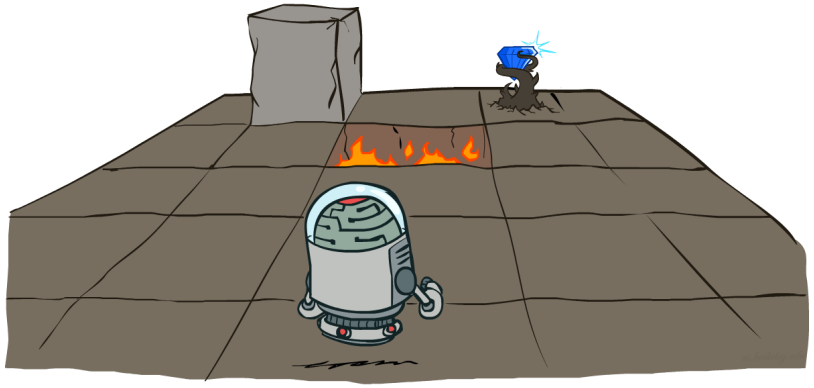
Markov Decision Processes II

CSE 473: Introduction to Artificial Intelligence



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cs.uw.edu/473
acknowledgements

Welcome!
How is Project 1 going?
sli.do **#cse473**

Administrivia

ANNOUNCEMENTS

- For each project, make sure to fill out the **AI usage reflection** (cs.uw.edu/473/projects)
- **Test 1** on Friday
  - One double-sided 8.5x11" note sheet allowed

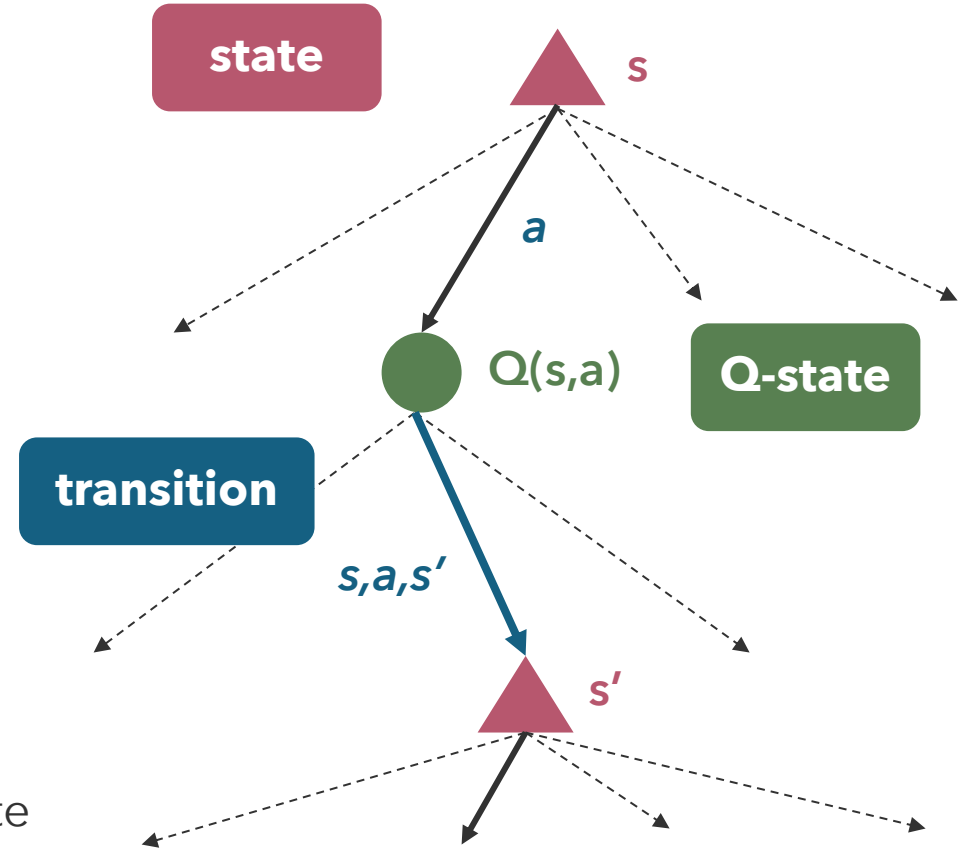
ASSIGNMENTS

- **Project 1 due** tomorrow! (7.9)
- **Homework 2** releases Friday
  - **due** in two weeks (7.23)

Recap: Optimal Quantities

FOUNDATIONS

- Expected utility (value) of policy π at s_0 :
  - $U^{\pi}(s) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t R(S_t, \pi(S_t), S'_t)]$
- Optimal Policy π^*
  - $\pi^* = \text{argmax}_{\pi} U^{\pi}(s)$
- Value of a State $U^*(s)$
  - Expected utility starting at state s and acting optimally
- Q-Value $Q(s, a)$
  - $Q^*(s, a)$ is the expected utility of taking action a from state s and acting optimally thereafter
  - $U^*(s) = \max_a Q^*(s, a)$



Recap: Bellman Equations

DEFINITION

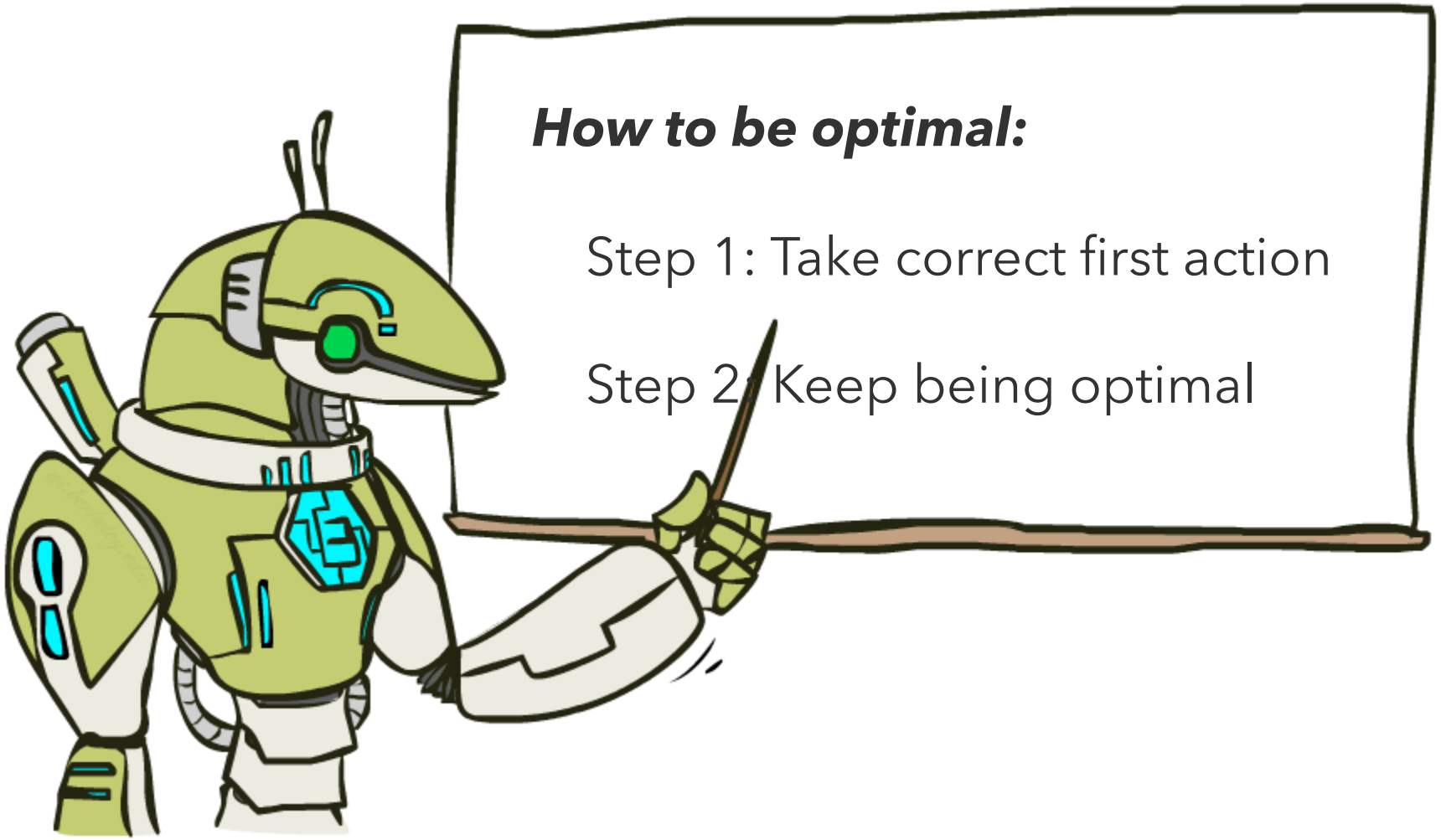
The utility of a state is the optimal reward for the next transition plus the utility of the next state

- Recursive definition of utility (Bellman Equation)

$$U(s) = \max_a \left[\sum_{s'} P(s'|s,a) \left[R(s,a,s') + \gamma U(s') \right] \right] = \max_a Q(s,a)$$

- Recursive definition of Q-value

$$Q(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U(s')]$$



It's Pretty Simple…

How to find $U^*(s)$?

