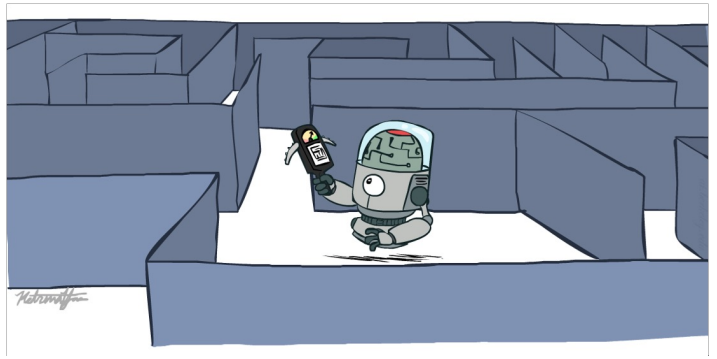
Informed Search

CSE 473: Introduction to Artificial Intelligence



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acknowledgements

Welcome!
Place in Washington you want to visit?
sli.do **#cse473**

Administrivia

ANNOUNCEMENTS

- For each project, make sure to fill out the **AI usage reflection** (cs.uw.edu/473/projects)
- Past quizzes, exams available on course website resources tab

ASSIGNMENTS

- **Homework 1 due** Friday (7.3)
- **Project 1 due** in next Thursday (7.9)

