SLAM: Simultaneous Localization and Mapping

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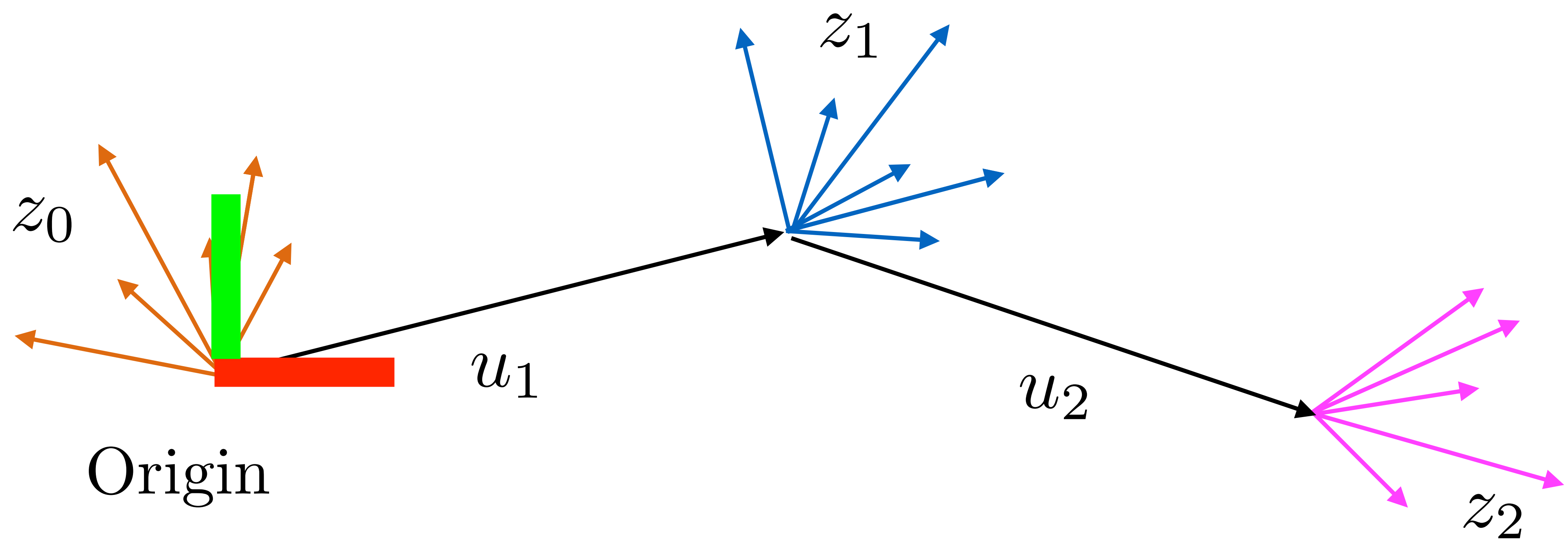
*Slides based on or adapted from Sanjiban Choudhury, Dieter Fox, Michael Kaess

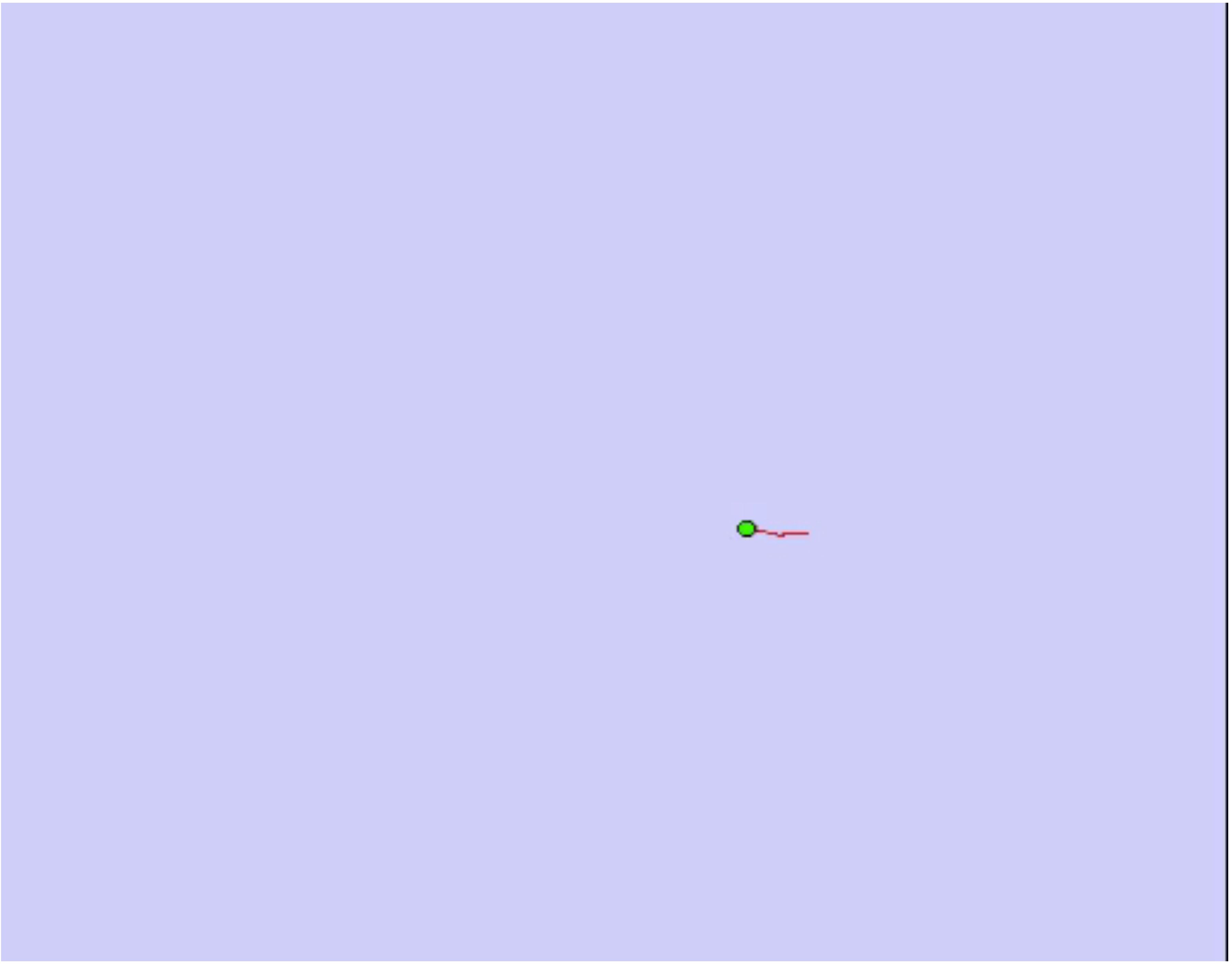
The SLAM problem

Robot is moving through a static unknown environment

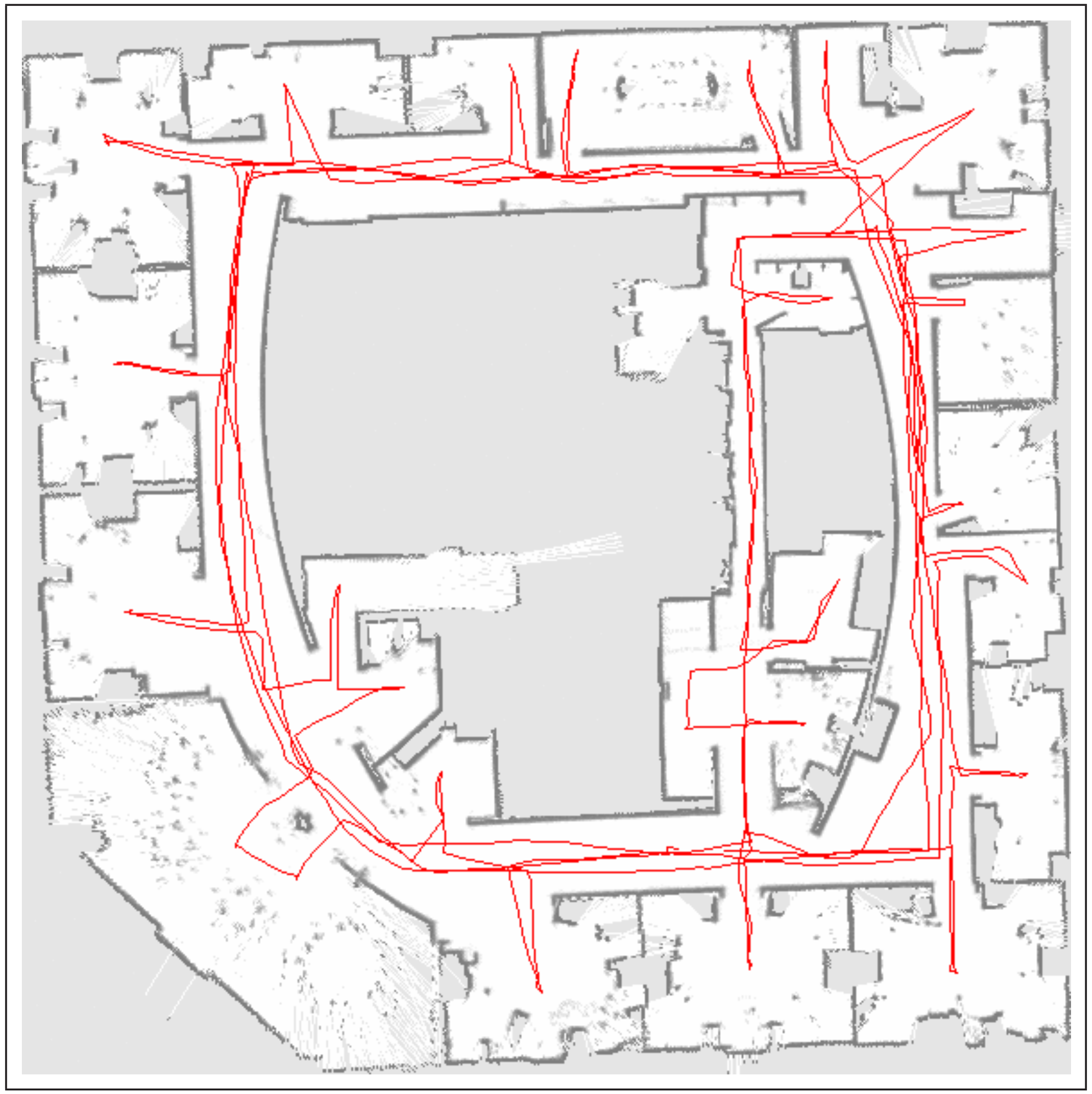


Given a series of **controls** and **measurements**, estimate **state** and **map**

What if I just integrate controls?



What we want ...