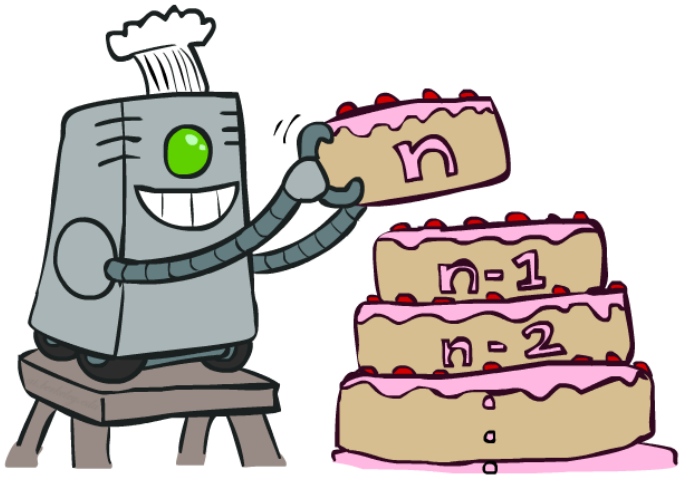
Value Iteration

FOUNDATIONS

Iteratively update *U(s)* values, stop using termination parameter ε

- Start with $U_0(s) = 0$
- Repeat until convergence (updates < ε):
  - Perform an update using Bellman equation:

$$U_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U_k(s')]$$

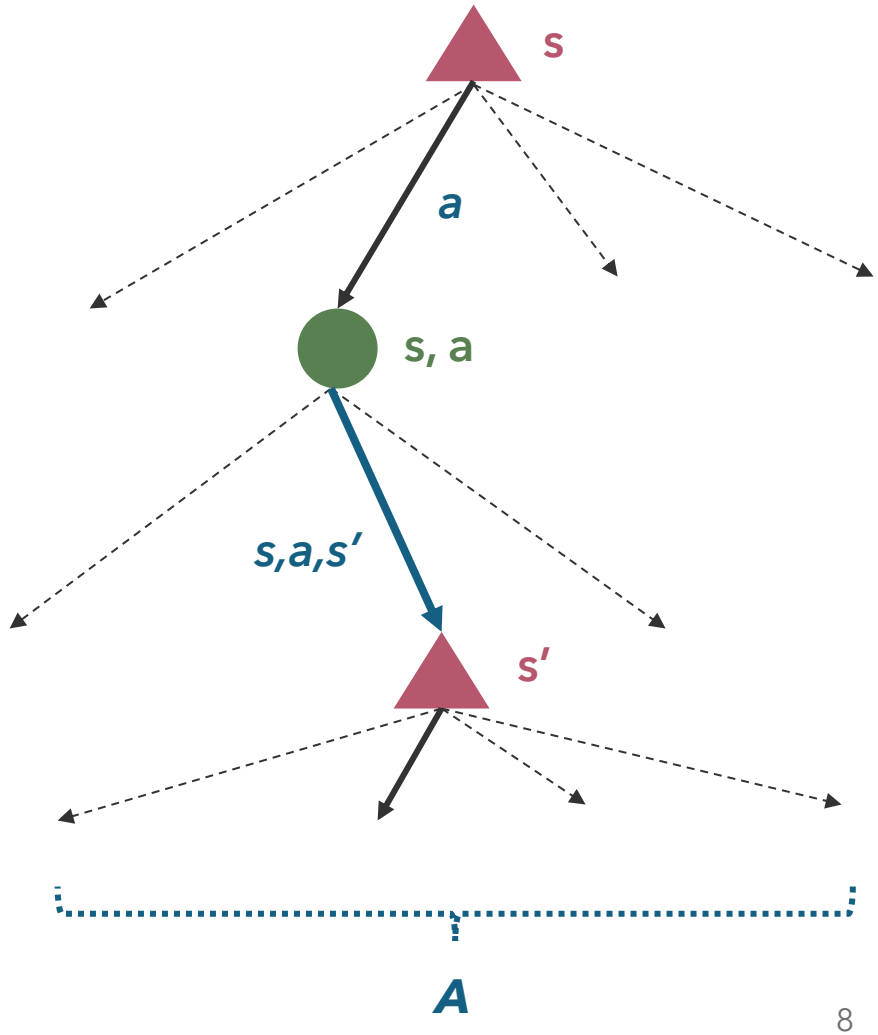


- **Theorem:** Value iteration will converge to optima
  - Approximations get refined towards optimal values
  - Policy may converge before values do

Properties of Value Iteration

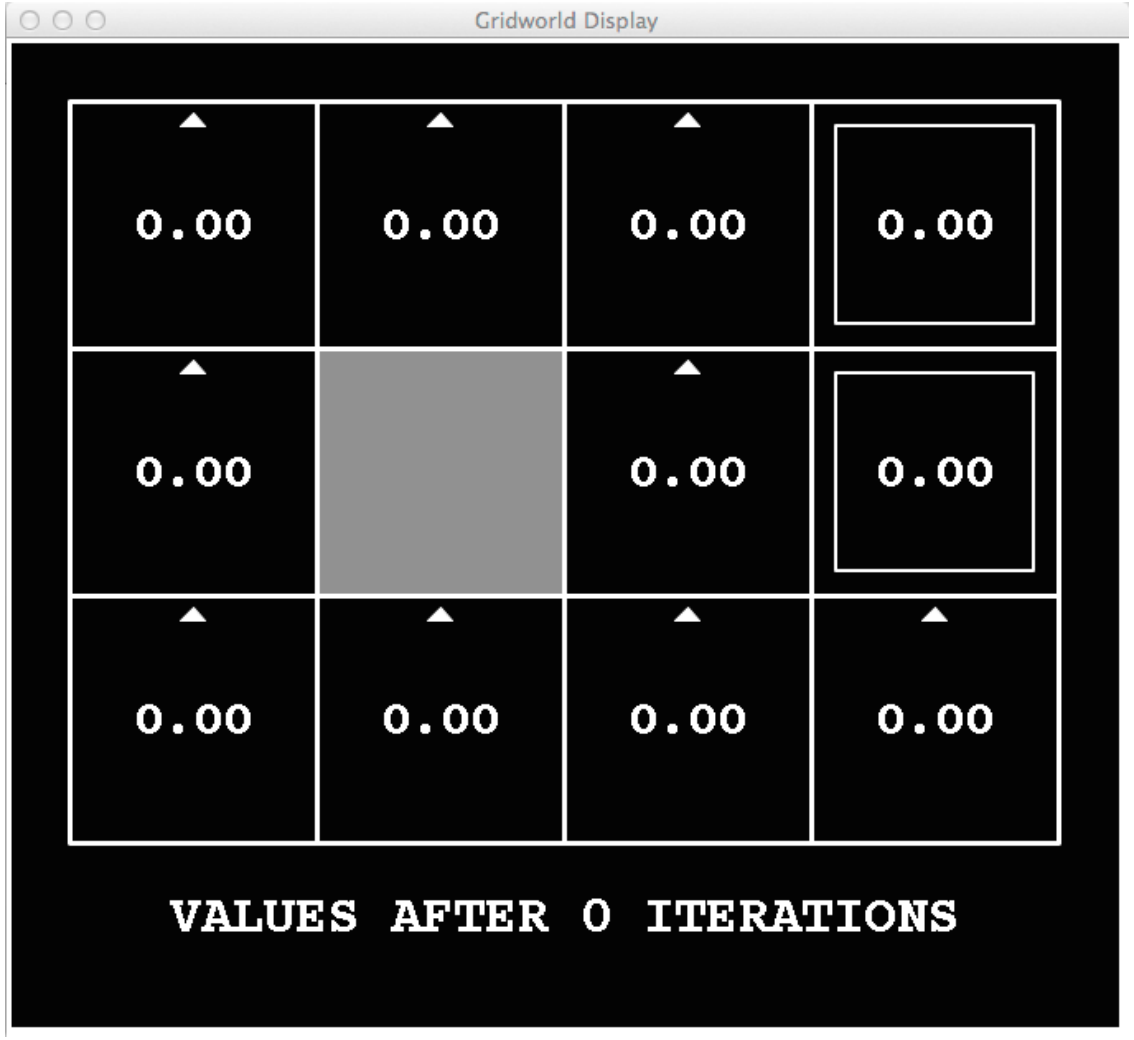
DEFINITION

- Bellman equations *characterize* optimal values
  - $U^*(s) = \max_a [\sum_{s'} P(s'|s,a)\,[R(s,a,s') + \gamma U^*(s')]]$
- Value iteration *computes* optimal values
  - $U_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U_k(s')]$
  - Value iteration is a fixed point solution method
- Complexity
  - Runtime per iteration: ***O(S²A)***



Grid World Value Iteration ($k = 0$)

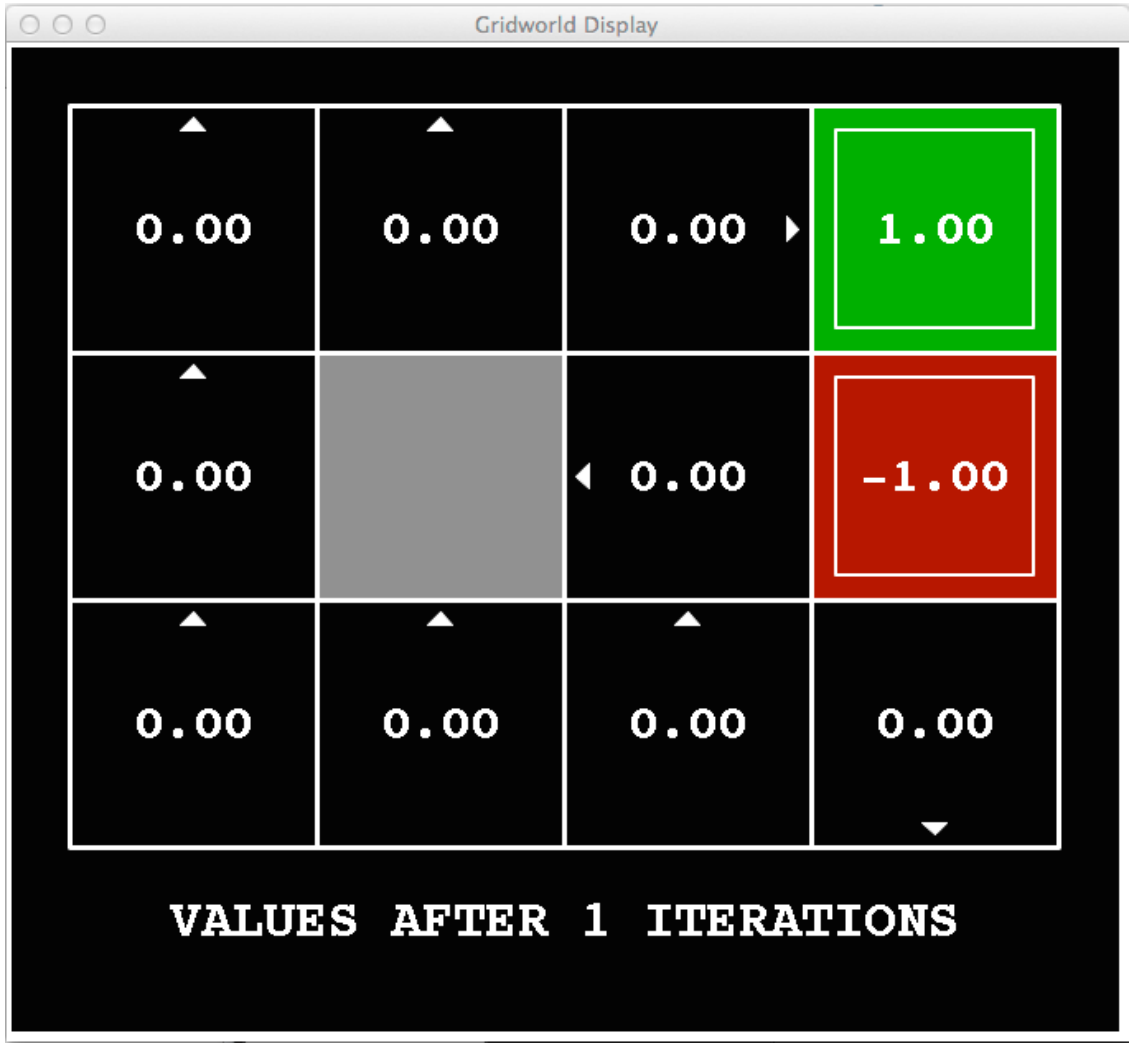
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 1$)

EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 2$)

EXAMPLE

Noise = 0.2
Discount = 0.9
Living Reward = 0

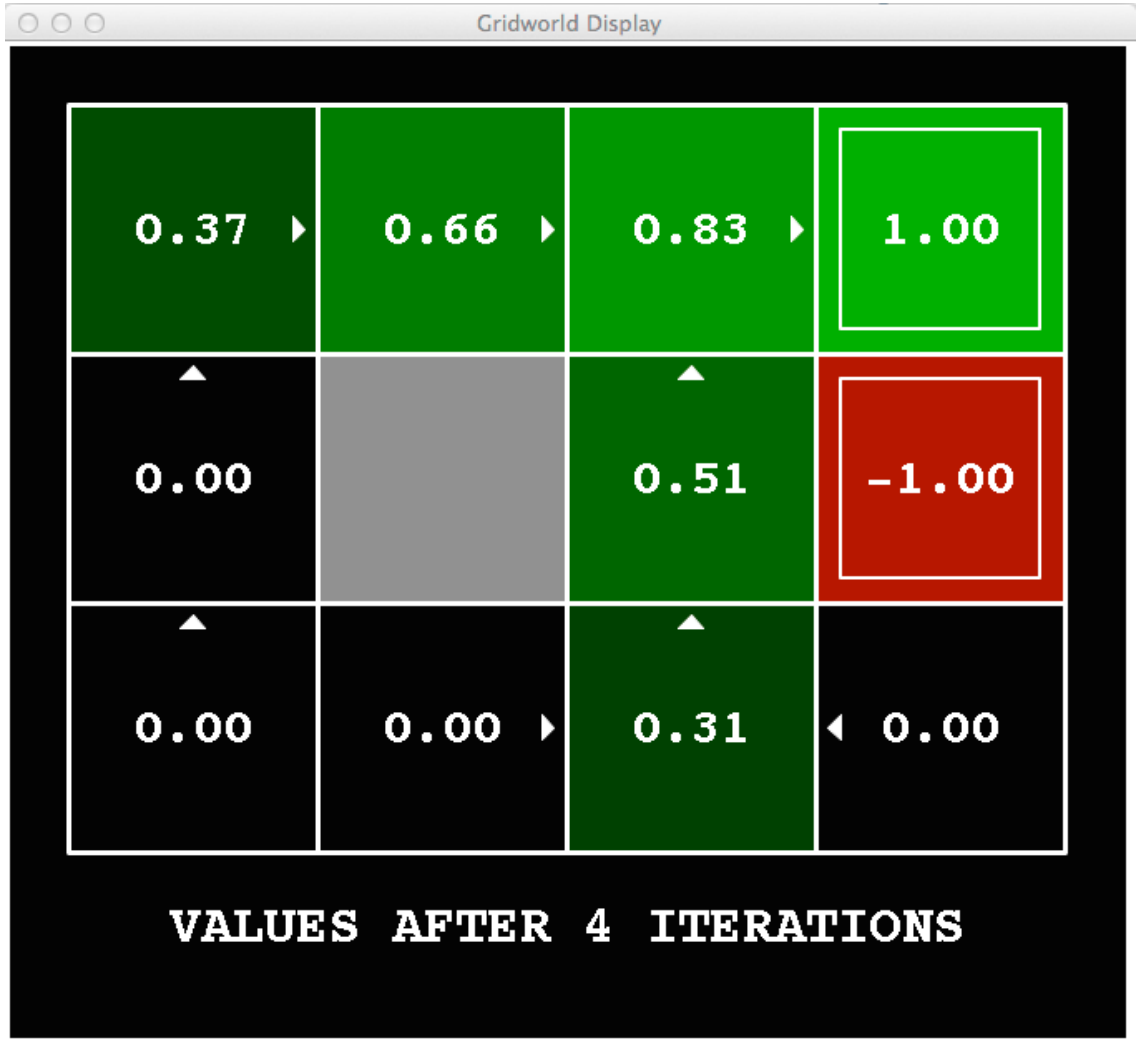
Grid World Value Iteration ($k = 3$)

EXAMPLE

Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 4$)

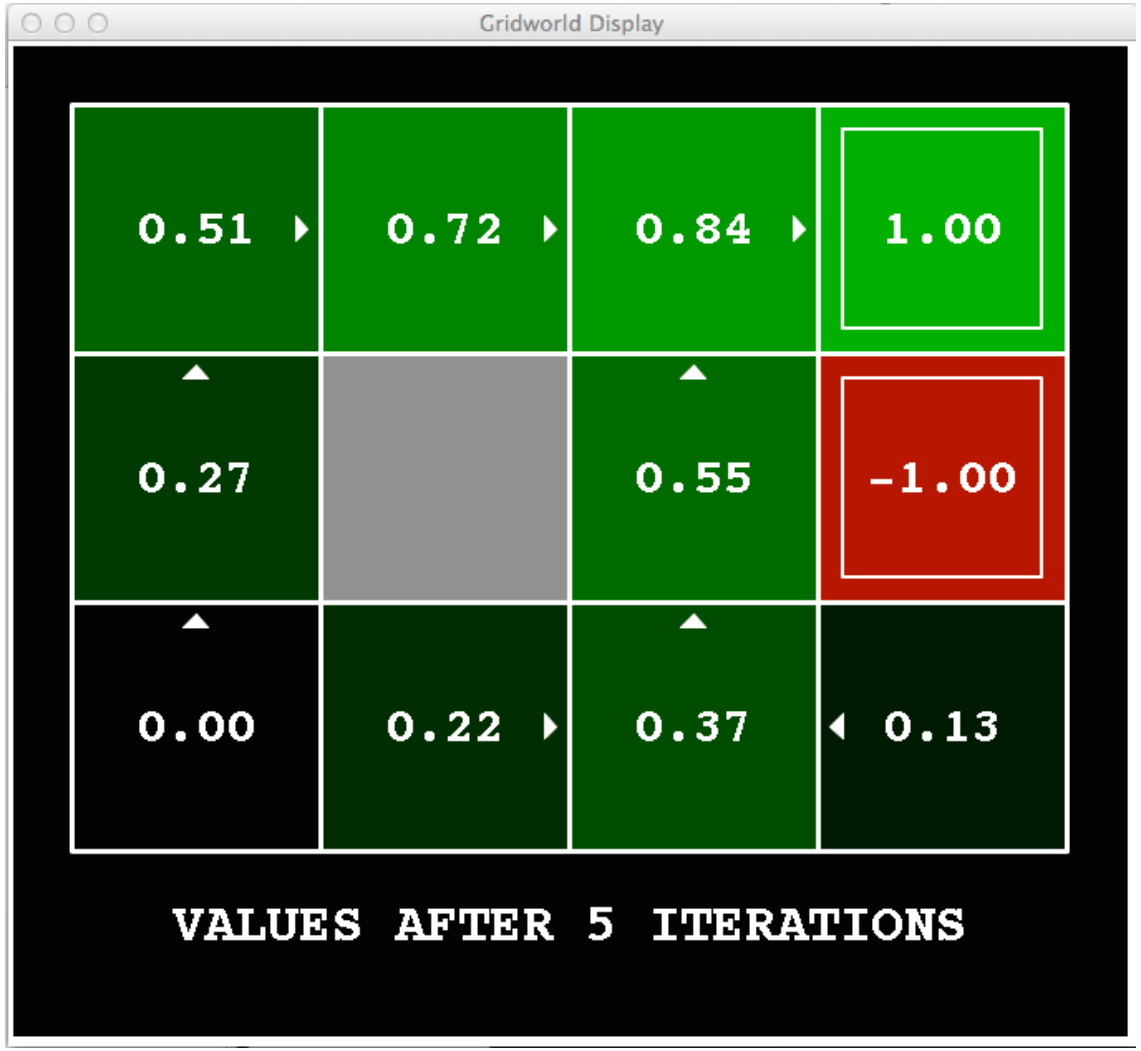
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 5$)