Search

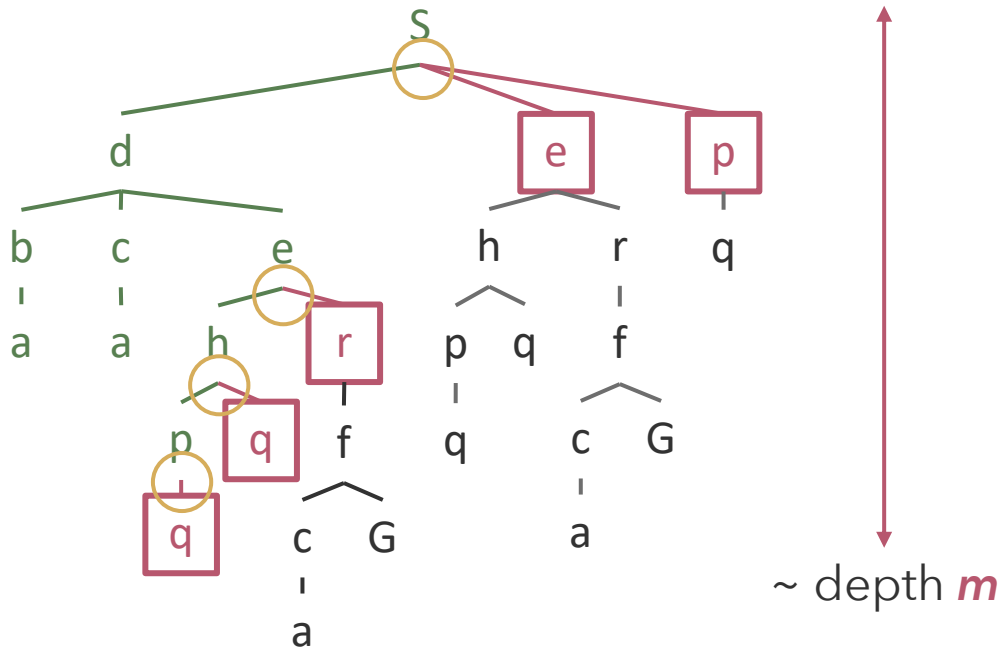
FOUNDATIONS

DFS vs. BFS

FOUNDATIONS

Depth-First Search

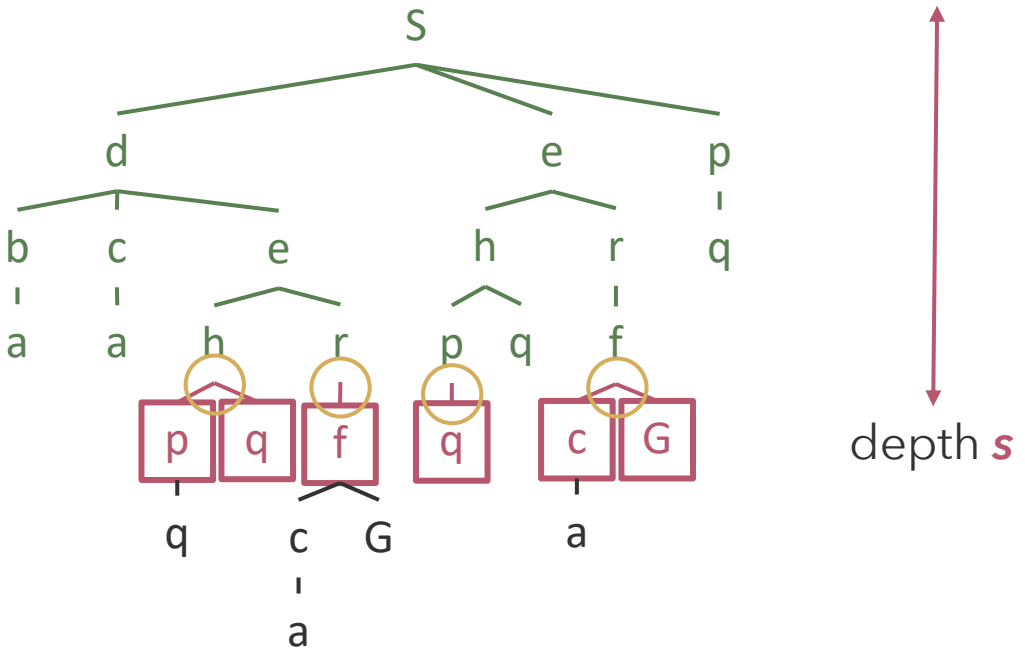
Fringe grows *linearly* with depth