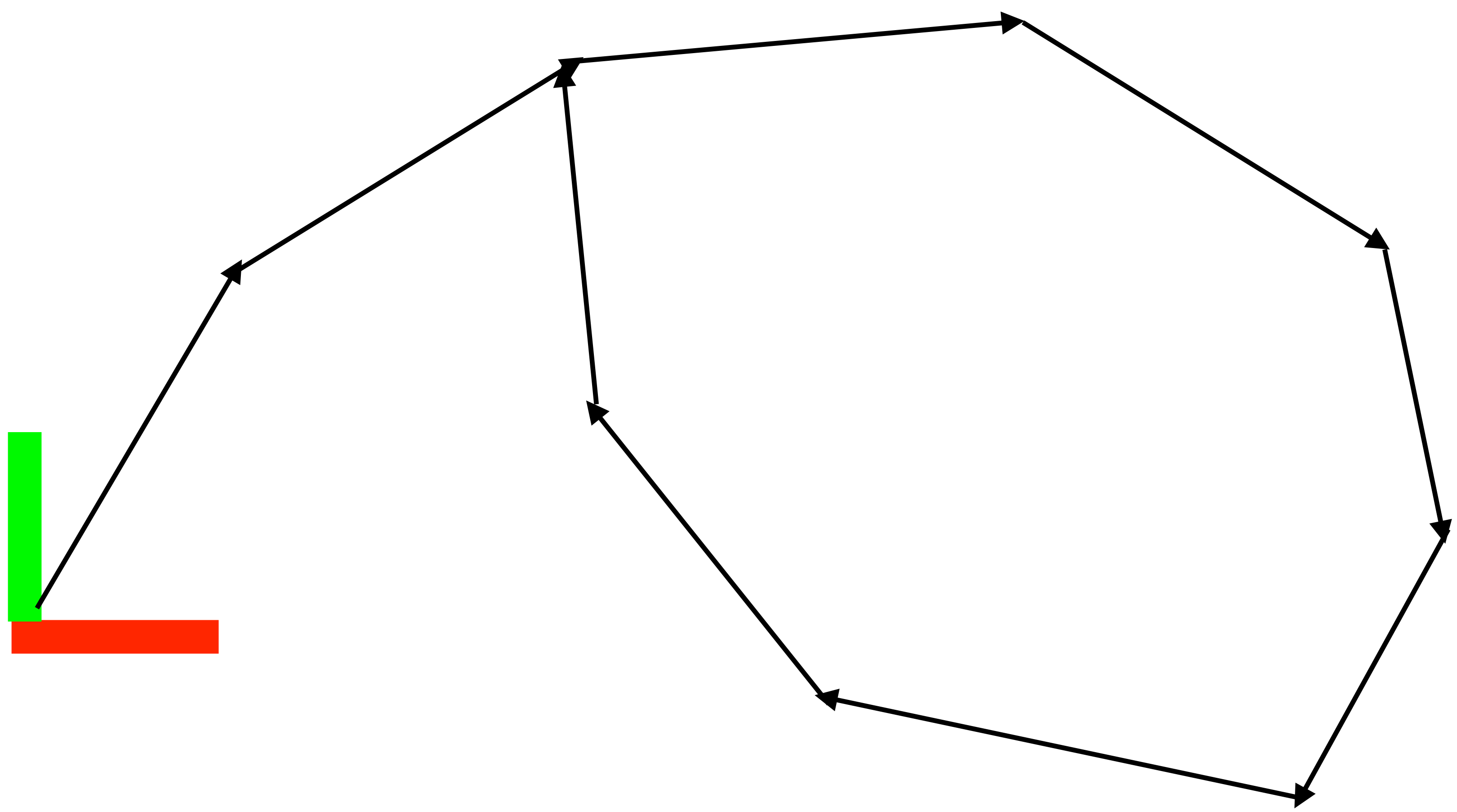
Need to figure out two things:

Correct relative movements between successive measurements

Closing the loop globally

Spilling the beans on SLAM

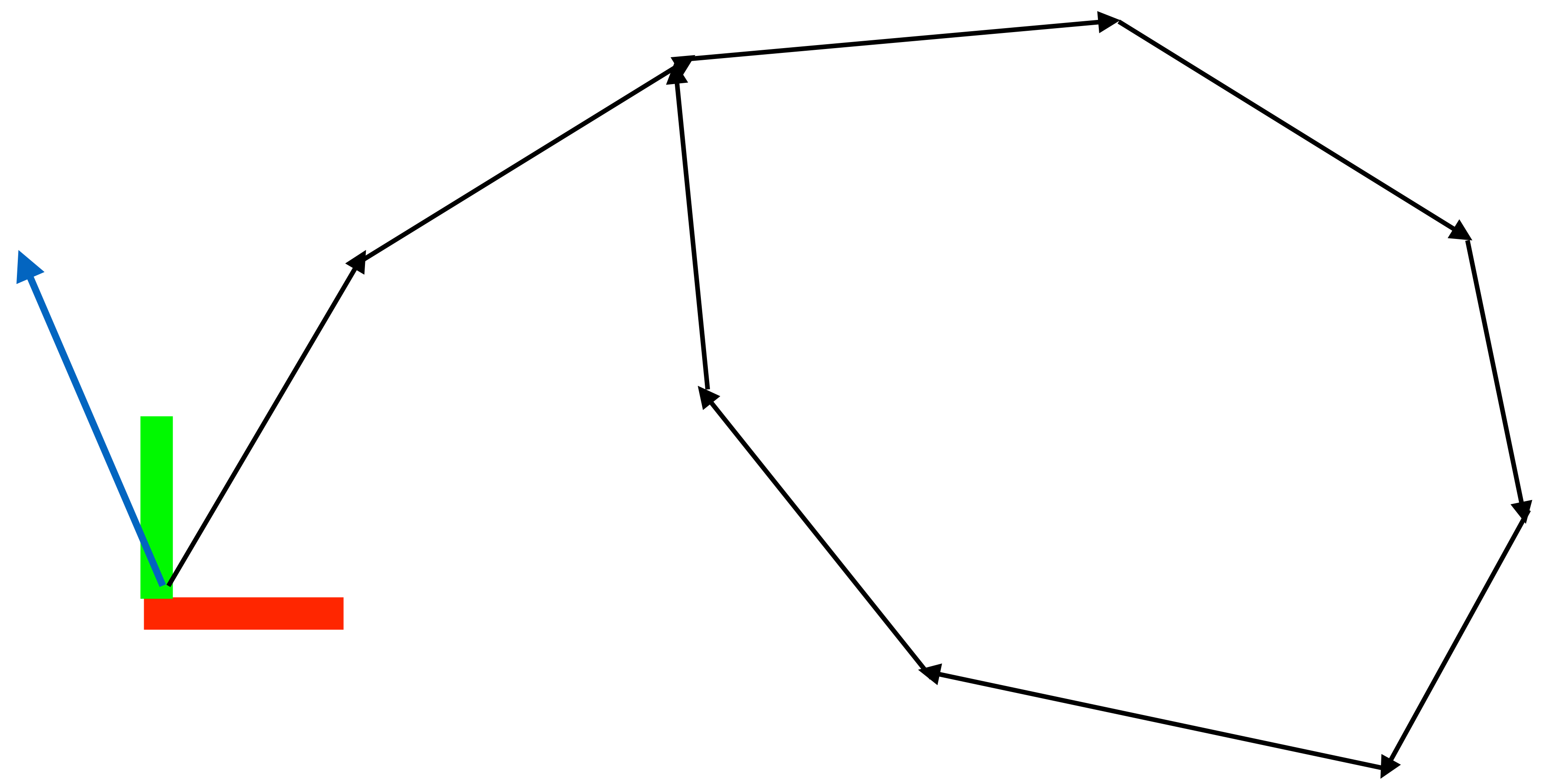
Let's assume this was the ground truth....



Ground truth

Spilling the beans on SLAM

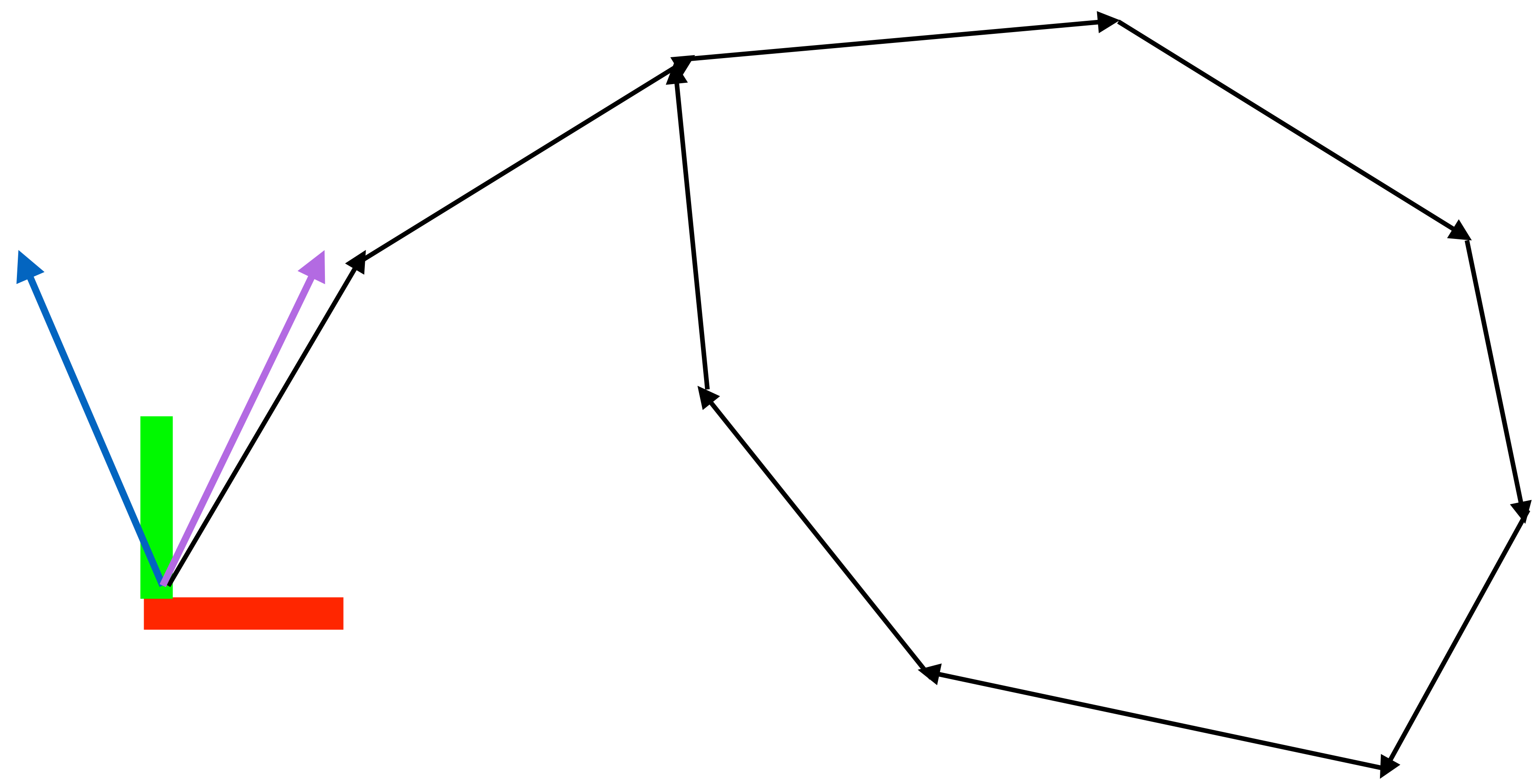
Odometry is really really noisy!



Ground truth   Odometry

Spilling the beans on SLAM

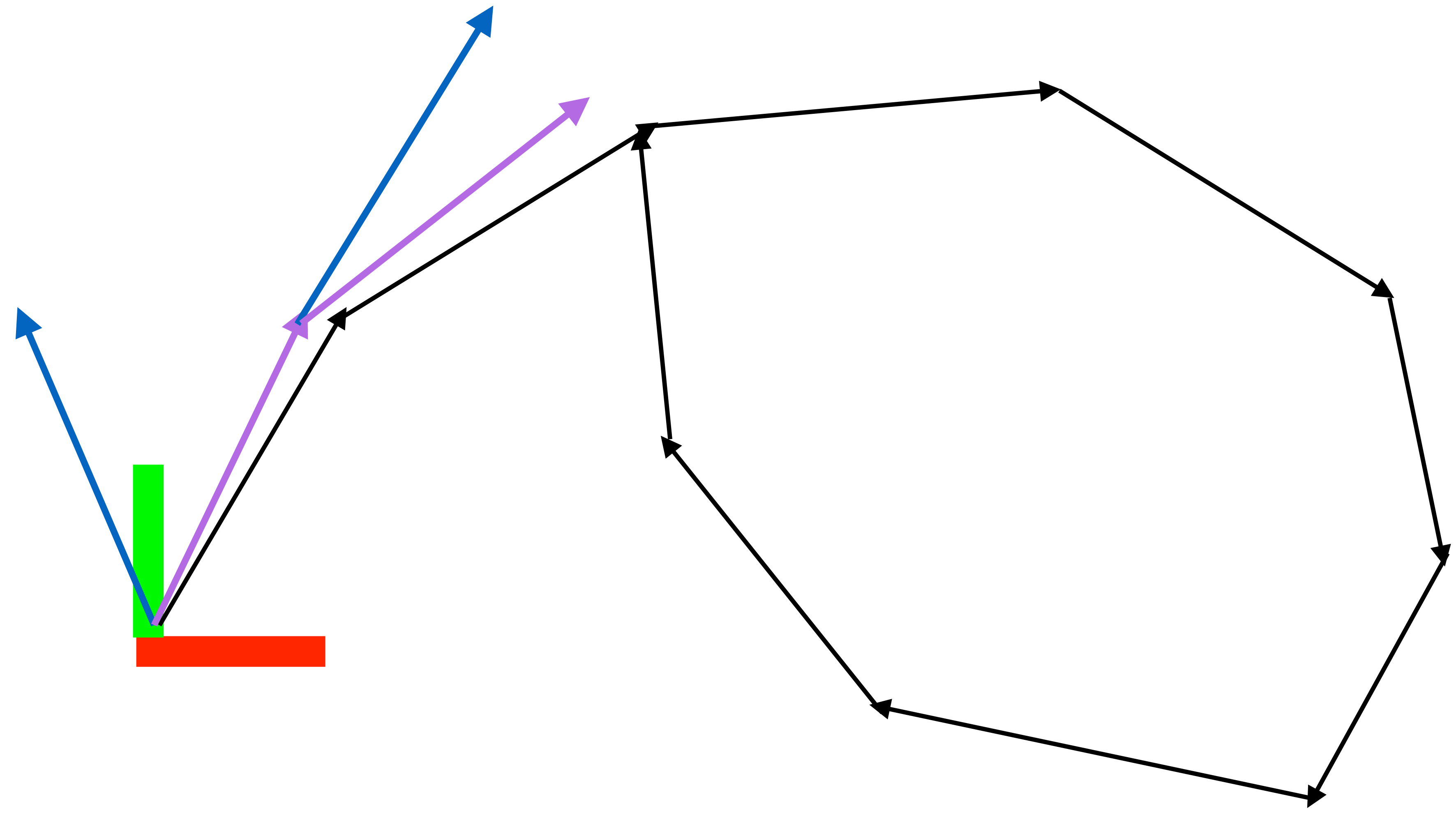
Measurements can help correct this somewhat