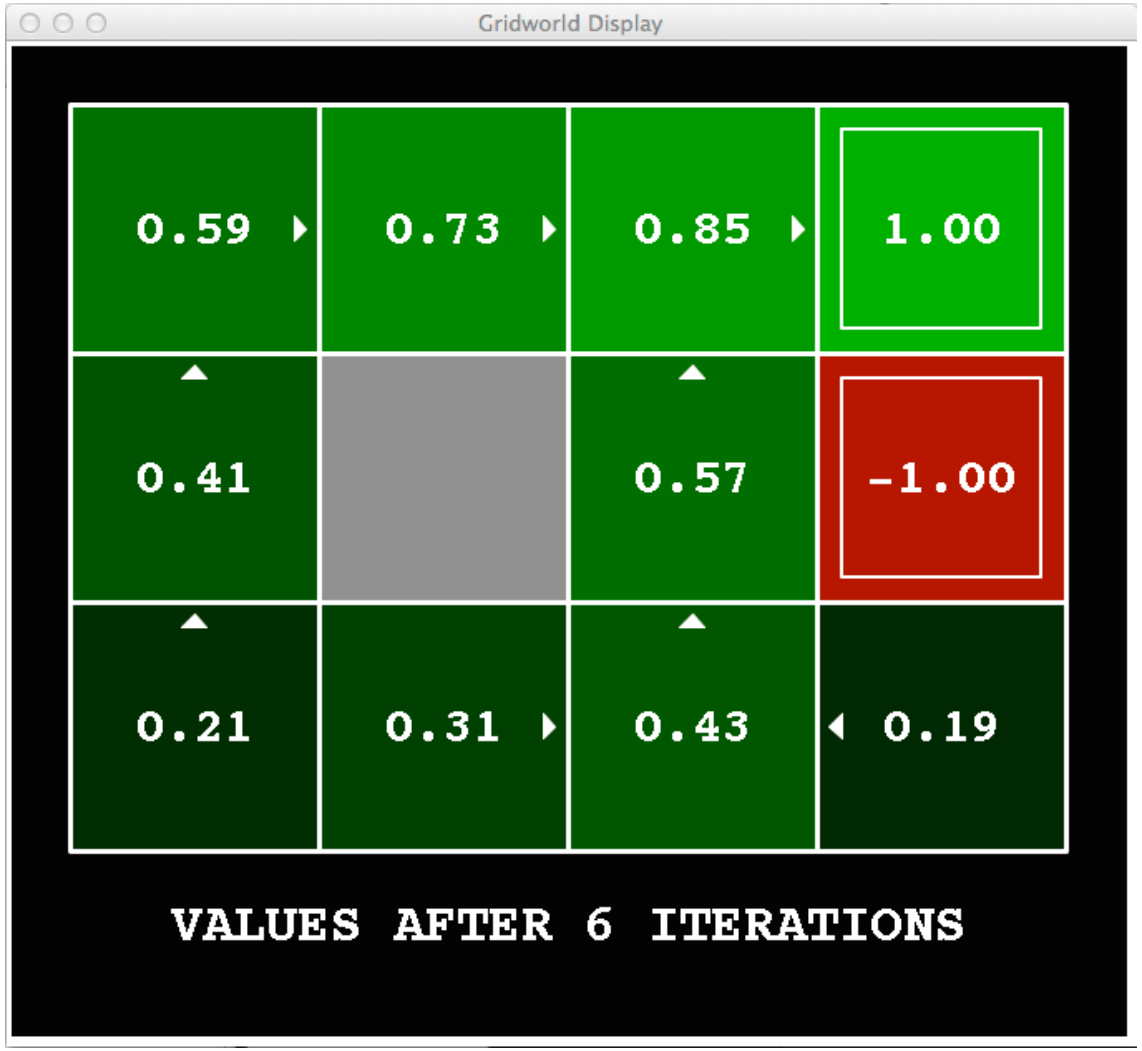
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 6$)

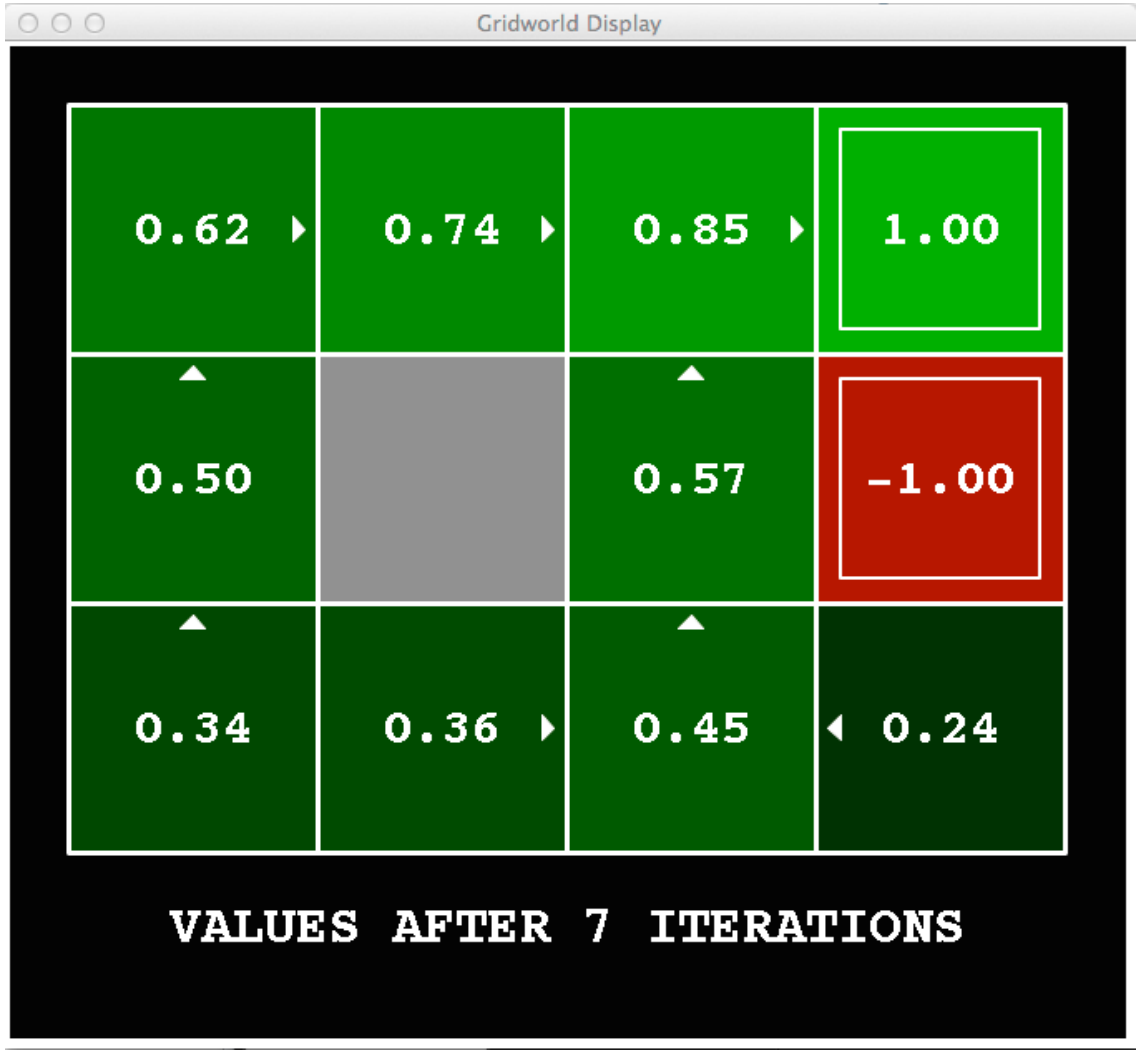
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 7$)

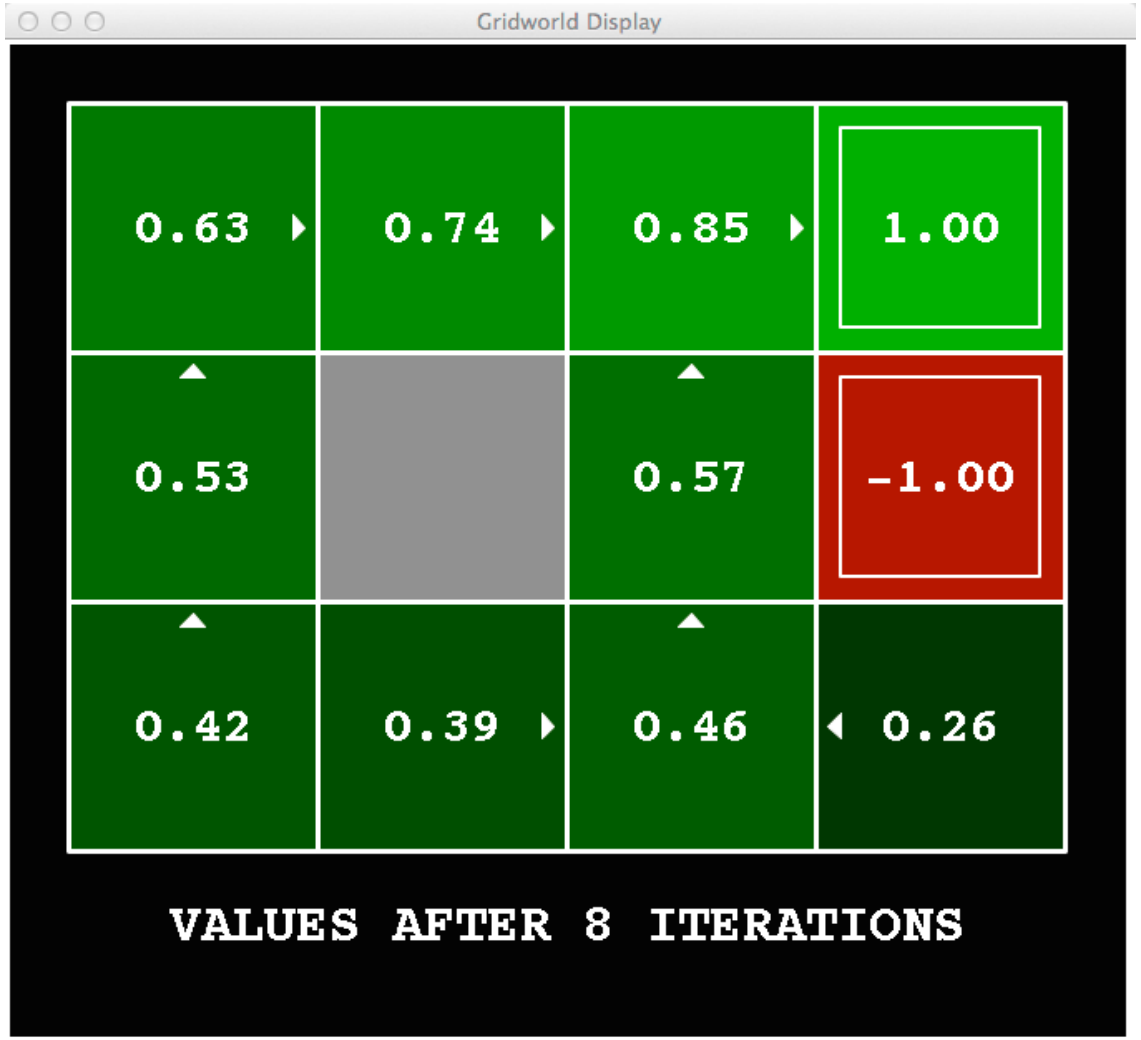
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 8$)