Space Complexity: ***O(bm)***

Breadth-First Search

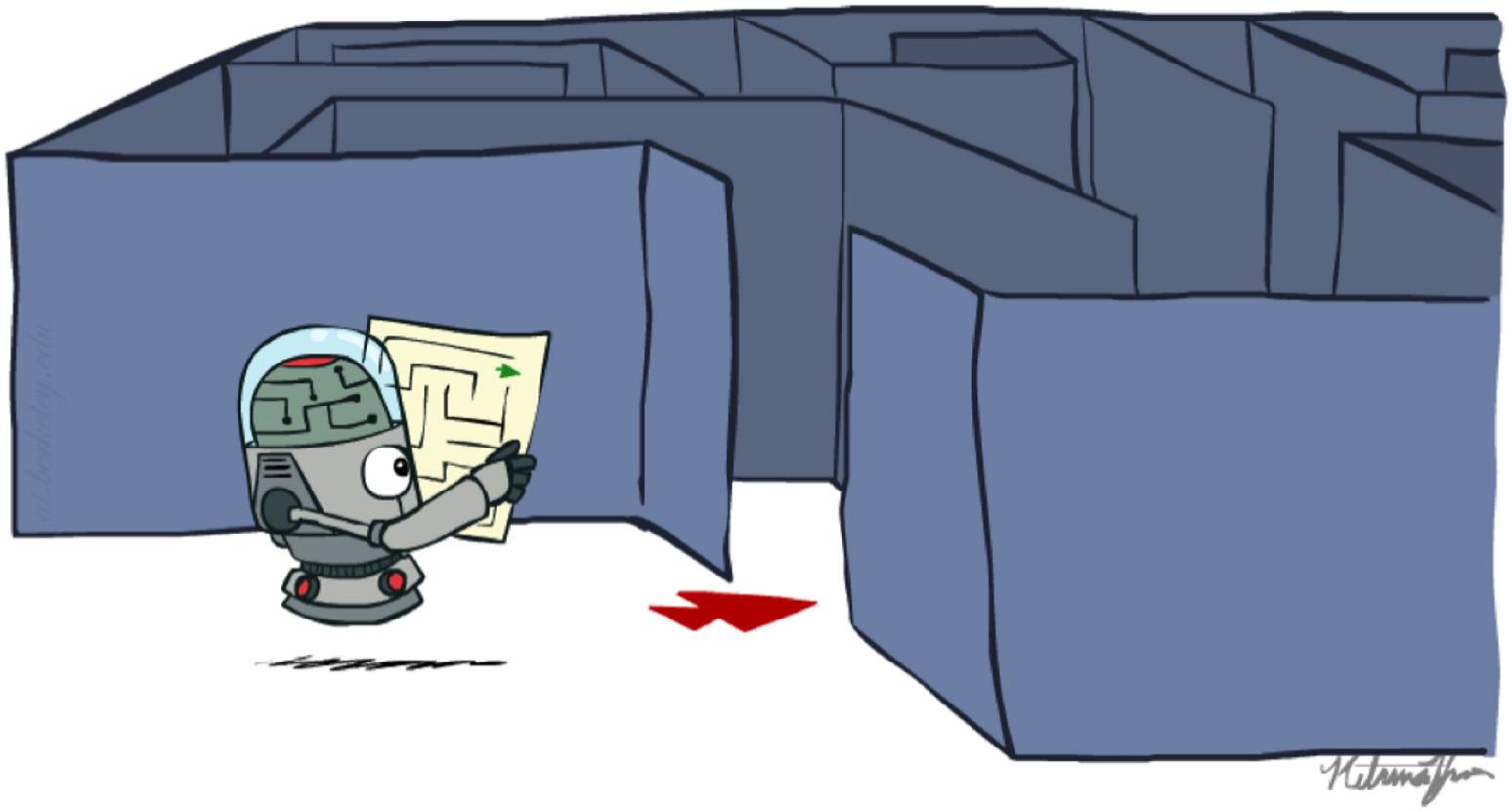
Fringe grows *exponentially* with depth



Space Complexity: ***O(b^s)***