Ground truth Odometry Measurement correction

Spilling the beans on SLAM

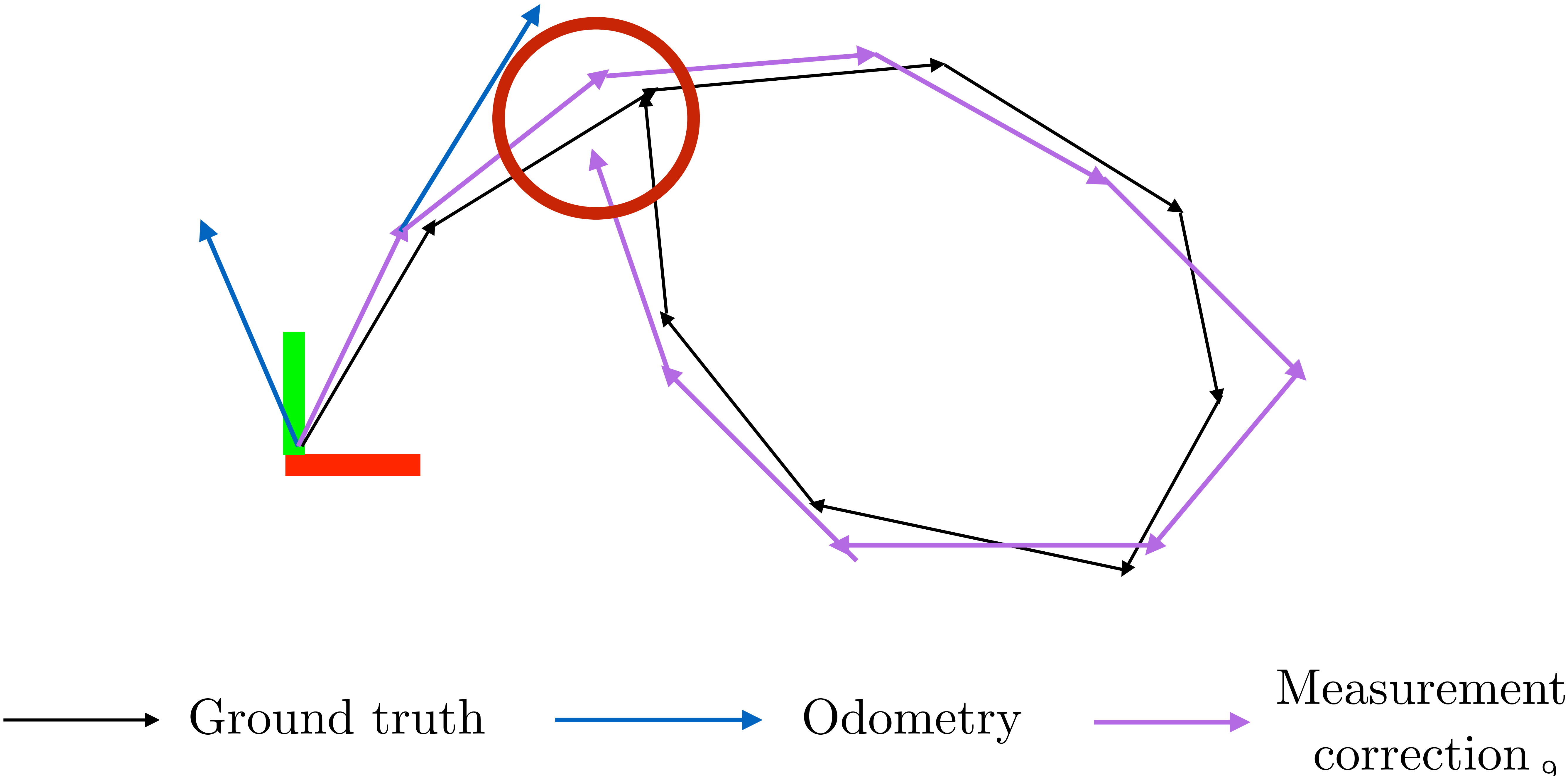
Relative error accumulates ...



Ground truth Odometry Measurement correction

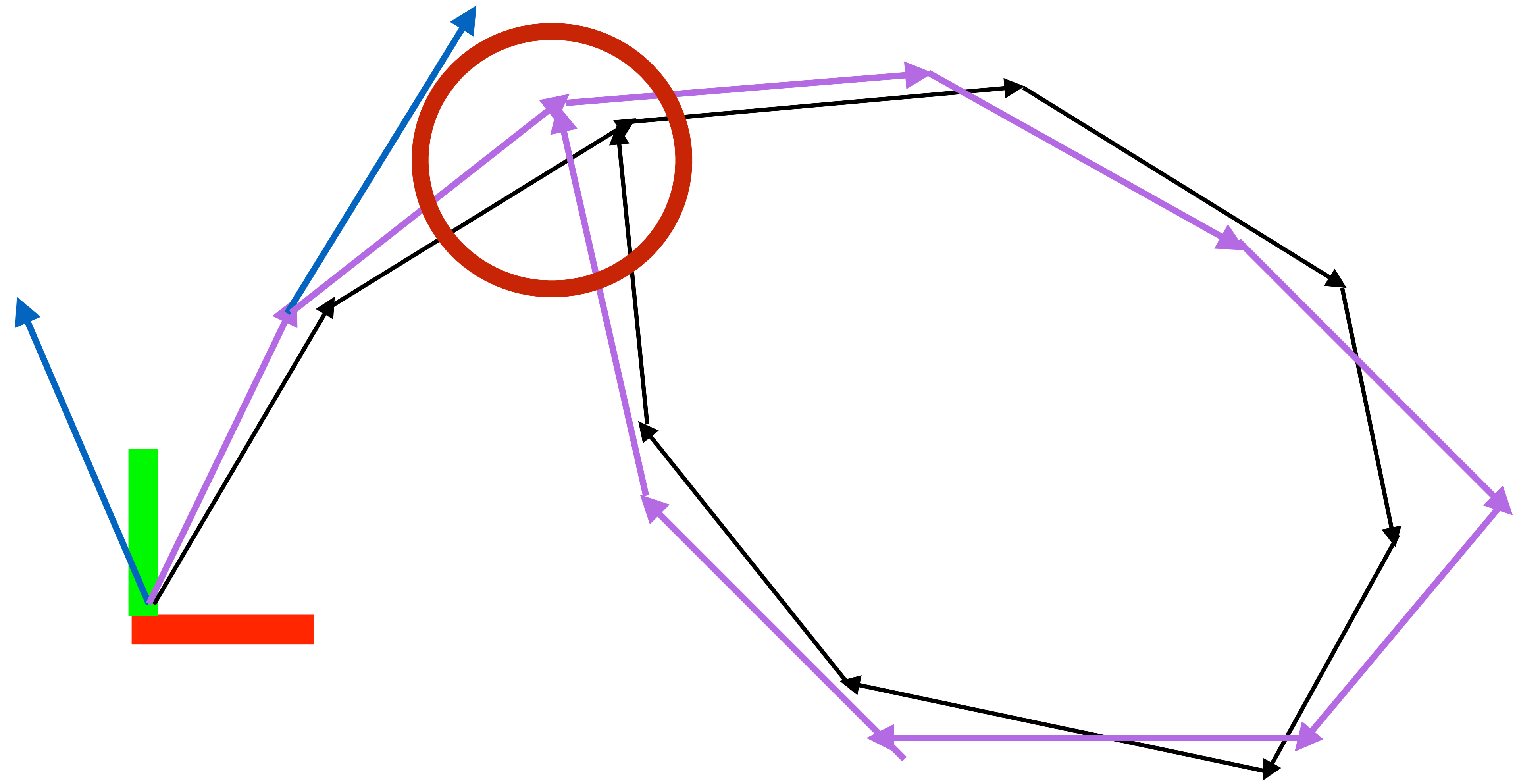
Spilling the beans on SLAM

Eventually robot comes back to a familiar place .... loop closure!



Spilling the beans on SLAM

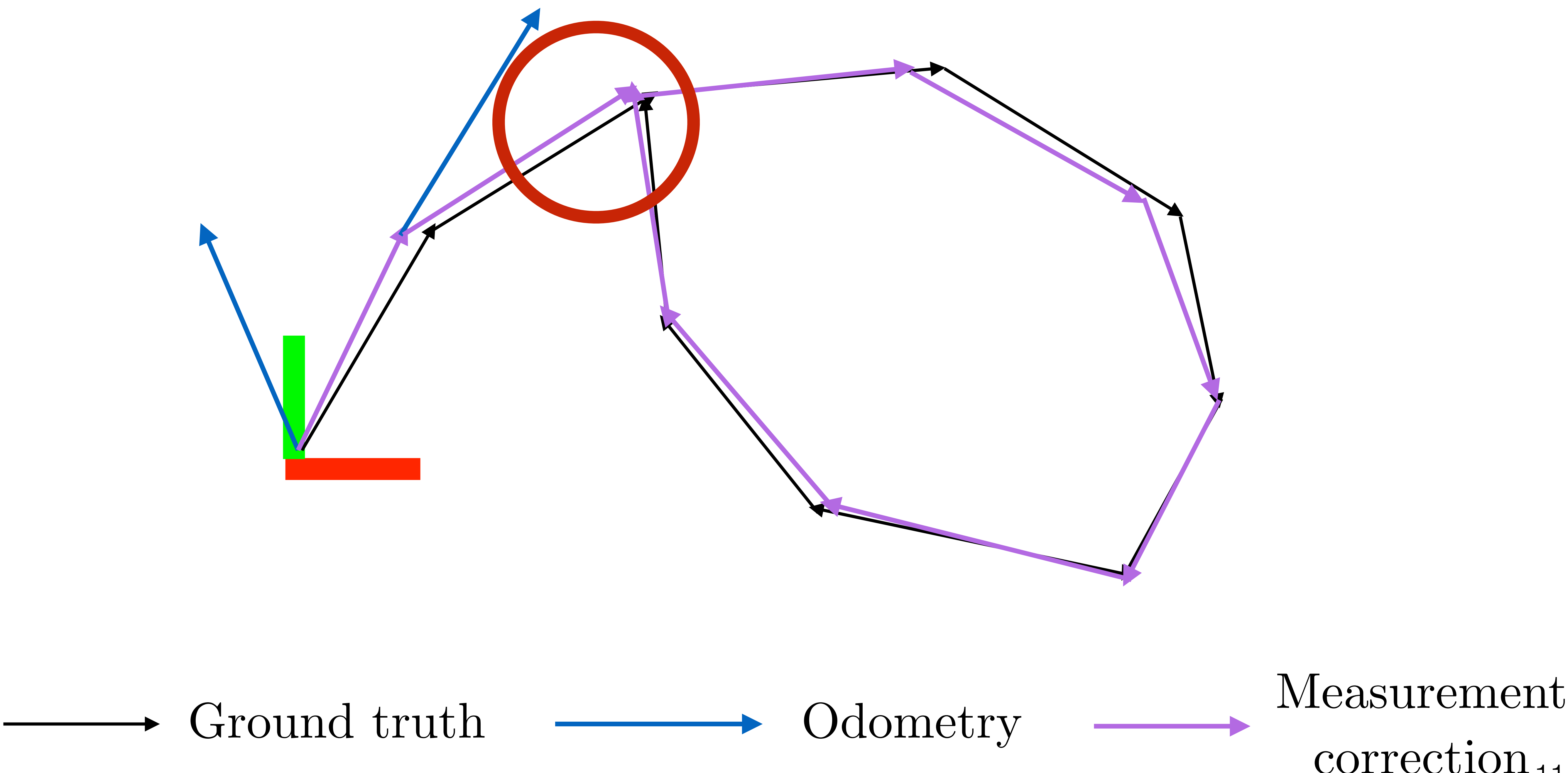
If we are filtering, we can only fix the last pose estimate



Ground truth — Odometry — Measurement correction

Spilling the beans on SLAM

If we are optimizing, we can backprop errors in time!