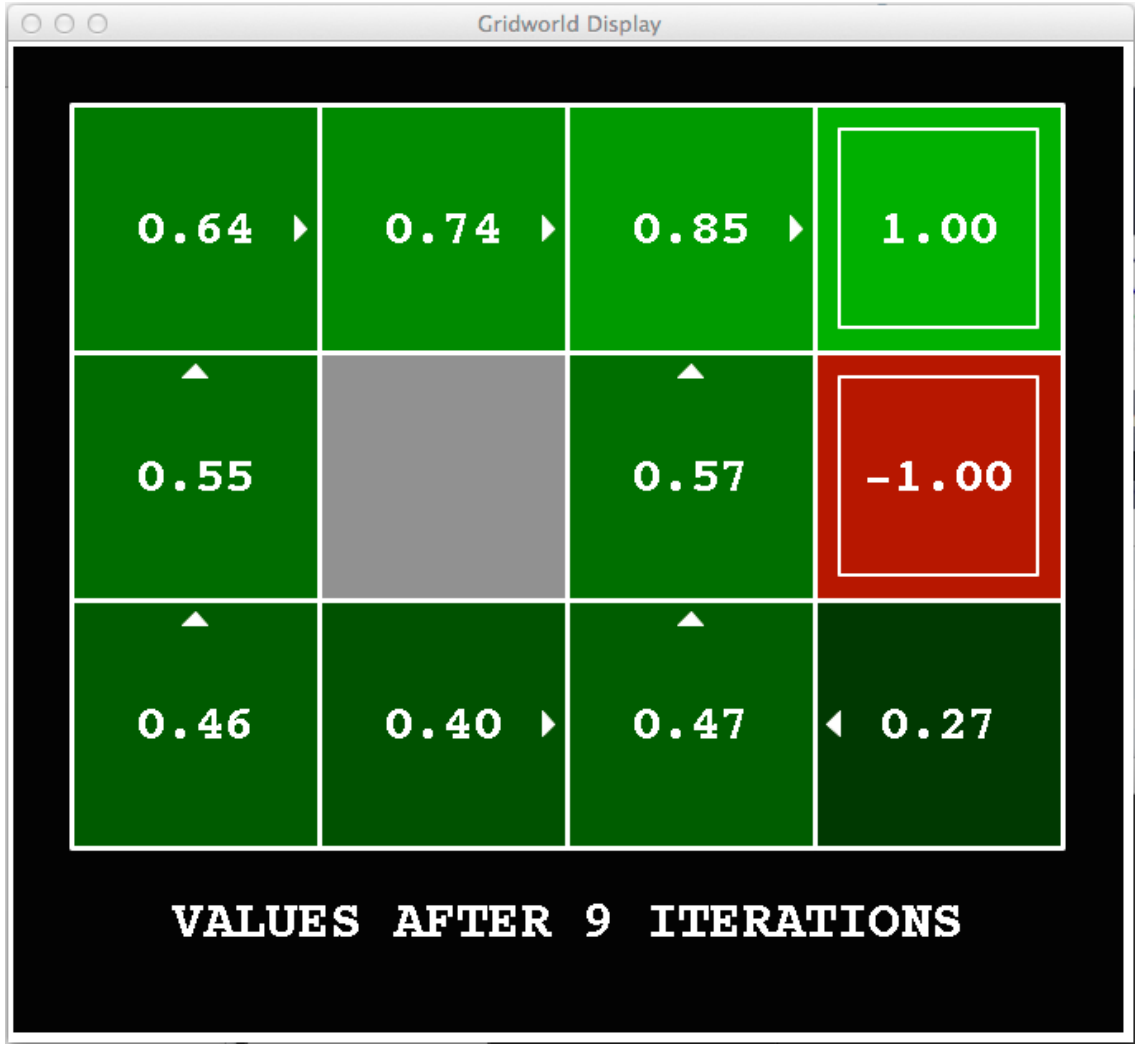
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 9$)

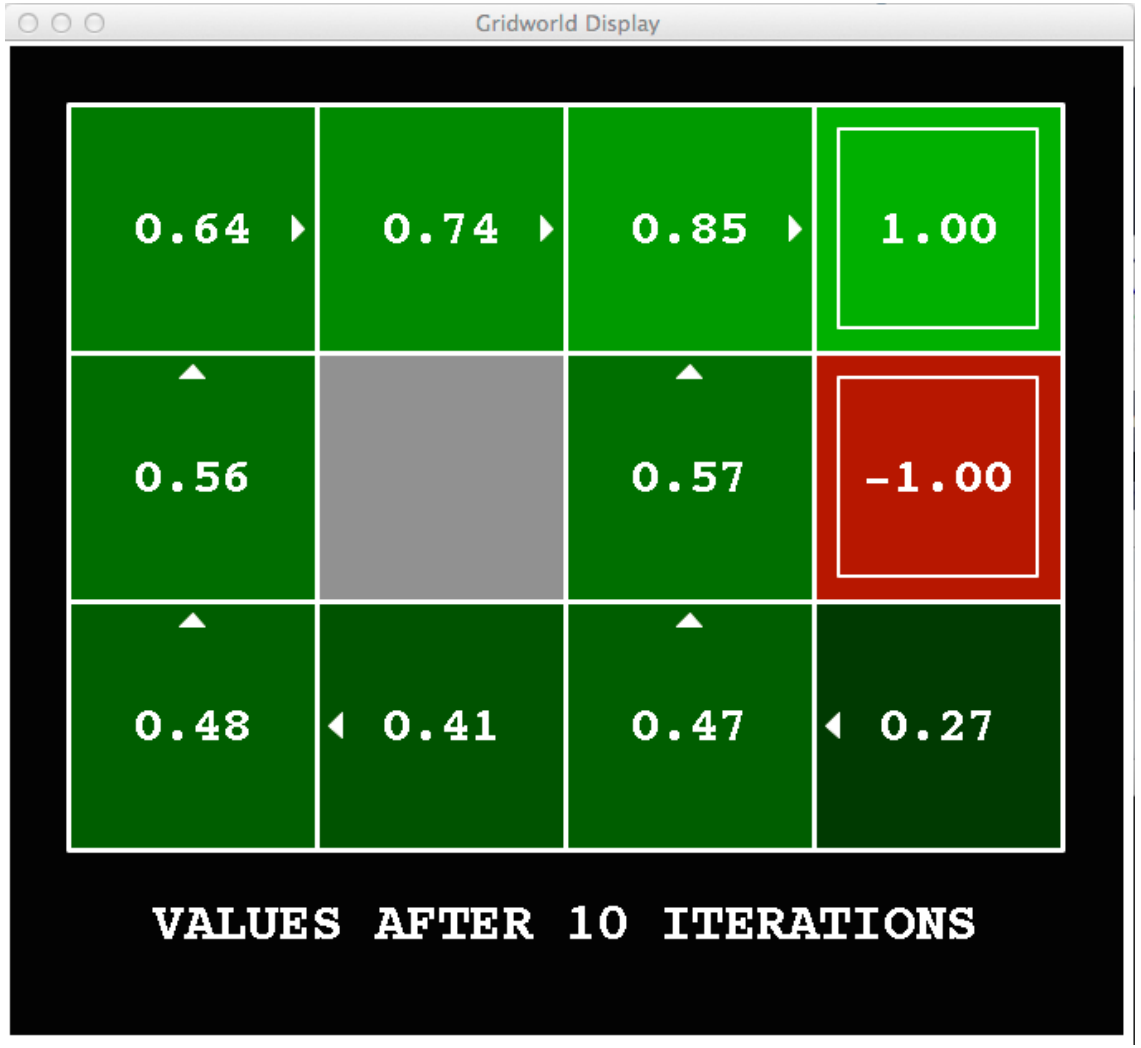
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration ($k = 10$)

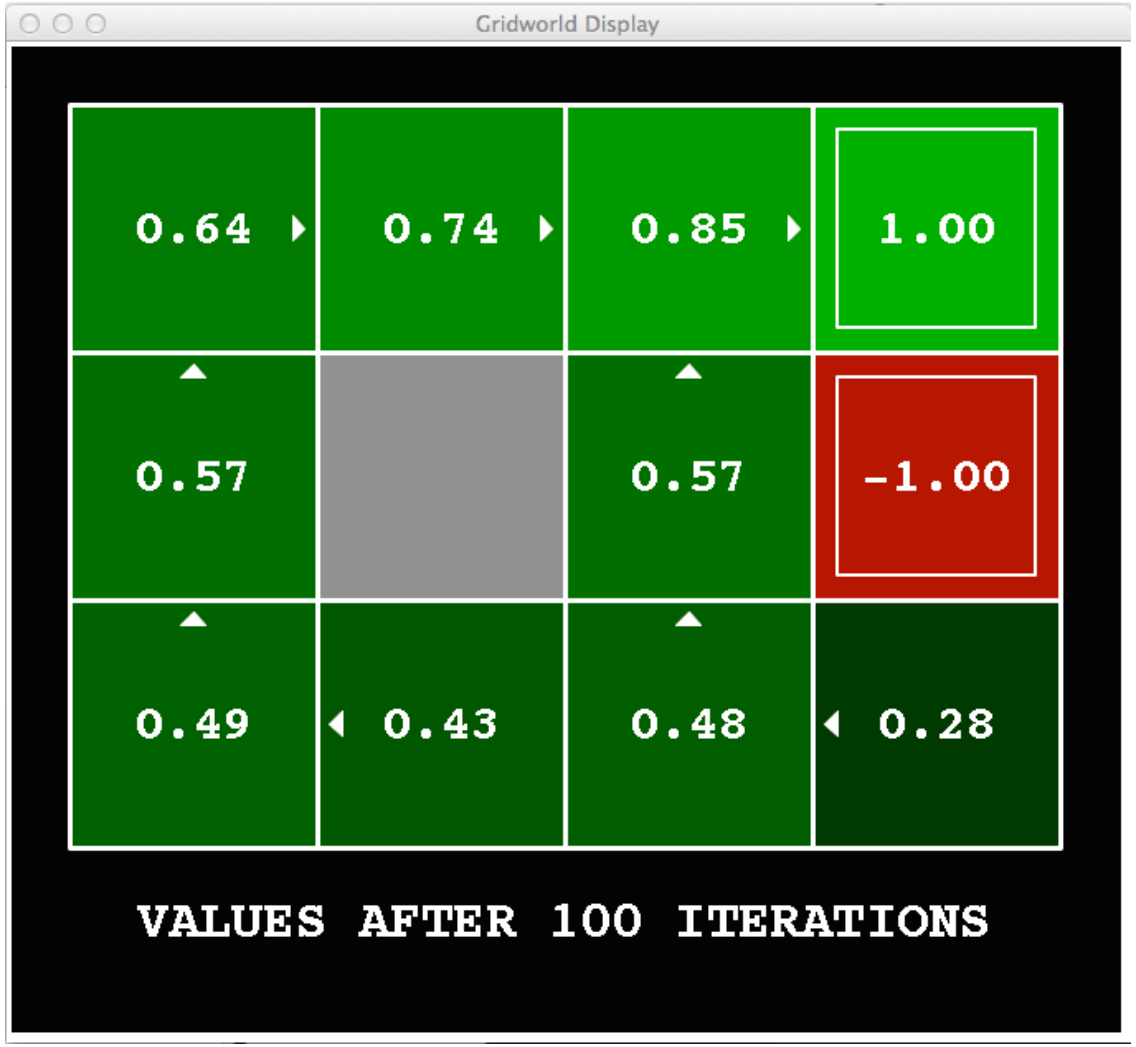
EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