Pacman Wayfinding

EXAMPLE

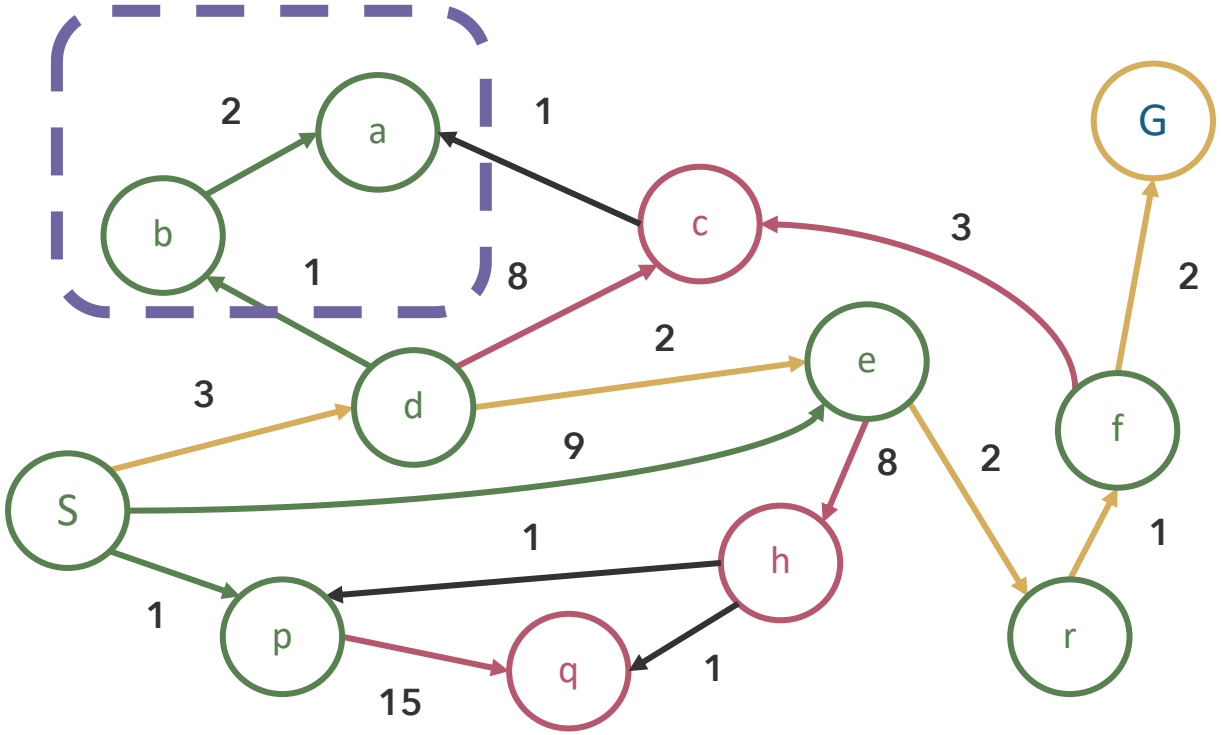


What's Next?

CONTEXT

dead end

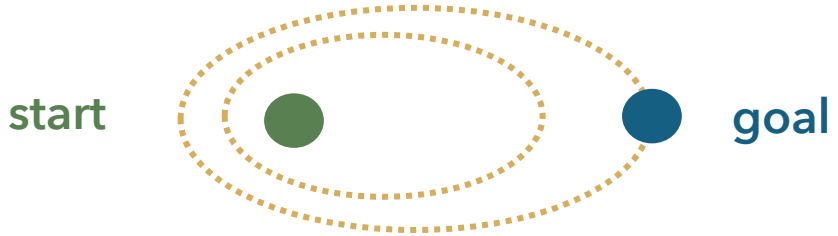
can we avoid it?