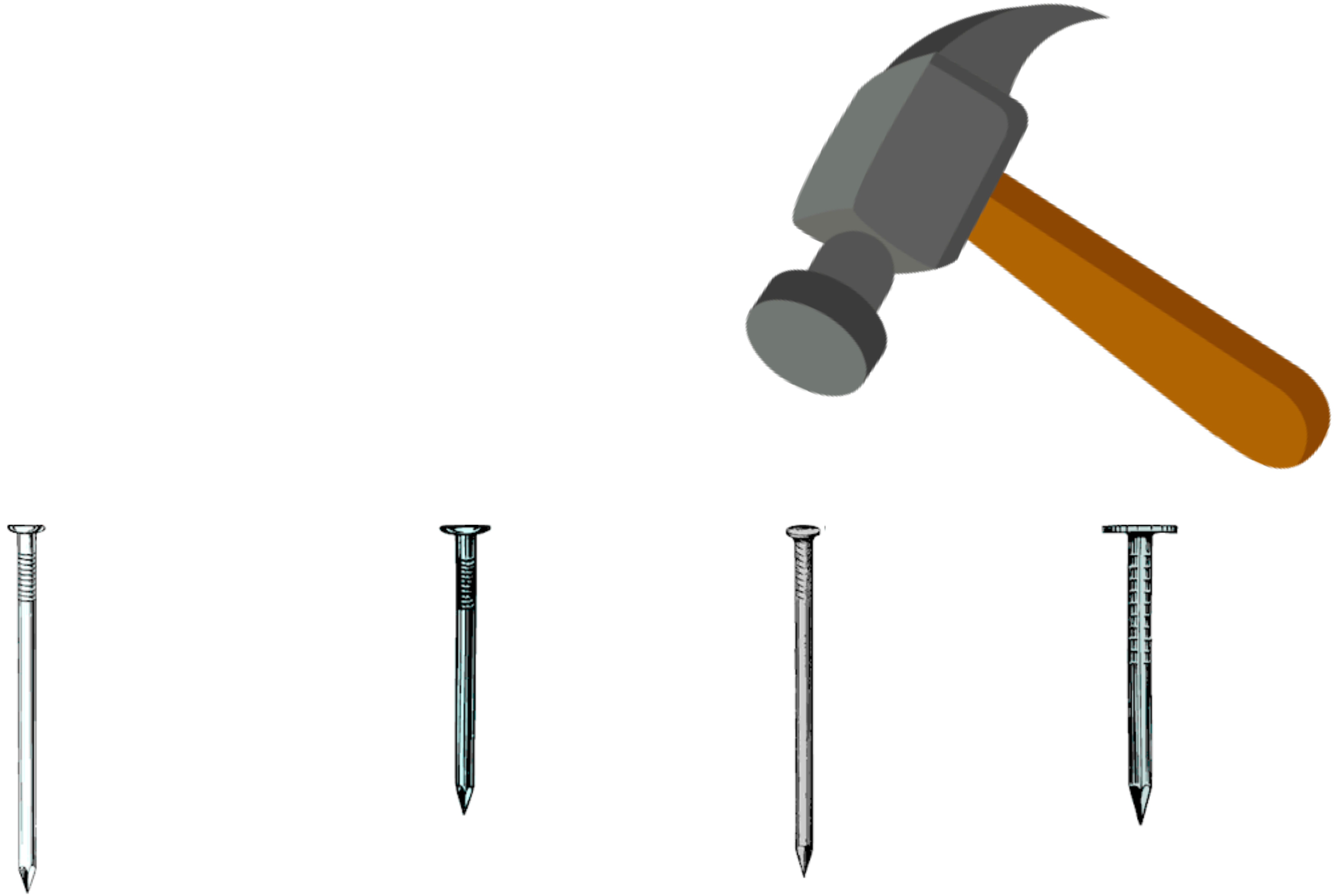


Today's objective

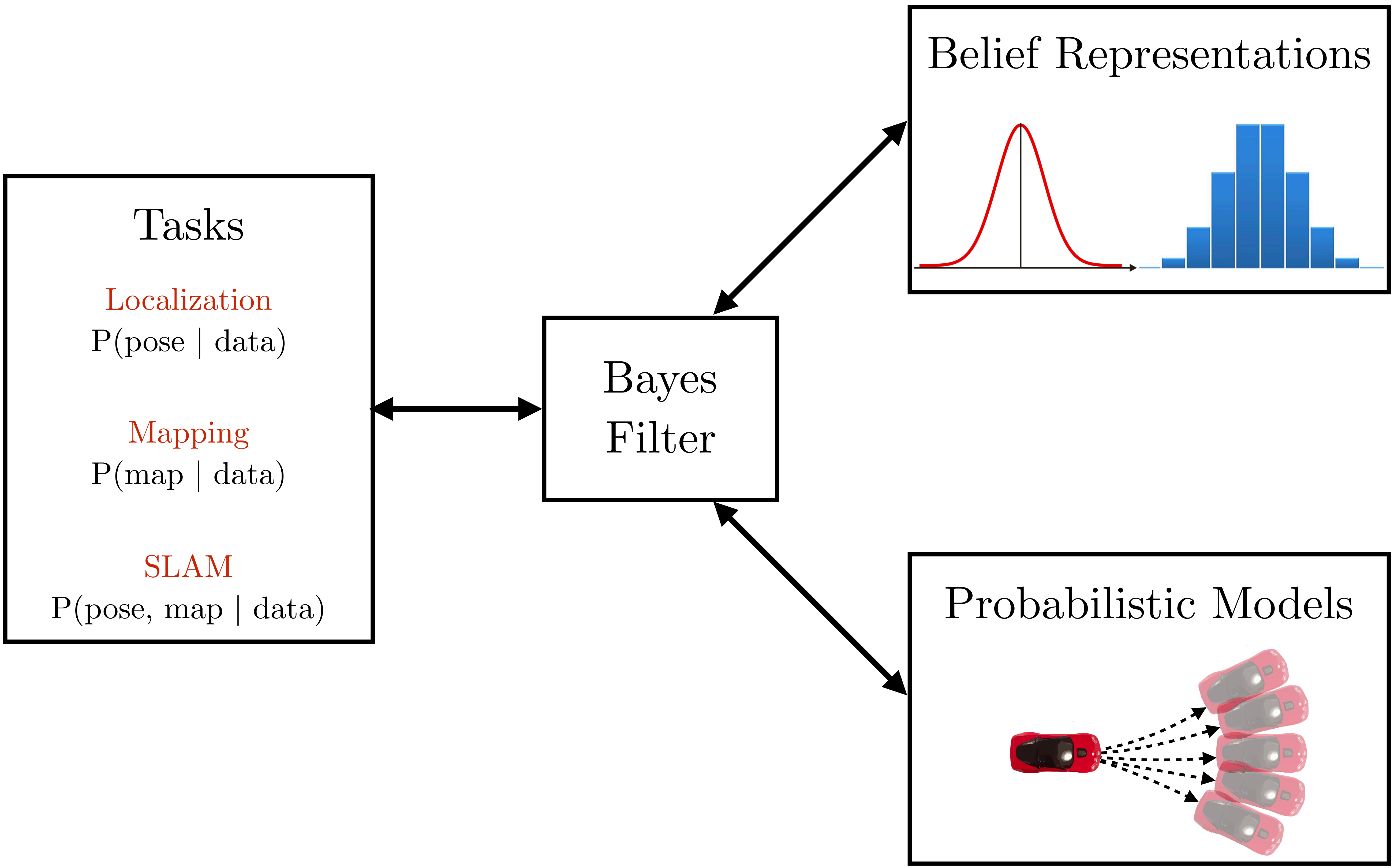
1. How do we solve the chicken-or-egg problem in SLAM?

2. SLAM as an optimization instead of filtering

Bayes filter is a powerful tool



Assembling Bayes filter



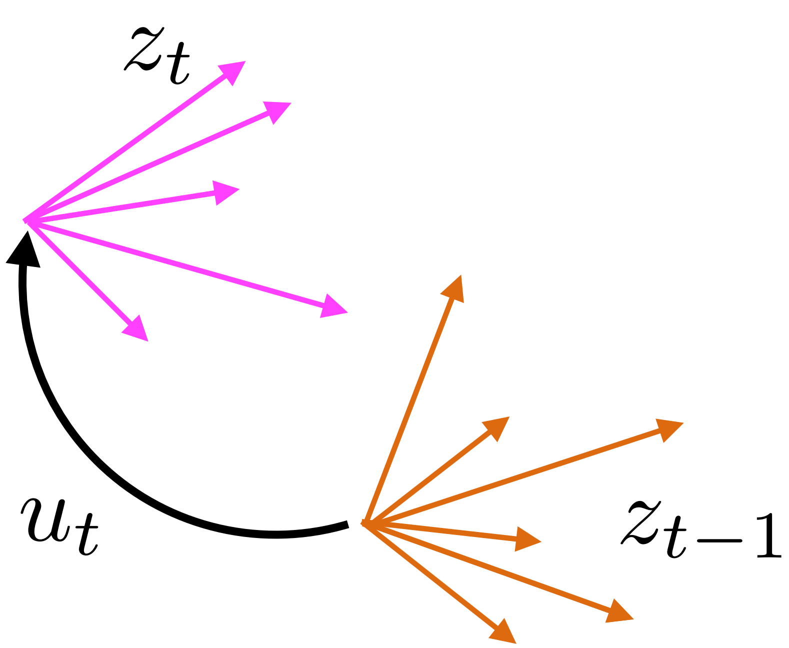
Tasks that we will cover

Tasks	Belief Representation	Probabilistic Models
Localization P(pose | data) (Week 3)	Gaussian / Particles	Motion model Measurement model
Mapping P(map | data) (Week 4)	Discrete (binary)	Inverse measurement model
SLAM P(pose, map | data) (Week 4)	Particles+GridMap (pose, map)	Motion model, measurement model, ***correspondence model***

SLAM as just another Bayes filtering problem

Task: SLAM
P(map, pose | data)

What is the data?

