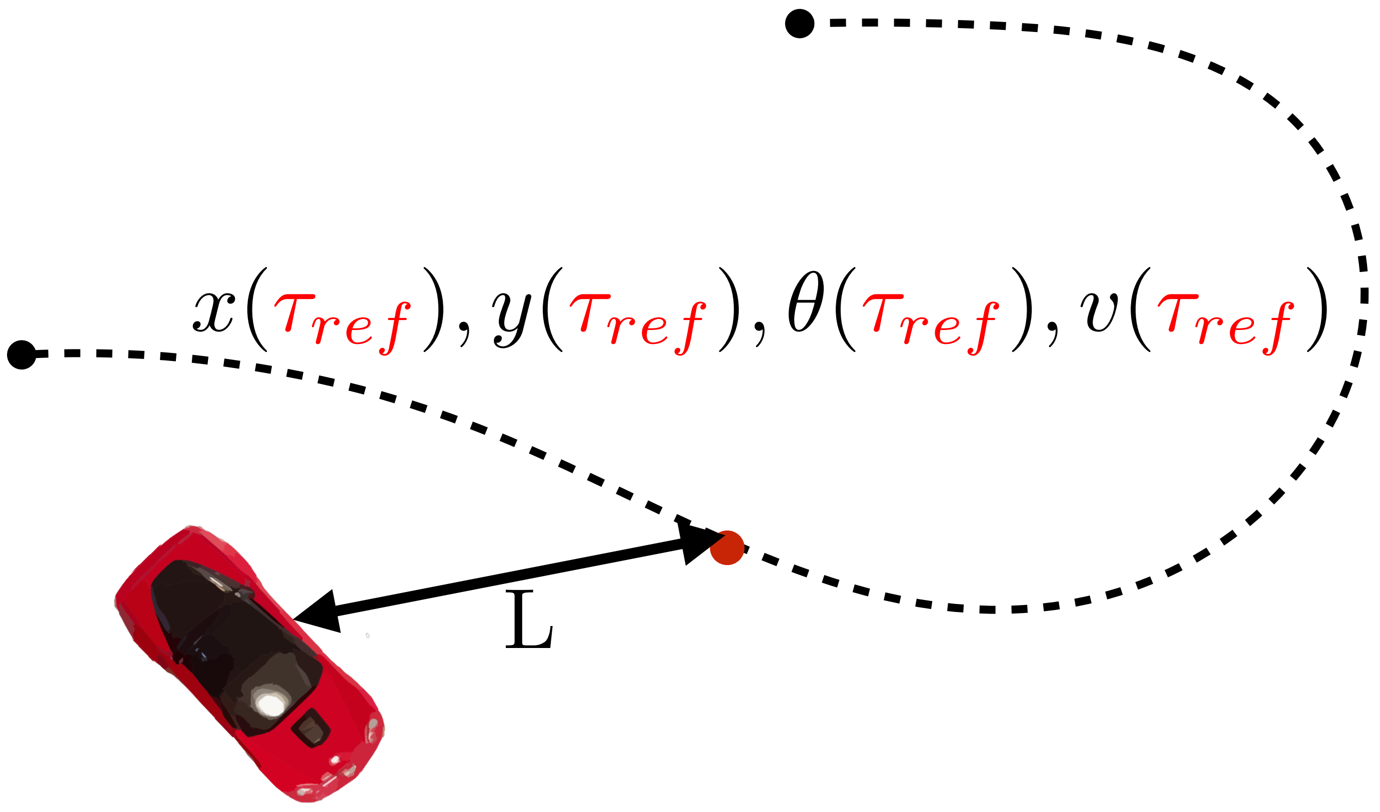


$$x(\tau_{ref}), y(\tau_{ref}), \theta(\tau_{ref}), v(\tau_{ref})$$

$$\tau_{ref} = \arg\min_{\tau} || \begin{bmatrix} x & y \end{bmatrix}^T - \begin{bmatrix} x(\tau) & y(\tau) \end{bmatrix}^T ||$$