Grid World Value Iteration (*k = 100*)

EXAMPLE



Noise = 0.2
Discount = 0.9
Living Reward = 0

QUESTIONS

Questions?

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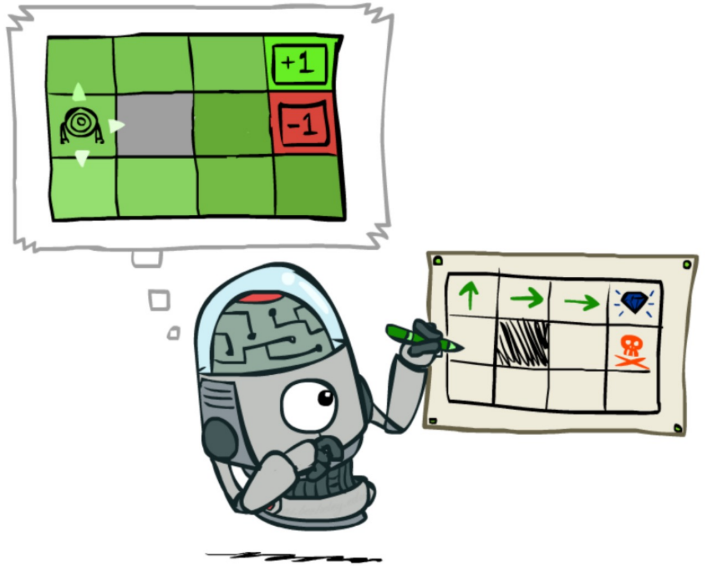
How to find policy from values?

Policy Extraction

FOUNDATIONS



- How should the agent act given $U(s)$?
  - Maximize expected utility!

$$\pi_U(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U(s')]$$

- This is **policy *extraction***, since it finds the policy π_U implied by U

Problems with Value Iteration

CONTEXT

Value iteration repeats Bellman updates until convergence

$$U_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U_k(s')]$$

- **Problem 1**: It's slow – ***O(S²A)*** *per* iteration
- **Problem 2**: argmax at each state barely changes
- **Problem 3**: Policy often converges long before values

QUESTIONS

Questions?

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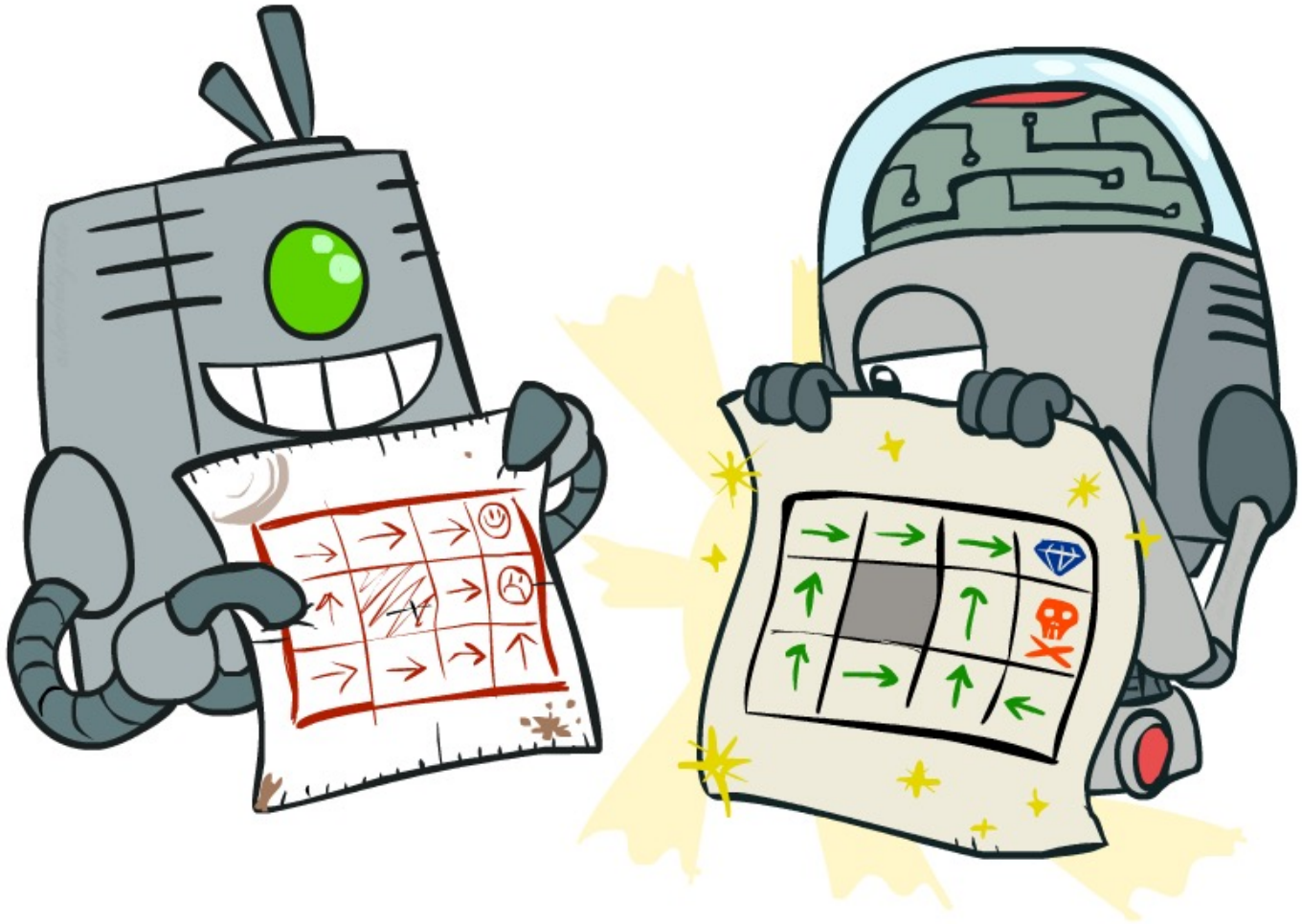


A Better Apporach…

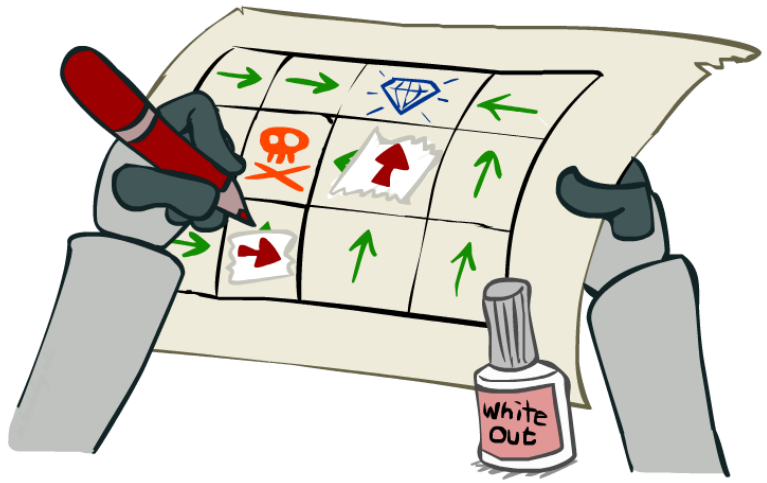
Policy Iteration

FOUNDATIONS

- Idea:
  - Make the policy implied by U explicit and compute long-term implications for values
- Repeat until no change in *policy*:
  - Step 1: **Policy Evaluation**