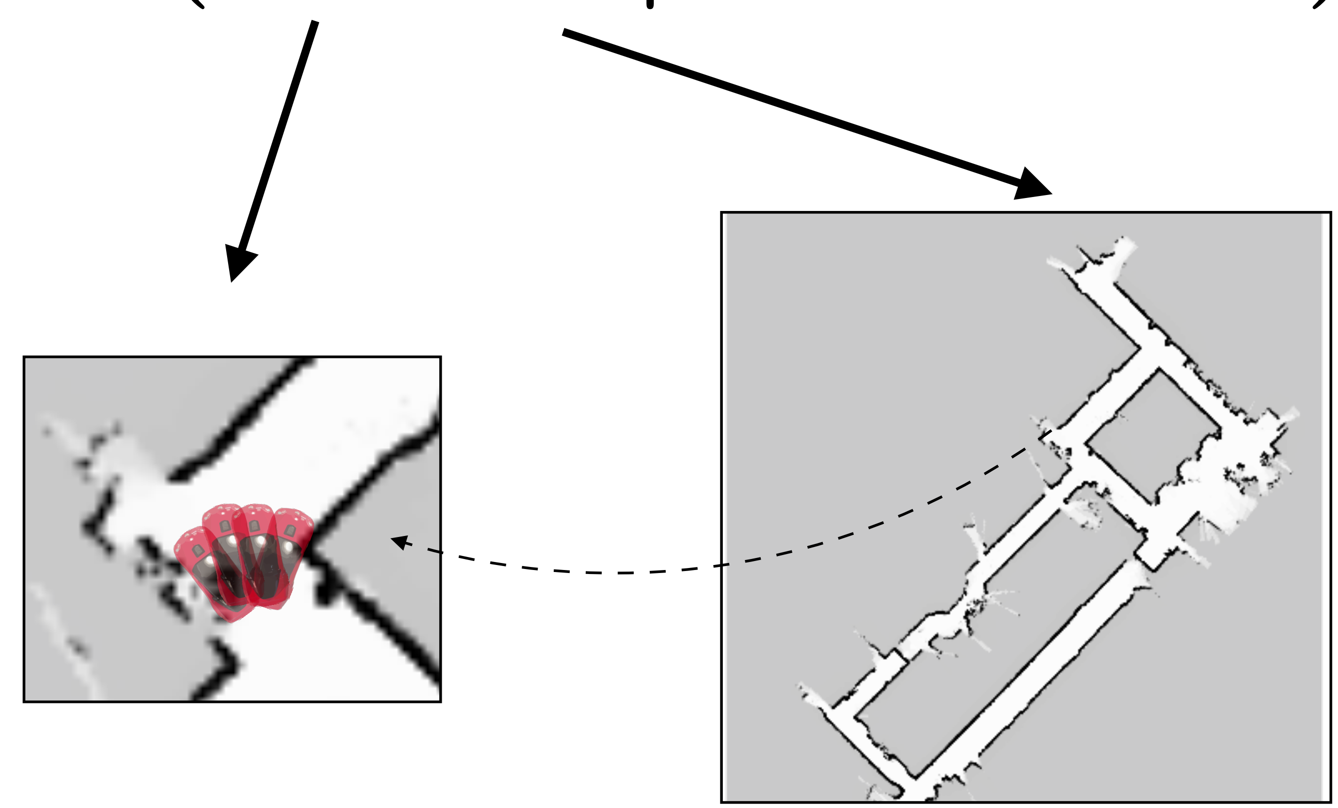


Stream of controls and measurements

$u_{1:t}, z_{1:t}$

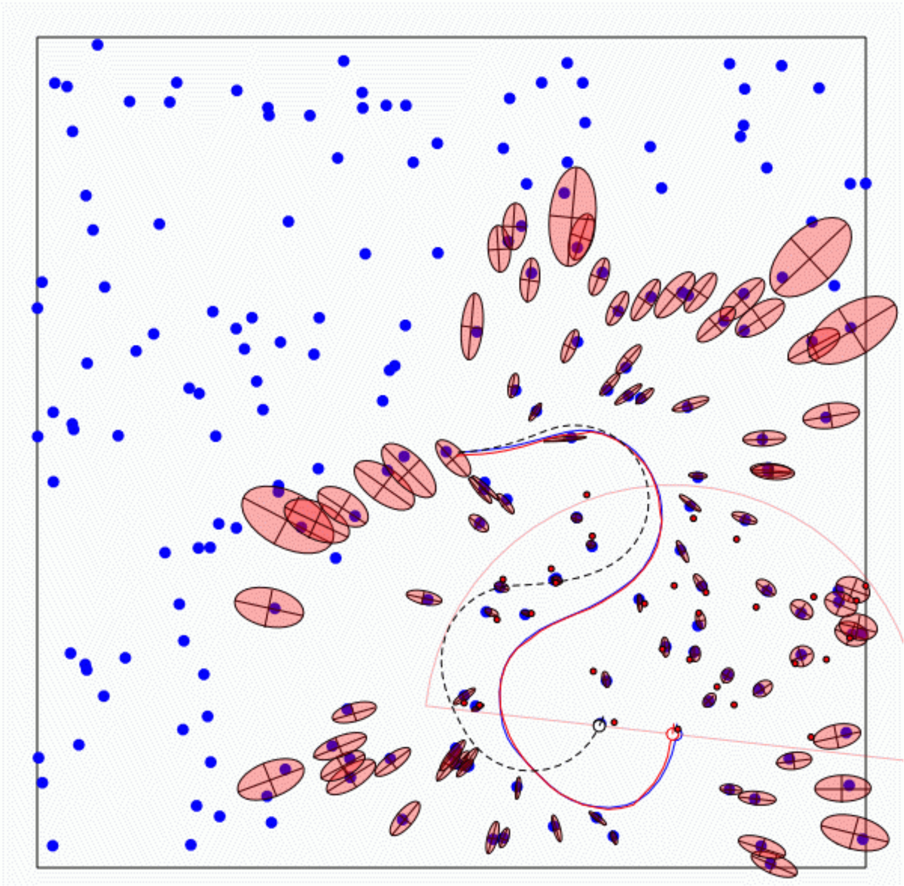
What is the belief representation?

$$P(x_t, m | u_{1:t}, z_{1:t})$$



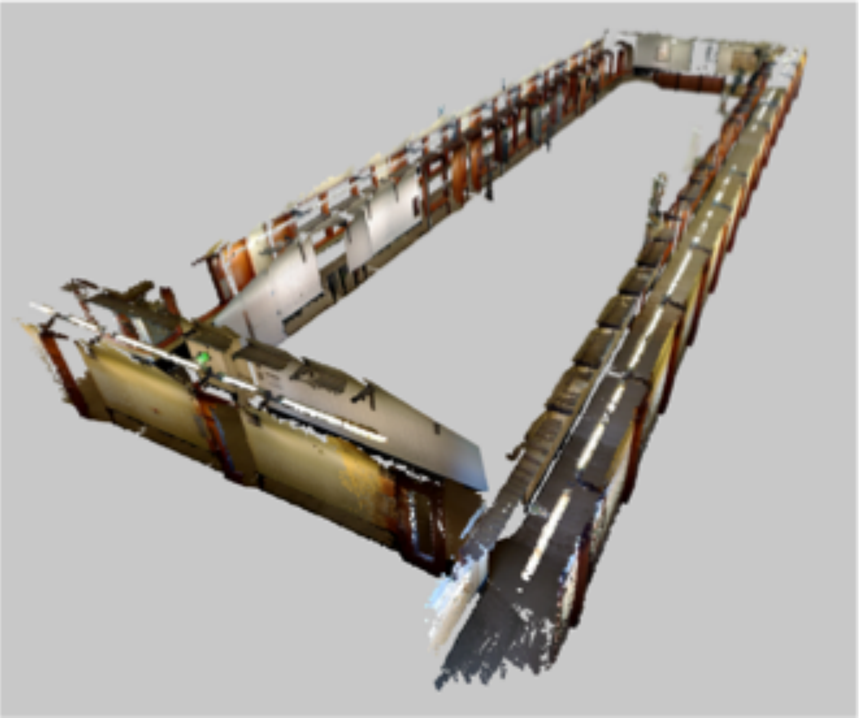
Different map representations

We are free to use any of the map representations we discussed



Feature maps

Grid maps



Surface maps

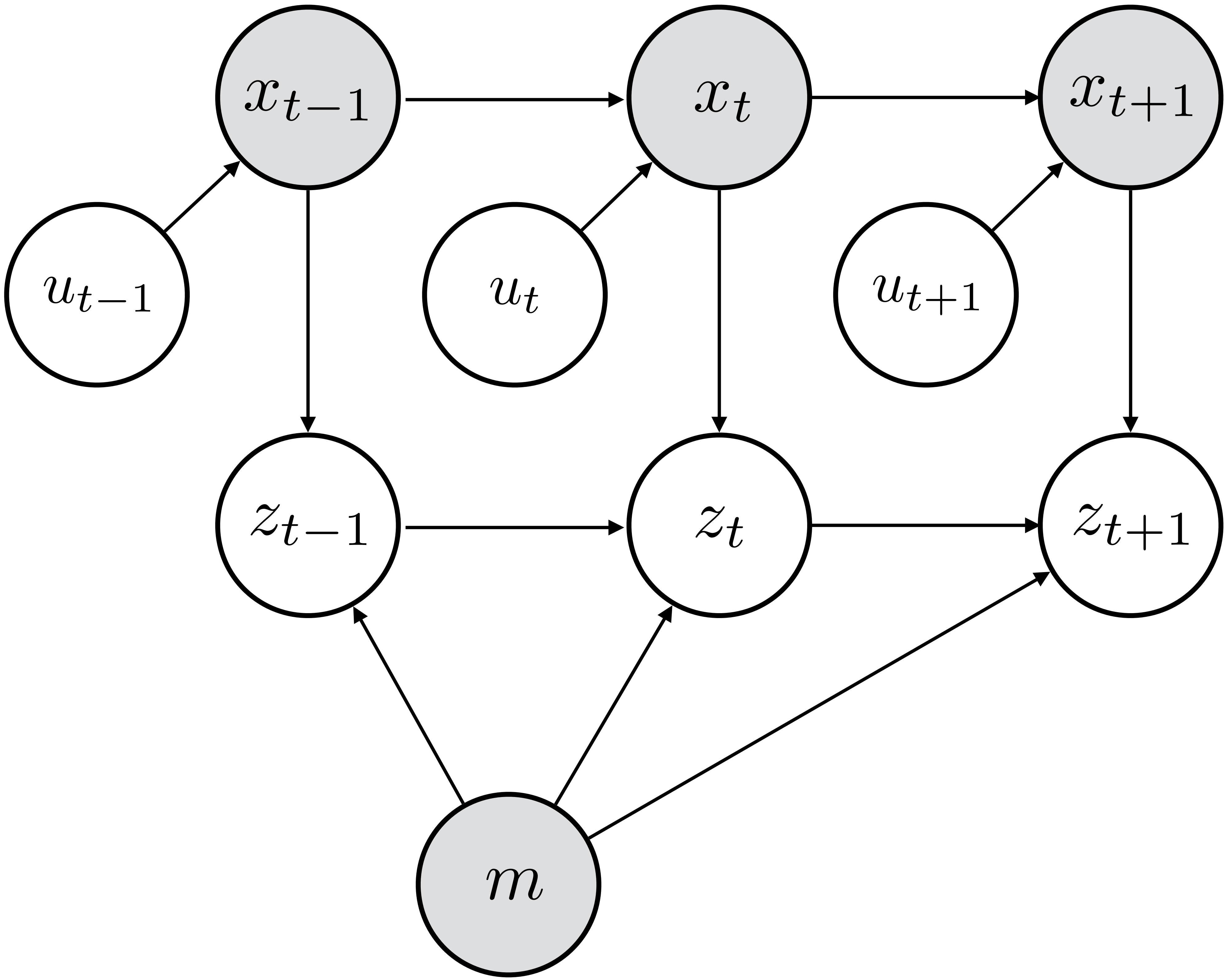
Why is SLAM hard?

Chicken-or-egg problem:

Given a map, we can localize

Given the pose, we can build map

Graphical model of SLAM



If SLAM can be expressed as a Bayes filtering problem, let's use our favorite Bayes filter…

Particle Filter!

Generalized particle filters

Step 0: Start with a set of particles

$$bel(x_{t-1}) = \{x_{t-1}^1, x_{t-1}^2, \ldots, x_{t-1}^M\}$$

Step 1: Sample particles from proposal distribution

$$\overline{bel}(x_t) = \{\bar{x}_t^1, \ldots, \bar{x}_t^M\}$$

Step 2: Compute importance weights

$$w_t^k = \frac{bel(x_t)}{\overline{bel}(x_t)} \left(= \eta P(z_t | x_t) \right)$$

Step 3: Resampling

Draw M samples from weighted distribution

Problem: Space of maps too large!!

$$P(x_t, m | u_{1:t}, z_{1:t})$$

How big is this space?