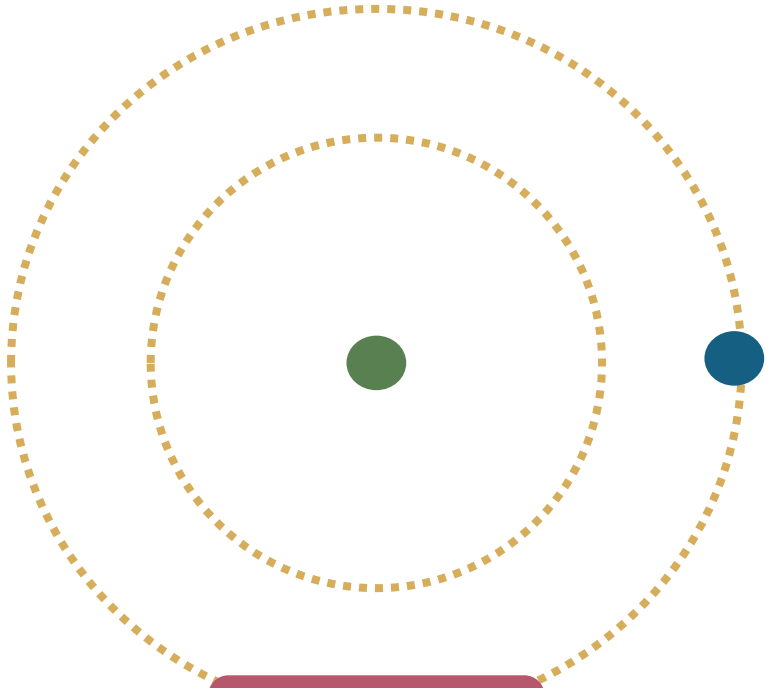
Directed Search

CONTEXT

Guide search in the ***direction*** of the goal, not in all directions



Informed

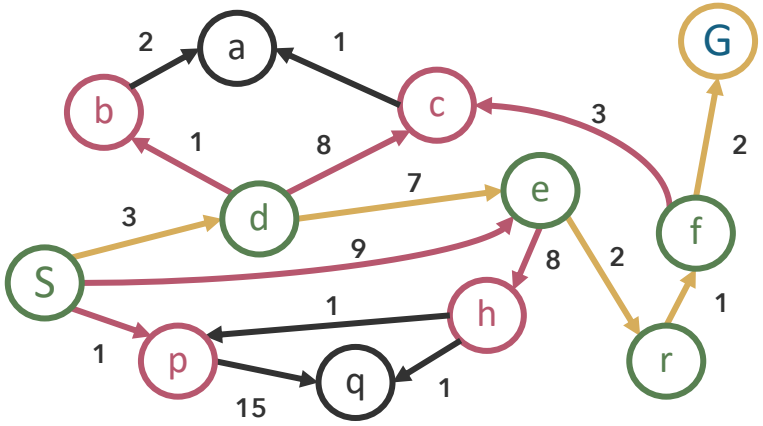


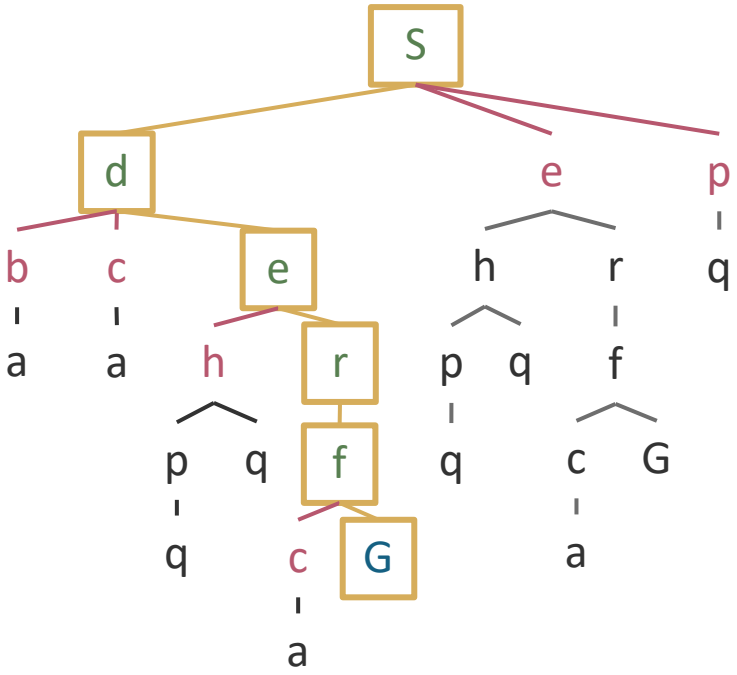
Uninformed

Greedy Search

FOUNDATIONS

- Strategy
  - Expand the node you *think* is closest to the goal
- Implementation
  - Use **heuristics** to estimate forward cost