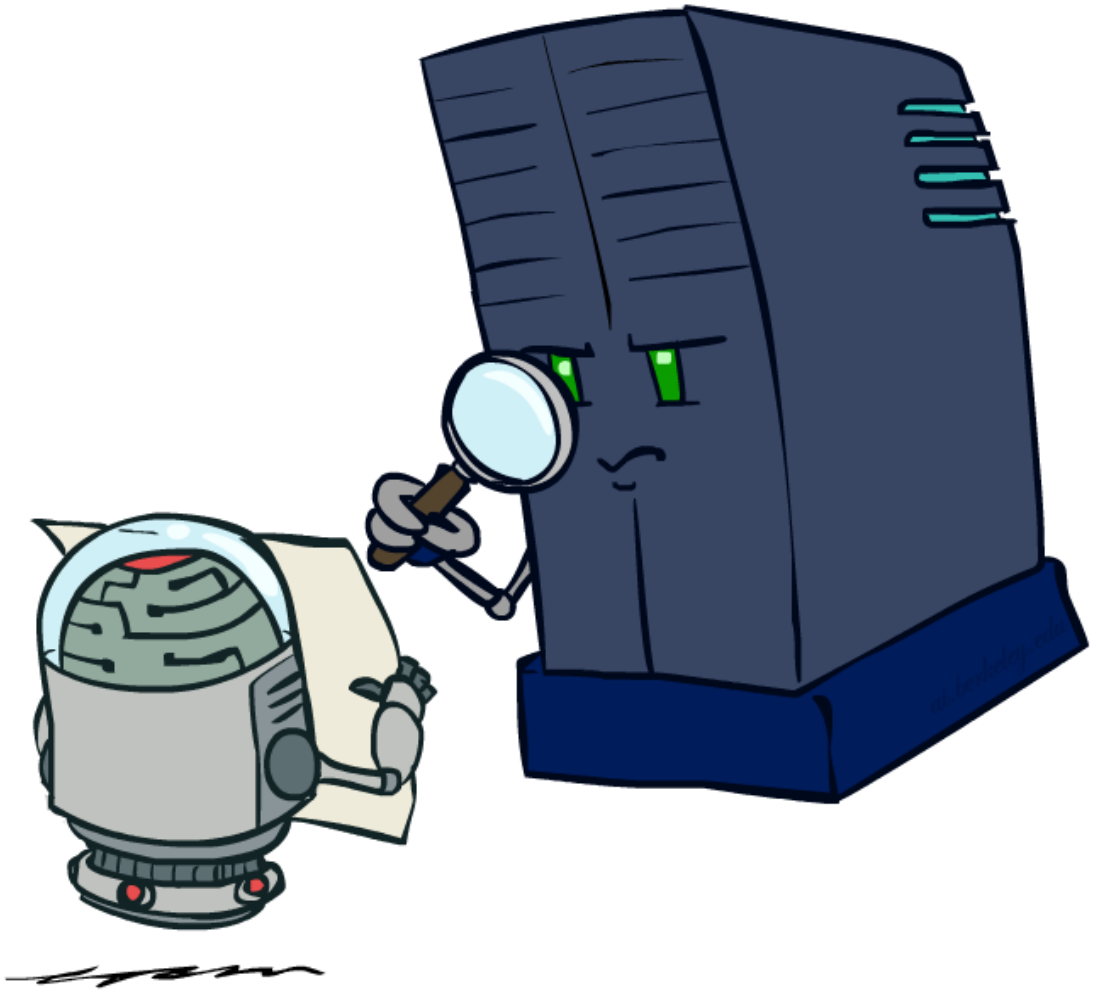
    calculate value U^{π_k} from current policy π_k
  - Step 2: **Policy Improvement**

    extract new implied policy π_{k+1} from U^{π_k}
- Properties:
  - Still optimal!
  - Can converge (much) faster under some conditions



How to Evaluate Policies?