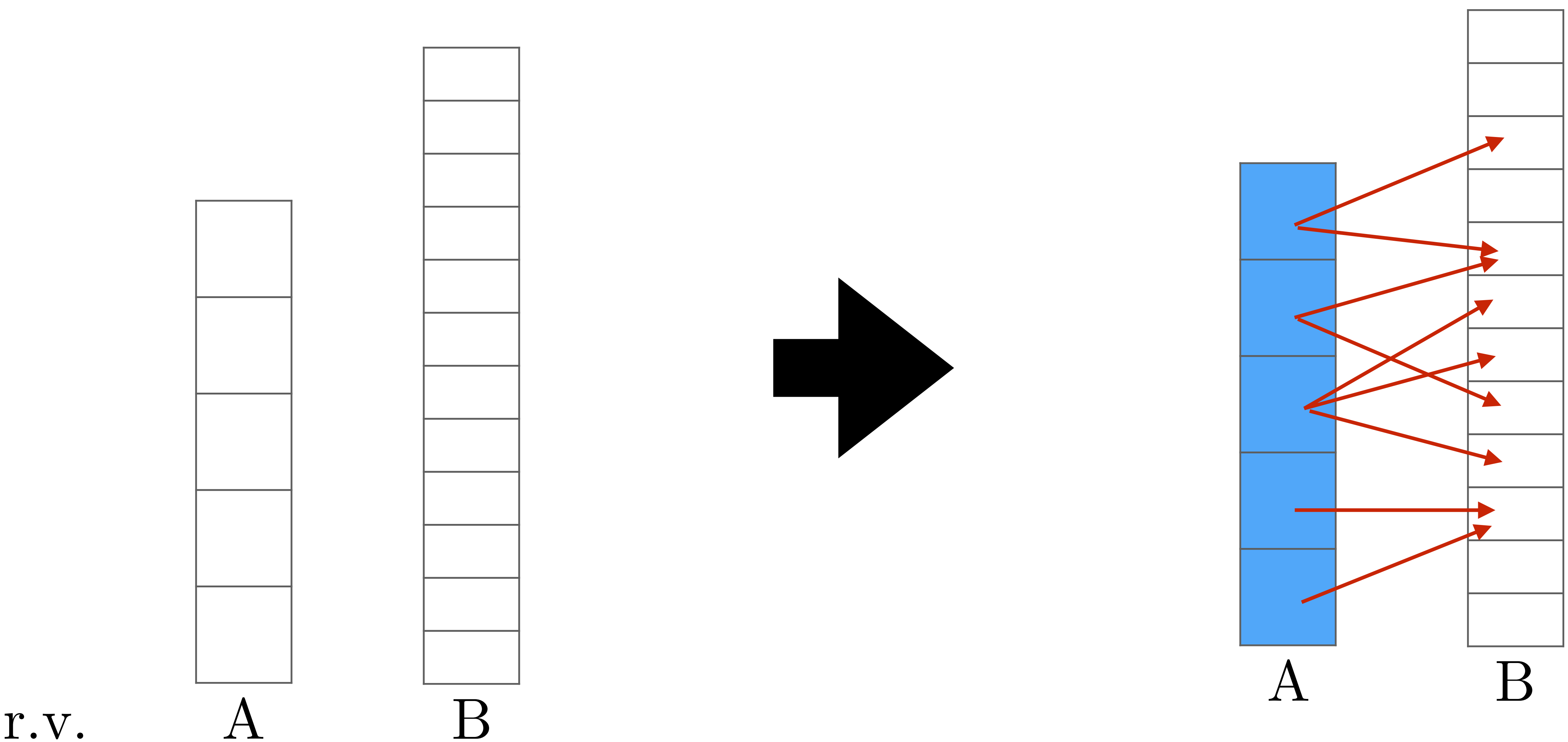
$$\left(\; 2^M \; \right)$$

If we were to sample particles, what is the likelihood that they would explain the measurements??

$$P(z_t | m, x_t)$$

Key idea: Exploit dependencies

Even if a space is absurdly large, dependencies within the space can significantly shrink the space of possibilities we should consider.



(need to store
5x12 = 60 numbers)

(need to store
5 + 9 = 14 numbers)

Key idea: Exploit dependencies

Is there a dependency in this gigantic combined state space?

Yes!

The map depends on the history of poses of the robot

Rao-Blackwellization

Factorization to exploit dependencies between variables

$$P(a, b) = P(b|a)P(a)$$

If P(b|a) can be computed efficiently

Represent

$$P(a)$$

with samples

Compute

$$P(b|a)$$

for every sample

Applying the factorization trick

$$P(x_{1:t}, m | z_{1:t}, u_{1:t})$$

state history   map   data

$$= P(x_{1:t} | z_{1:t}, u_{1:t}) \; P(m | x_{1:t}, z_{1:t}, u_{1:t})$$

state history   data   map   state history   data

$$= P(x_{1:t} | z_{1:t}, u_{1:t}) \; \prod_{i=1}^{N} P(m_i | x_{1:t}, z_{1:t})$$

state history   data

(Particle filter to estimate this)   (Occupancy map)

How do we compute a PF over state history?

For simplicity, each particle only stores the current state at timestep t

(just like you did in you assignment)

However the weights of the particle are computed based on it's path through time.

How do we compute a PF over state history?

Let's jump straight to the importance sampling step

$$w_t = \frac{bel(x_{1:t})}{\overline{bel}(x_{1:t})} = \frac{P(x_{1:t}|z_t, z_{1:t-1}, u_{1:t})}{P(x_{1:t}|z_{1:t-1}, u_{1:t})}$$

$$\propto \frac{P(z_t|x_{1:t}, z_{1:t-1}, u_{1:t})P(x_{1:t}|z_{1:t-1}, u_{1:t})}{P(x_{1:t}|z_{1:t-1}, u_{1:t})}$$

$$\propto P(z_t|x_t, x_{1:t-1}, z_{1:t-1}, u_{1:t})$$

$$\propto \sum_m P(z_t|x_t, m)P(m|x_{1:t-1}, z_{1:t-1}, u_{1:t})$$

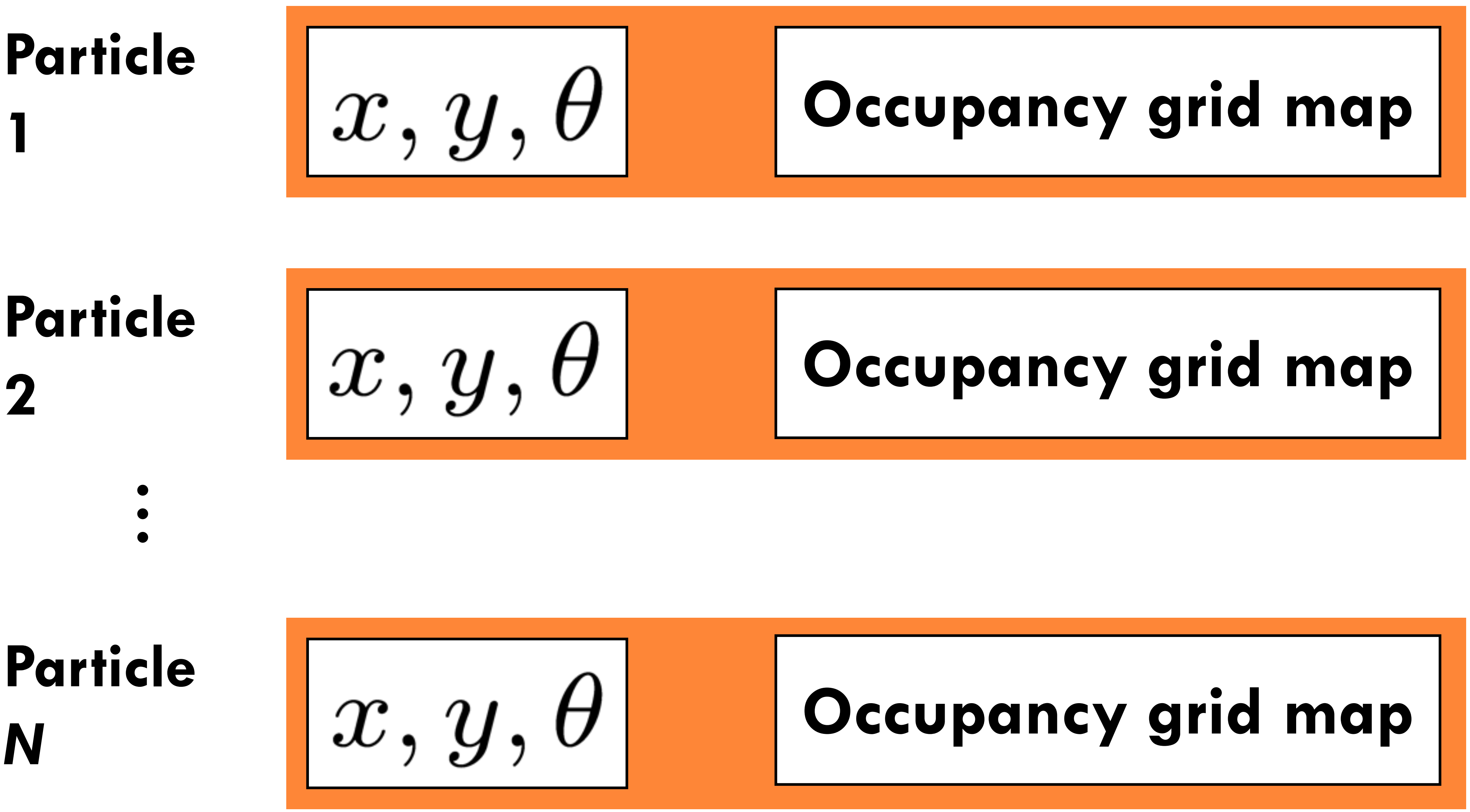
$$\approx P(z_t|x_t, \hat{m})$$

(meas) (most likely map from prev timestep)

where $\hat{m} = \arg\max_m P(m|x_{1:t-1}, z_{1:t-1}, u_{1:t-1})$

FastSLAM

Proposed by Montemerlo 2002



FastSLAM Algorithm

Algorithm FastSLAM_occupancy_grids($\mathcal{X}_{t-1}, u_t, z_t$**):**

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $k = 1$ *to* M *do*

$x_t^{[k]} = \textbf{sample\_motion\_model}(u_t, x_{t-1}^{[k]})$

$w_t^{[k]} = \textbf{measurement\_model\_map}(z_t, x_t^{[k]}, m_{t-1}^{[k]})$

$m_t^{[k]} = \textbf{updated\_occupancy\_grid}(z_t, x_t^{[k]}, m_{t-1}^{[k]})$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[k]}, m_t^{[k]}, w_t^{[k]} \rangle$

endfor

for $k = 1$ *to* M *do*

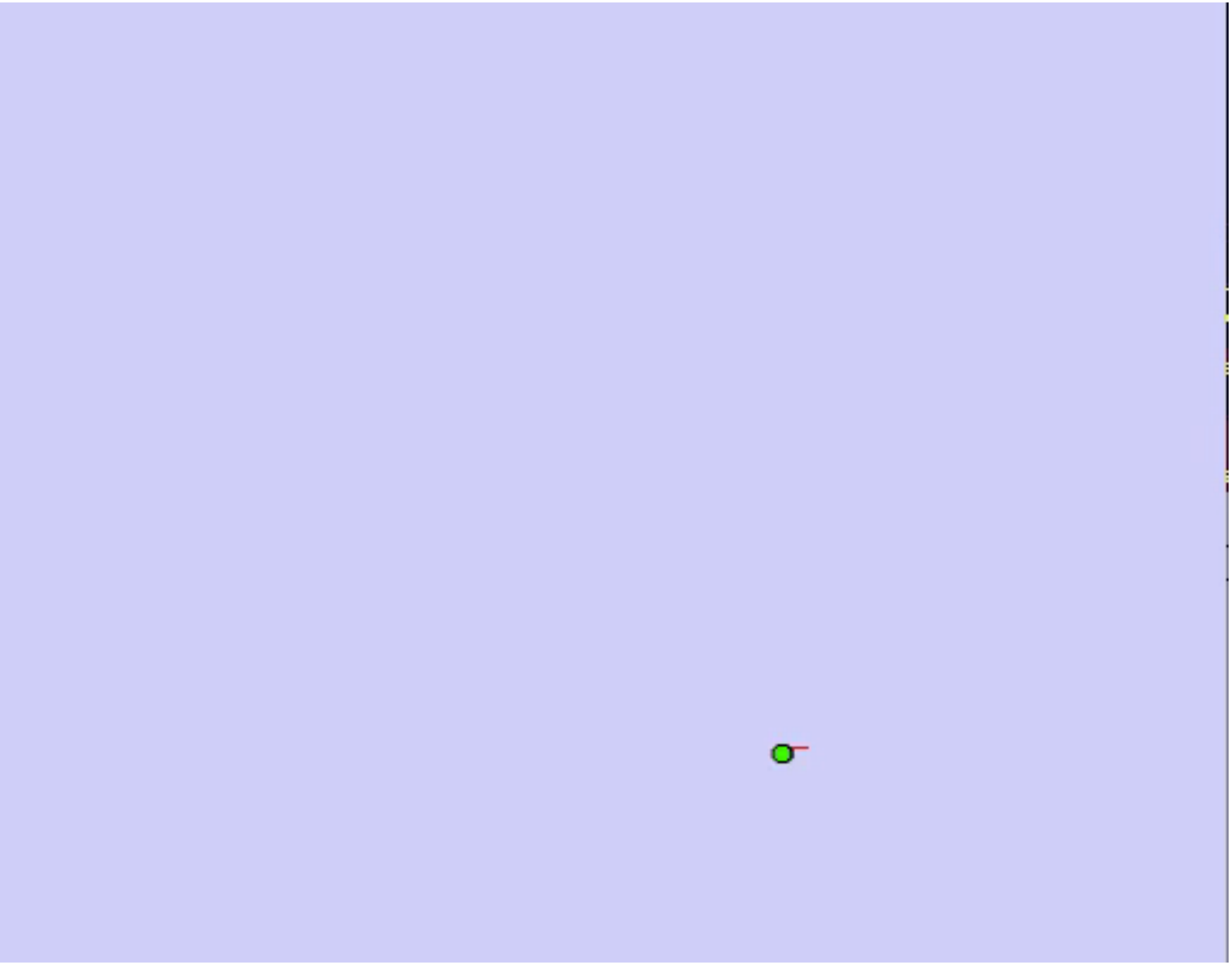
draw i *with probability* $\propto w_t^{[i]}$

add $\langle x_t^{[i]}, m_t^{[i]} \rangle$ *to* $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

FastSLAM in action!