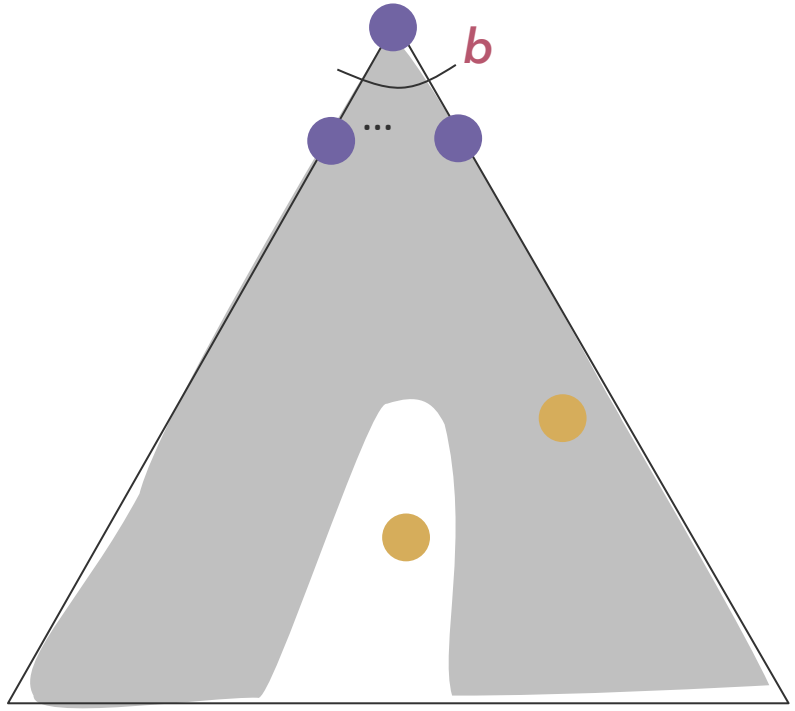


more directed, but not optimal

Properties of Greedy Search

DEFINITION

- Strategy
  - Expand the node you *think* is closest to the goal
  - Use **heuristics** to estimate forward cost
- Common Case
  - Expand straight to the (suboptimal) goal
- Worst Case
  - Search around optimal goal
  - Like a poorly-guided DFS



QUESTIONS

Questions?

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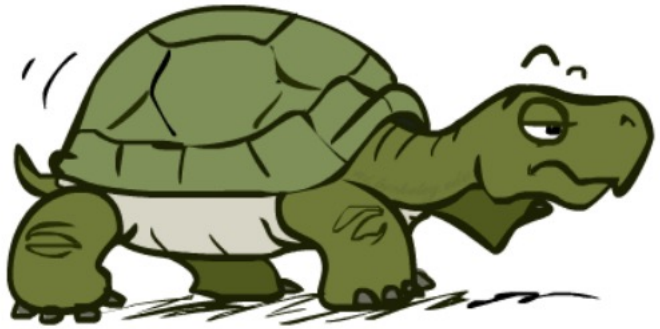
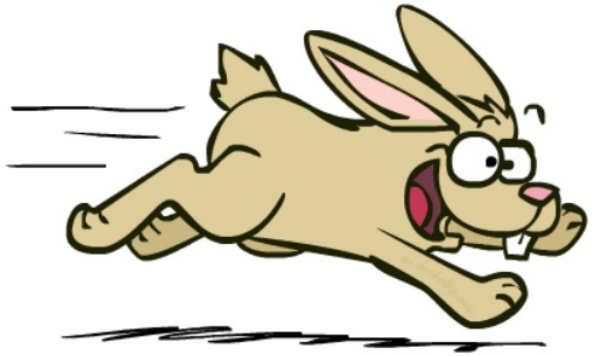
We're getting closer, though…