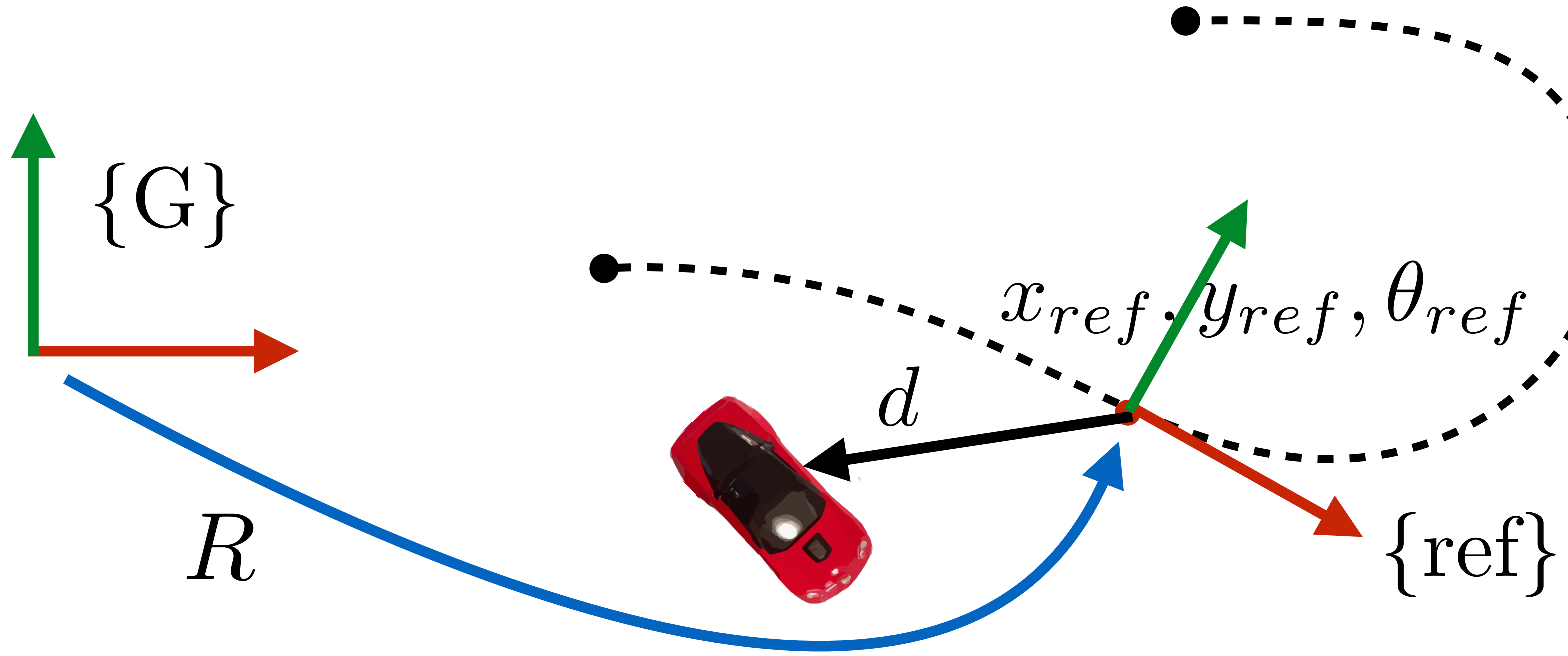
Step 2: Pick a point on the reference

Option 2: Pick a lookahead point on the path



$$|| \begin{bmatrix} x & y \end{bmatrix}^T - \begin{bmatrix} x(\tau_{ref}) & y(\tau_{ref}) \end{bmatrix}^T || = L$$

Step 3: Compute error to reference point



1. Define error vector

$$d = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix}$$

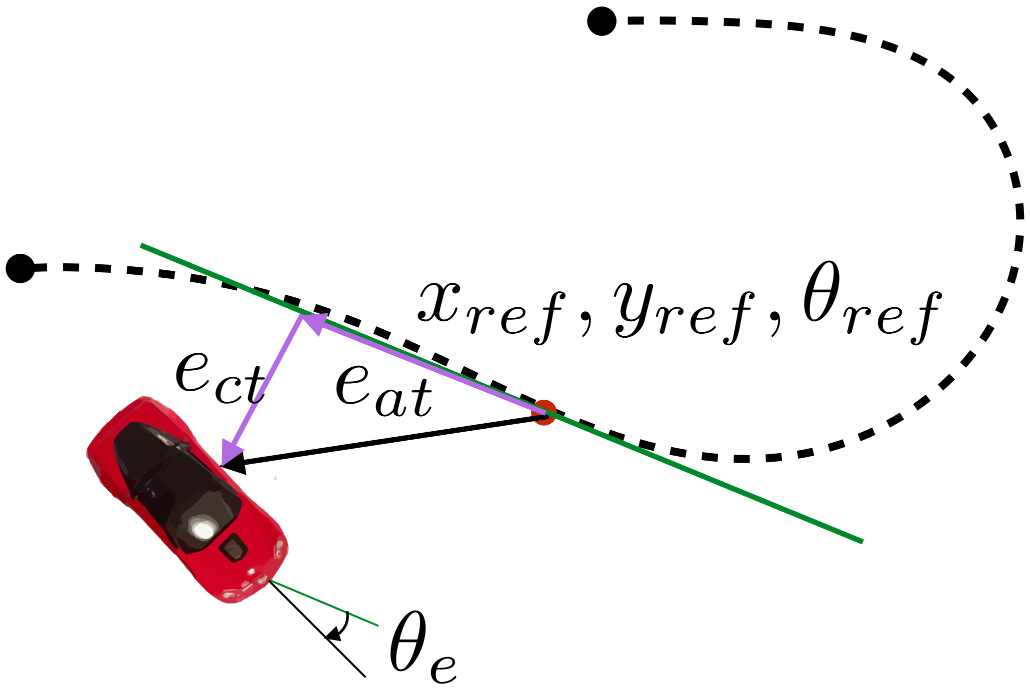
2. Rotate error vector to be in the **reference** frame