Haehenl et al.

SLAM resources

OpenSLAM

Give your algorithm to the community

https://openslam-org.github.io/

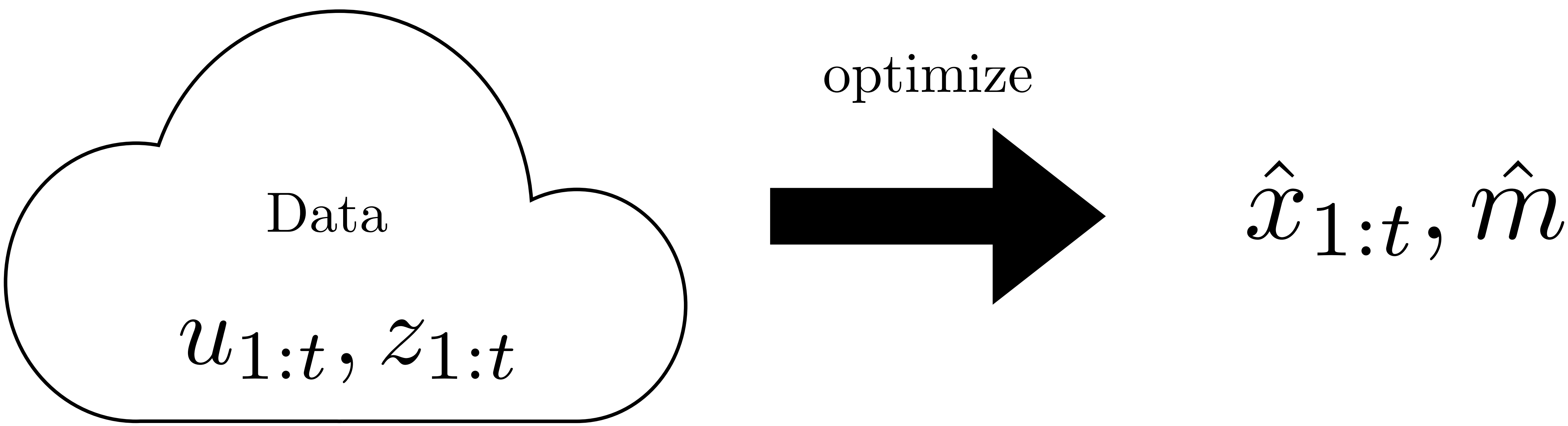
Do we really need a probability distribution?

$$P(x_{1:t}, m)$$

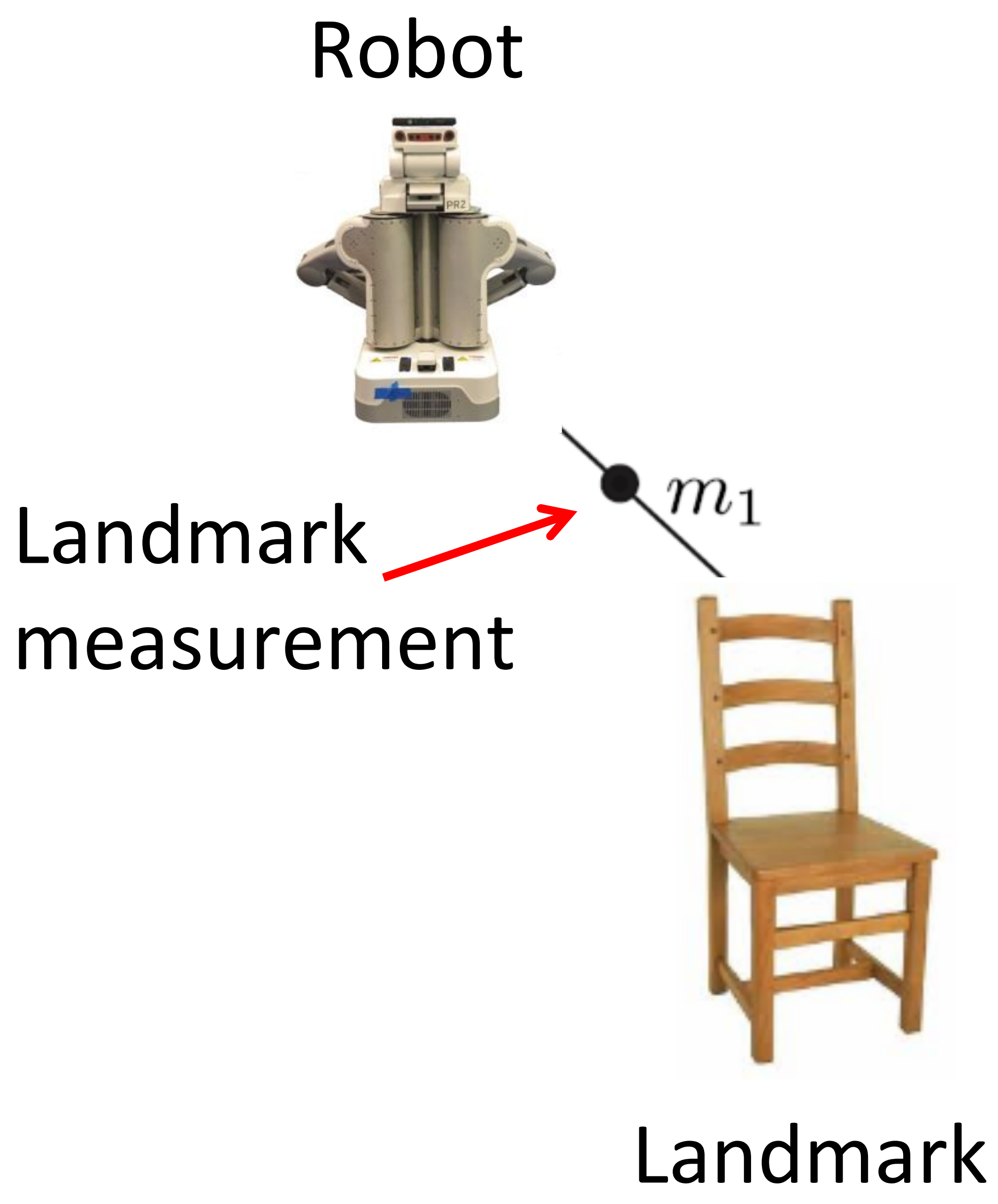
Or are does a maximum a posteriori estimate suffice?