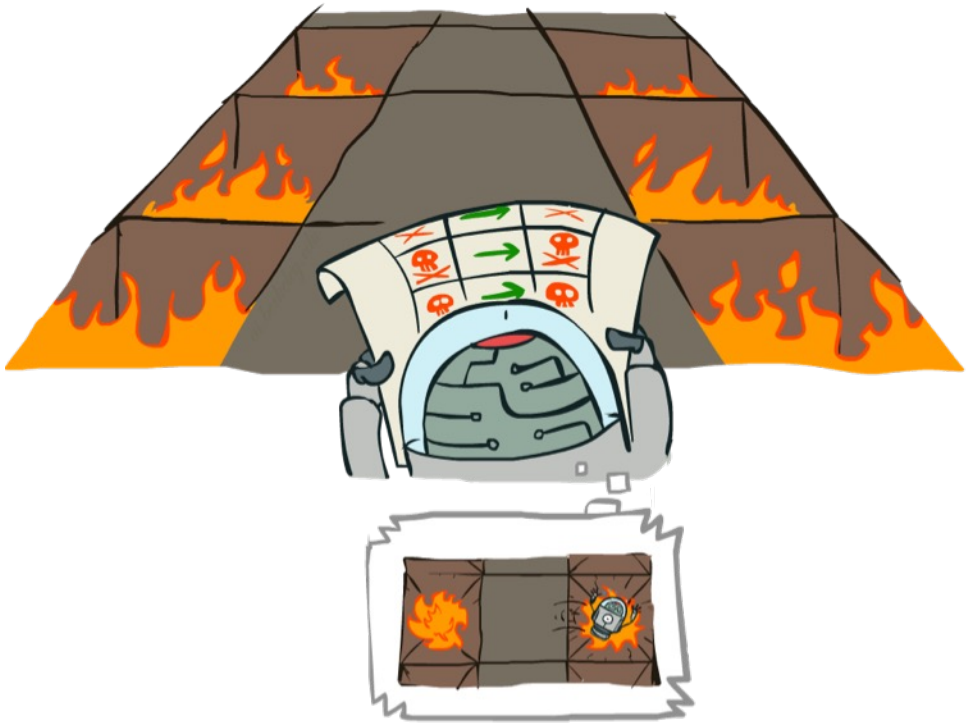
CONTEXT



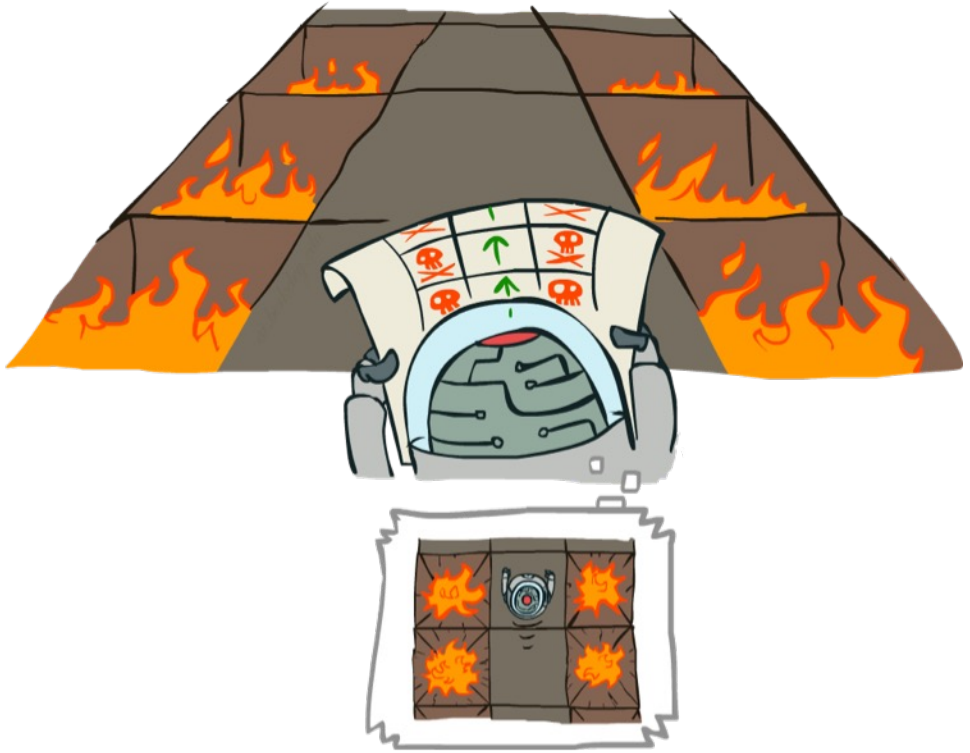
Evaluating Policies

EXAMPLE

π: *always go right*



π: *always go straight*

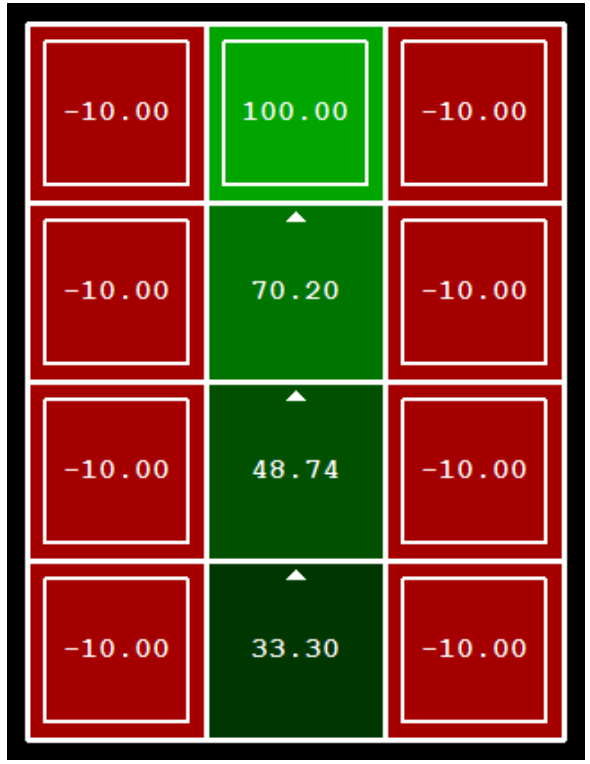


Evaluating Policies from Utilities

EXAMPLE

π: *always go right*

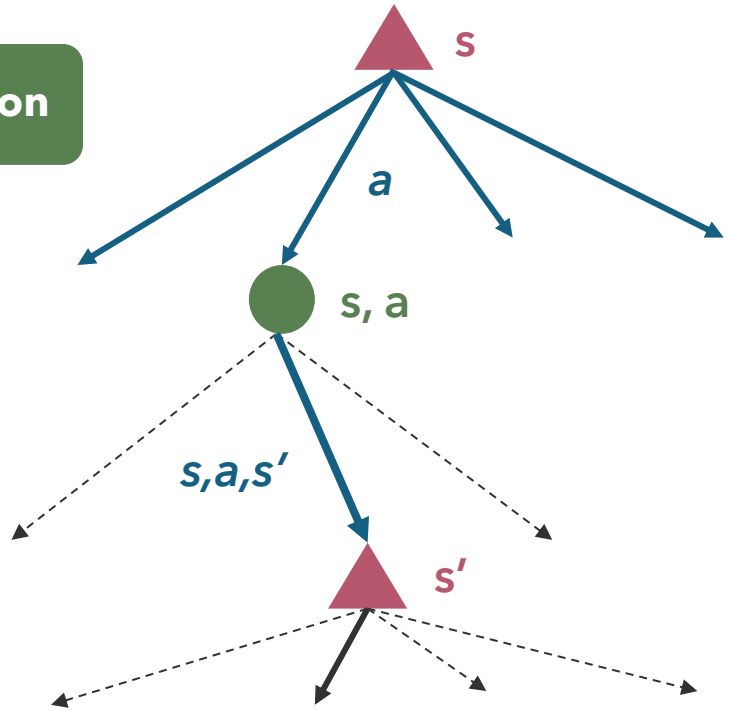
π: *always go straight*



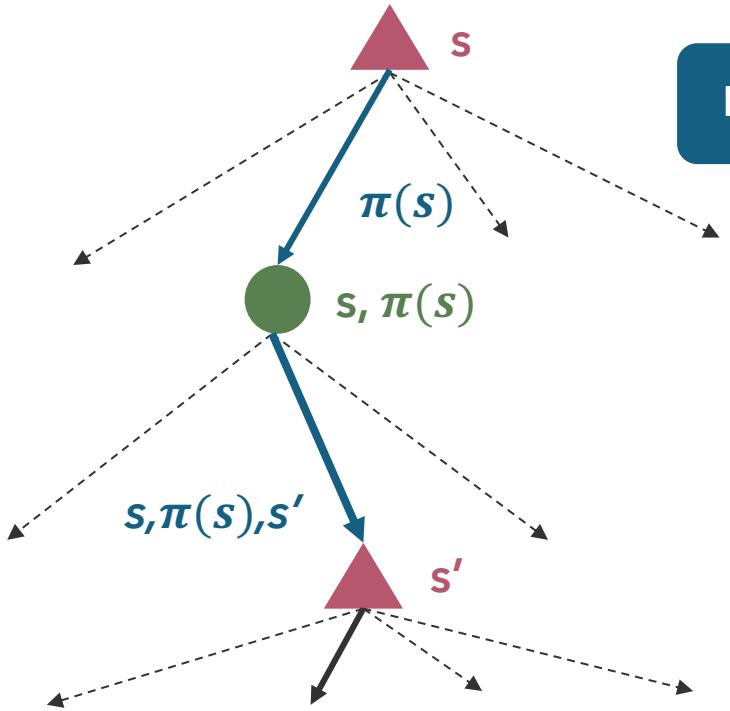
Fixed Policies

CONTEXT

Optimal Action



Fixed Policy π



- Expectimax over all actions is complicated
- If we fixed some policy $\pi(s)$ per state, the tree would be much simpler…

Fixed Policy Utilities

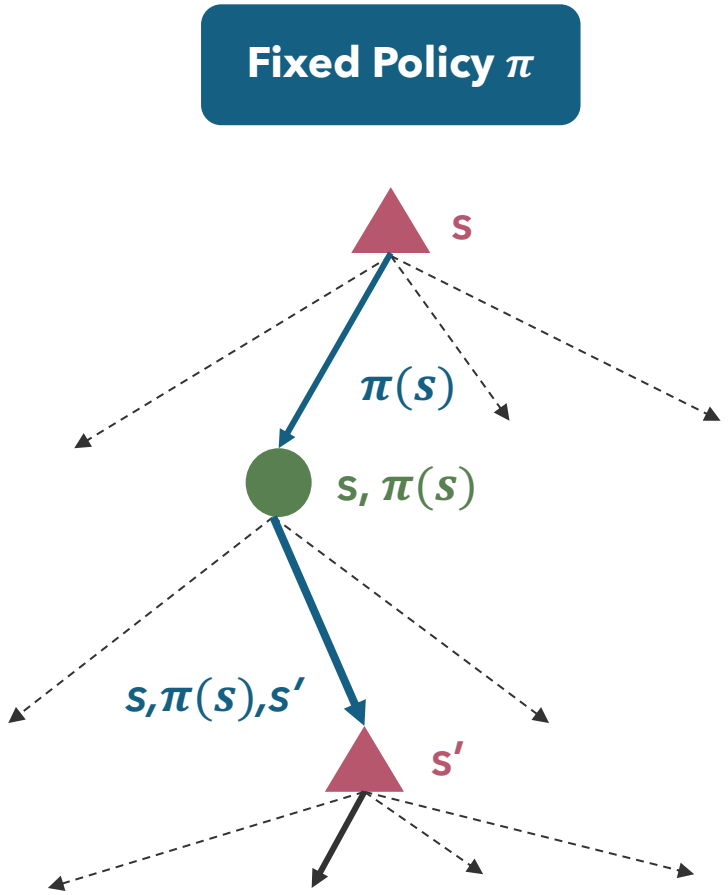
CONTEXT

- Define the utility of state s under fixed policy π
  - $U^{\pi}(s)$ is the expected total utility starting at s and following π
- Recursive relationship:

$$U_{i+1}(s) \leftarrow \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s' + \gamma U_i(s')]$$

- Approximation of system of linear equations:

$$U_i(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma U_i(s')]$$



Policy Evaluation

FOUNDATIONS

Calculating U^{π} for fixed policy π:

- **Option 1**: Recursive Bellman Updates

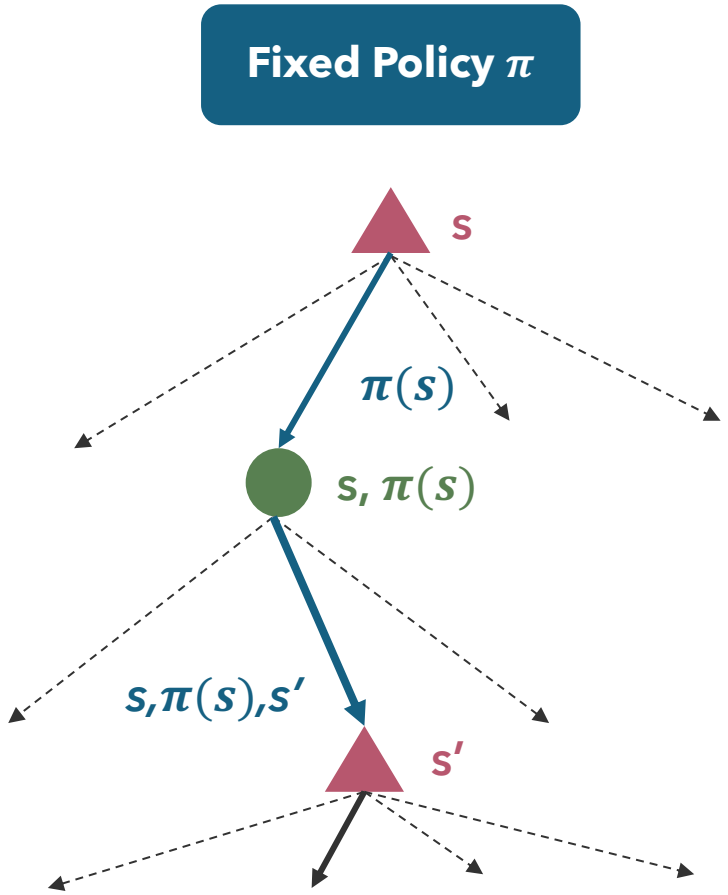
$$U_0^{\pi}(s) = 0$$

$$U_{i+1}^{\pi}(s) \leftarrow \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s' + \gamma U_i(s')]$$

Efficiency: $O(S^2)$ per iteration

- **Option 2**: Solve the Linear System of Equations

$$U_i(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma U_i(s')]$$



Policy Iteration Updates

FOUNDATIONS

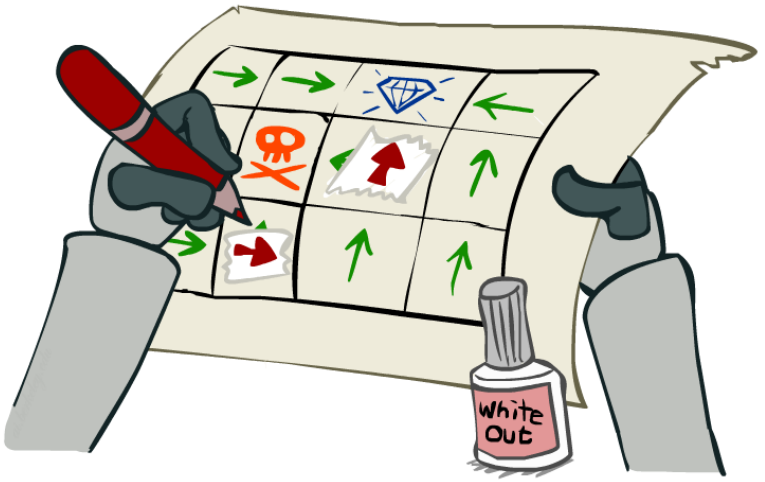
- **Policy Evaluation**
  - Iterate until values converge:

$$U_{i+1}(s) \leftarrow \sum_{s'} P(s'|s, \pi_i(s)[R(s, \pi_i(s), s') + \gamma U_i(s)]$$

  - Quicker: ***O(S²)***
- **Policy Improvement**
  - One-step lookahead:

$$\pi_{i+1}(s) = \text{argmax}_a \sum_{s'} P(s'|s, a)[R(s, a, s') + \gamma U^{\pi_i}(s')]$$

  - Slower: ***O(S²A)***



QUESTIONS

Questions?

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Comparison: Value vs. Policy Iteration

CONTEXT

Value Iteration

$$U_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U_k(s')]$$

- Every iteration updates the utilities and (implicitly) the policy
- Doesn't track the policy, but taking max over actions implicitly re-computes it
- Therefore, value iteration is expensive

Policy Iteration

$$U_{i+1}(s) \leftarrow \sum_{s'} P(s'|s,\pi_i(s)[R(s,\pi_i(s),s') + \gamma U_i(s)]$$

$$\pi_{i+1}(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma U^{\pi_i}(s')]$$

- Update the utilities using a fixed policy, saving computational time
- After policy is evaluated, update policy using new utilities

that's it for today!

SUMMARY

- Value iteration computes values by taking the max over possible actions
- Policy iteration considers a fixed policy, then improves the policy using calculated values

UPCOMING

- Reinforcement Learning: learning by doing!

REMINDERS

- Complete **Practice Problem 7**
- Do **Project 1** (due 7.9)