Rotation Matrix

$$e = R^T(\theta_{ref}) \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} \right)$$

$$e = \begin{bmatrix} \cos(\theta_{ref}) & \sin(\theta_{ref}) \\ -\sin(\theta_{ref}) & \cos(\theta_{ref}) \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} \right)$$

Step 3: Compute error to reference point



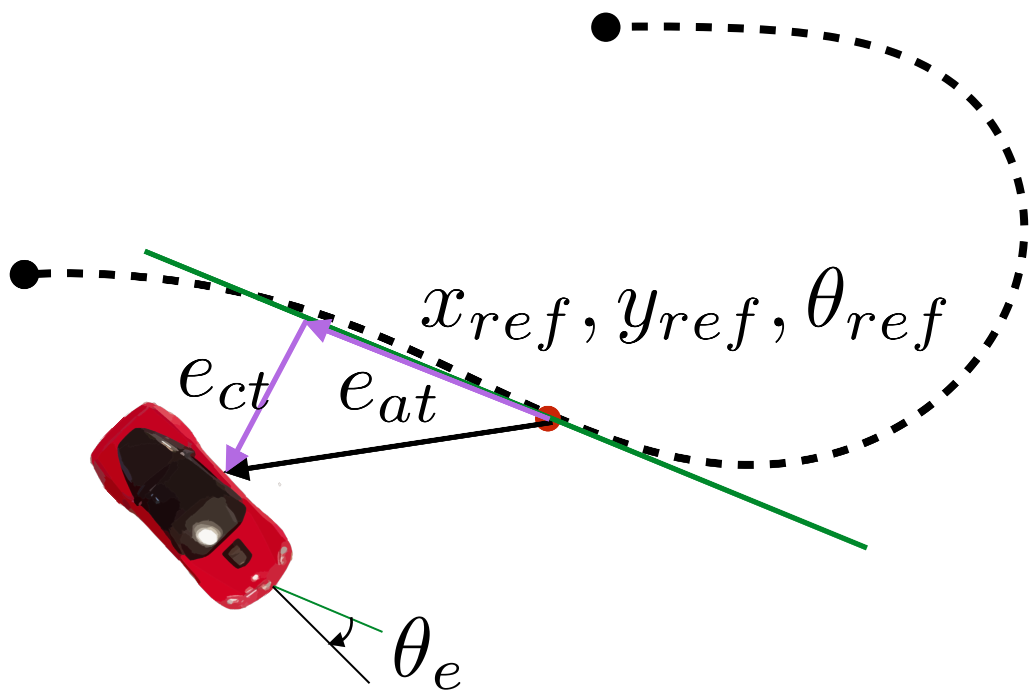
Error has two components: along-track and cross-track

$$e = \begin{bmatrix} e_{at} \\ e_{ct} \end{bmatrix} \quad \begin{matrix} \text{(along track)} \\ \text{(cross track)} \end{matrix}$$

$$\theta_e = \theta - \theta_{ref} \quad \text{(heading error)}$$

Let's consider only the cross track error for now

Step 3: Compute error to reference point



Cross-track error

$$e_{ct} = -\sin(\theta_{ref})(x - x_{ref}) + \cos(\theta_{ref})(y - y_{ref})$$

Derivative of cross-track error

$$\begin{aligned}\dot{e}_{ct} &= -\sin(\theta_{ref})\dot{x} + \cos(\theta_{ref})\dot{y} \\ &= -\sin(\theta_{ref})V\cos(\theta) + \cos(\theta_{ref})V\sin(\theta) \\ &= V\sin(\theta - \theta_{ref}) = V\sin(\theta_e)\end{aligned}$$

Step 4: Compute control law

Compute control action based on instantaneous error

$$u = K(\mathbf{x}, e)$$

control — state — error

(steering angle, speed)

Apply control action, robot moves a bit, compute new error, repeat

Different laws have different trade-offs

We assume that control speed is set to be equal to reference speed

Hence control = steering angle

Different control laws

1. Bang-bang control
2. PID control
3. Pure-pursuit control
4. Lyapunov control
5. Linear Quadratic Regulator (LQR)
6. Model Predictive Control (MPC)