$$\hat{x}_{1:t}, \hat{m} = \arg\max_{x_{1:t}, m} P(x_{1:t}, m)$$

SLAM as a pure optimization problem

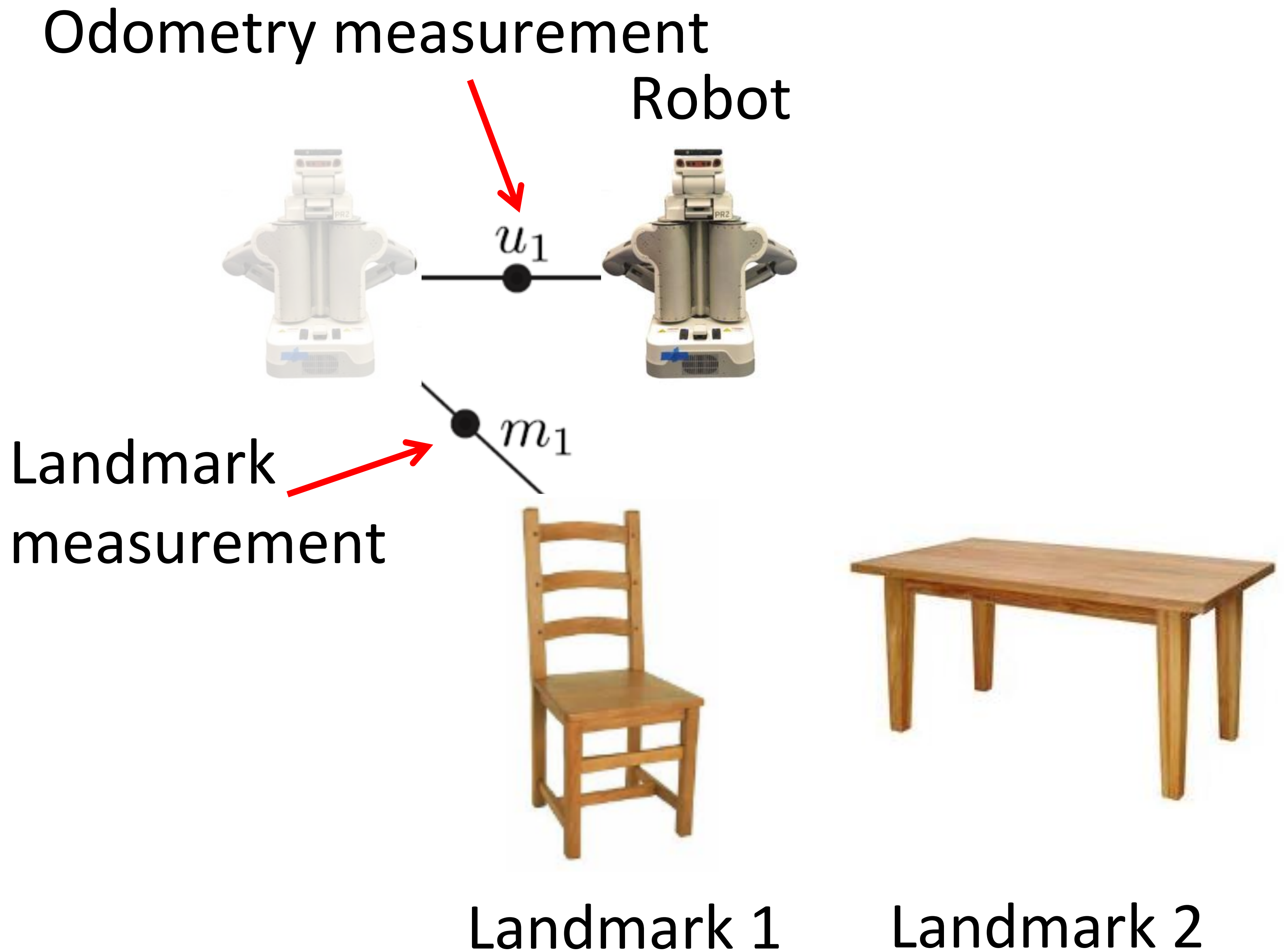


SLAM as a set of relationships



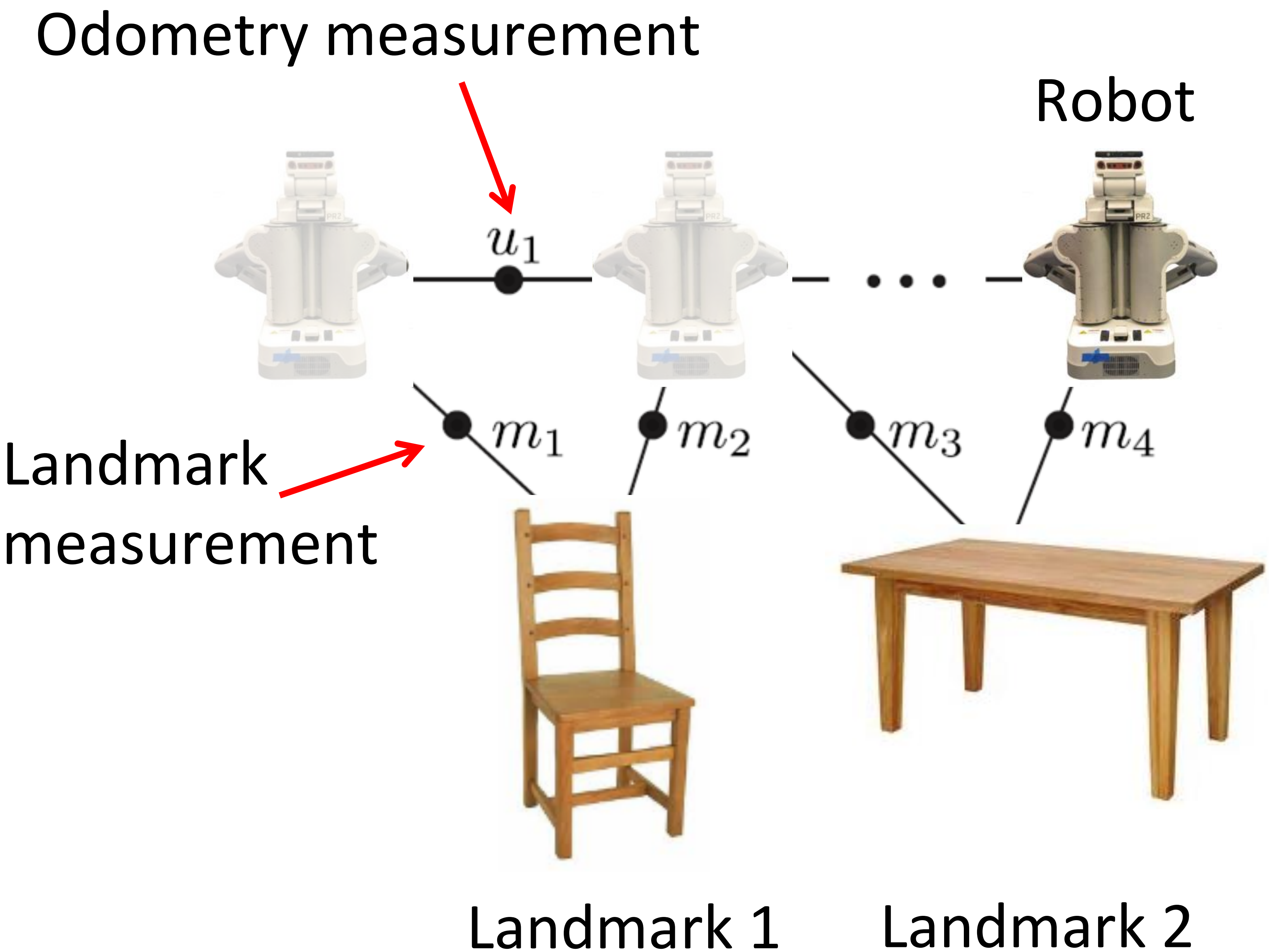
Courtesy M.Kaess

SLAM as a set of relationships



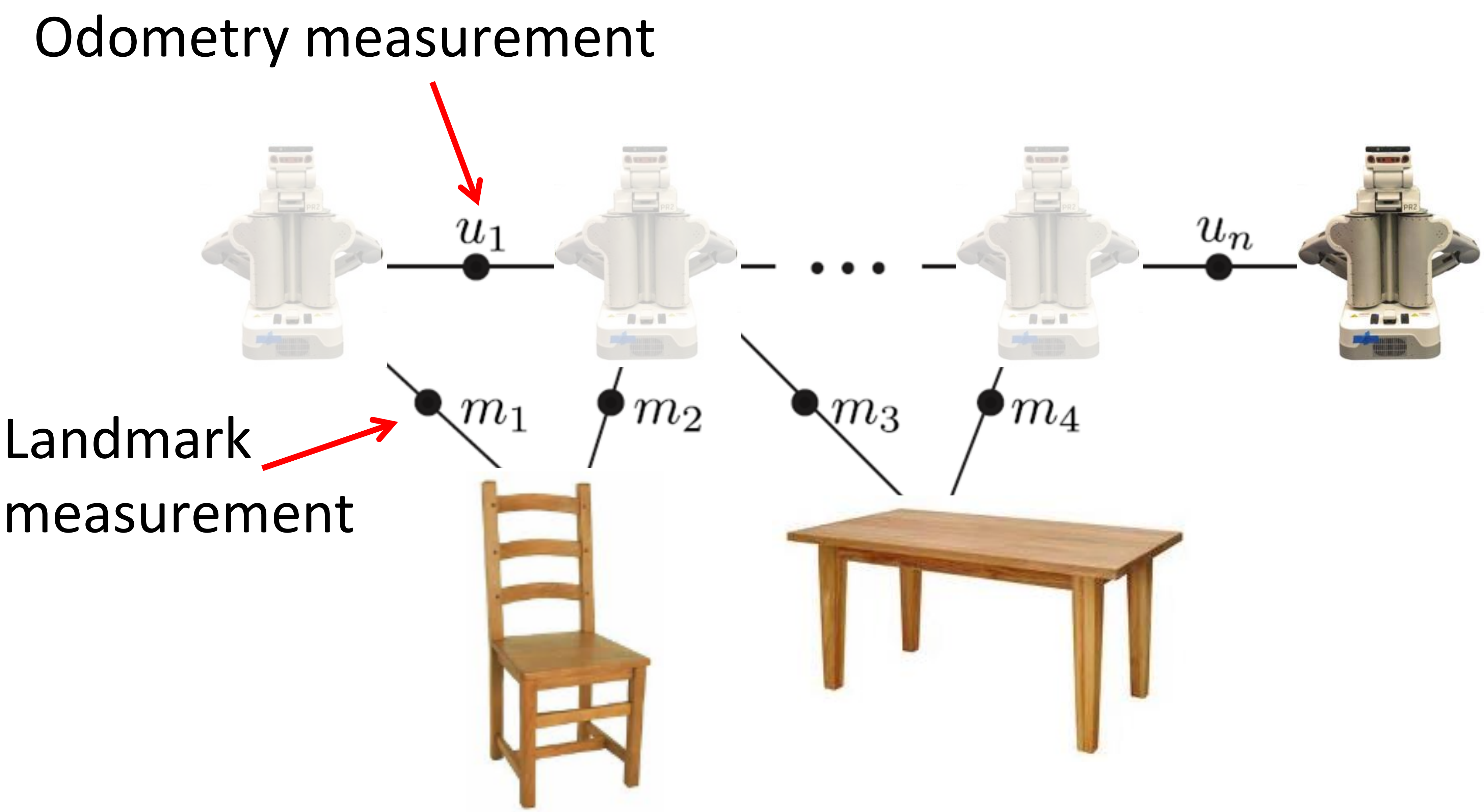
Courtesy M.Kaess

SLAM as a set of relationships



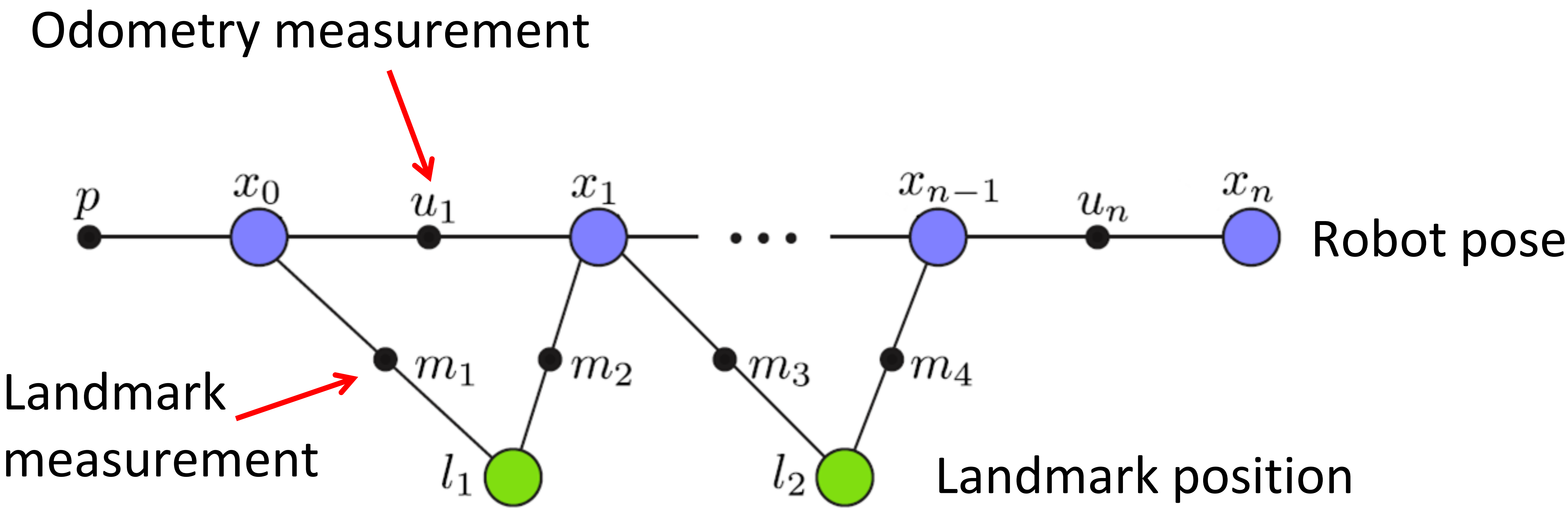
Courtesy M.Kaess

SLAM as a set of relationships

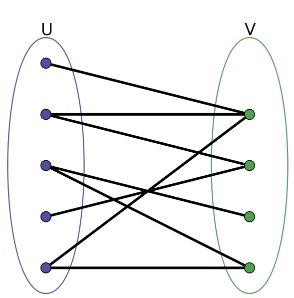


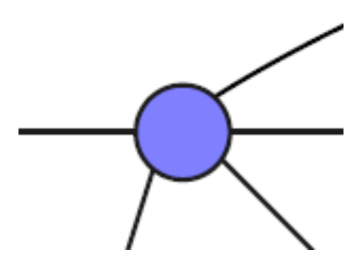
Courtesy M.Kaess

Factor Graph representation of SLAM



Bipartite graph with ***variable nodes*** and ***factor nodes***





Courtesy M.Kaess

Factor Graph representation of SLAM

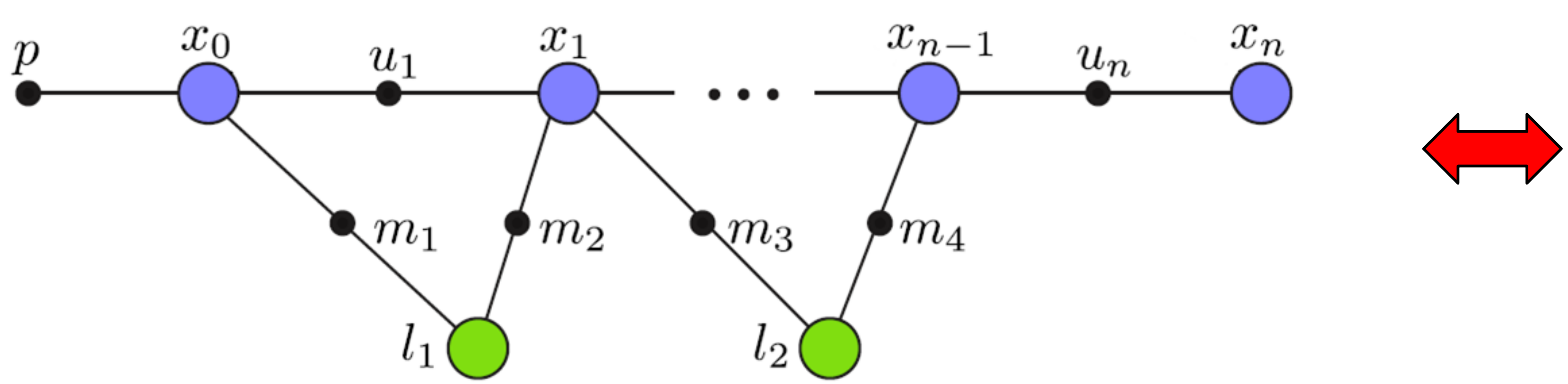
The variables in the optimization are poses of robot at all time steps and all landmarks

$$\theta = [x_1, x_2, \ldots, x_n, l_1, \ldots, l_m]$$

Factorization of probability density

- Conditional independence:

$$p(z_1 z_2|\Theta) = p\left(z_1|\Theta\right) p\left(z_2|\Theta\right)$$



$$\text{argmax}_\Theta \prod_{z \in Z} p\left(z|\Theta\right)$$

$$\text{argmax}_\Theta \; p(p|\Theta)\, p\left(u_1|\Theta\right) \cdots p(u_n|\Theta)\, p\left(m_1|\Theta\right) \cdots p(m_4|\Theta)$$

Courtesy M.Kaess

How do we solve such optimization?

Large scale sparse non-linear least squares optimization	+	incrementally growing factors

Option 1: Using sparse matrix algebra

(Kaess et al. 2008)

Option 2: Using probabilistic graphical models

(Kaess et al. 2011)

Top: The Bayes tree.
The parts affected by the new variables are colored in red.

Bottom: Color coded trajectory.
Green corresponds to a low number of variables, red to a high number.

iSAM2, Kaess et al. 2012

Application: Kintinuous 2.0 (Whelan et al.)