Combining UCS and Greedy Search

CONTEXT





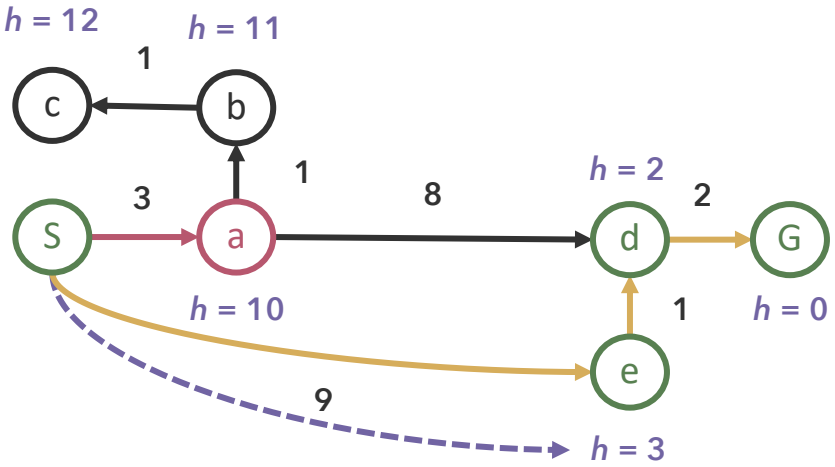
Uniform Cost Search

Greedy Search

A* Search

A* Search

FOUNDATIONS



- Goal
  - Avoid unnecessary searching (and backtracking)
- Strategy
  - Expand the node most likely to be on the **optimal path**
  - Combine **UCS** and **Greedy Search**
- Implementation
  - ***g(n)*** = cost from root to n
  - ***h(n)*** = heuristic estimate of cost from n to goal
  - Use **priority queue** with ***f(n) = g(n) + h(n)***

Key: good **heuristic**

Heuristics

DEFINITION

- We want to know the **forward cost *h*(n)*** from ***n*** to the goal
  - If we know (and can quickly calculate) h*(n), our search is straightforward
  - …but that doesn't happen (often) in the 'real world'…
- Instead, let's model a **simplified forward cost**
- A **heuristic** is a function that estimates proximity to goal

$$h(n) \approx h^*(n)$$

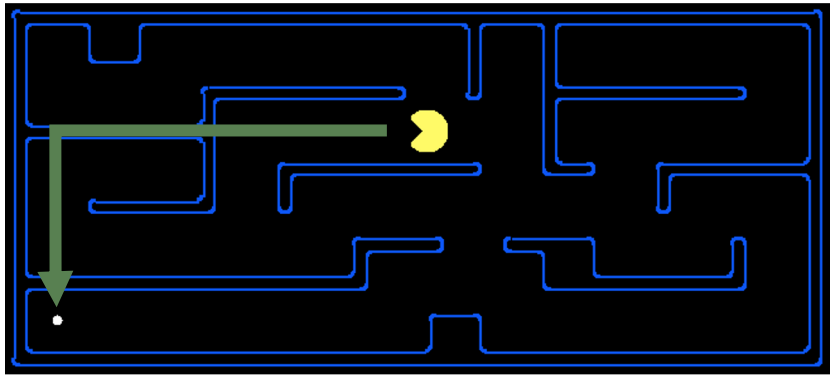
  - Designed for the specific search problem
  - Easy to calculate
  - An *underestimate* of the true forward cost

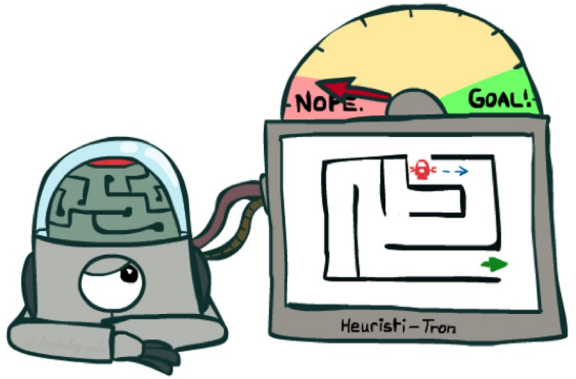
Pacman Pathing

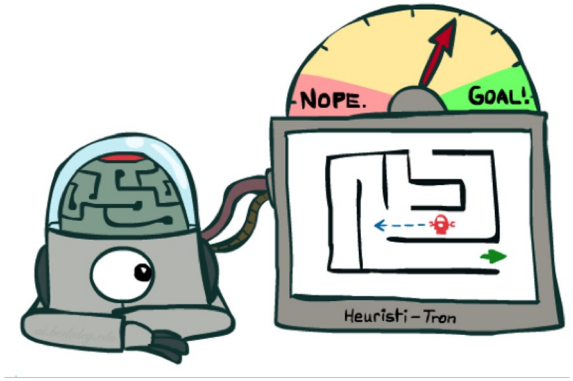
EXAMPLE

For Pacman maze navigation, **Manhattan distance** is one possible heuristic

- $\text{manhattan}(n) = |\Delta x| + |\Delta y|$
- Manhattan distance cannot overestimate the cost to get to the goal (walls!)

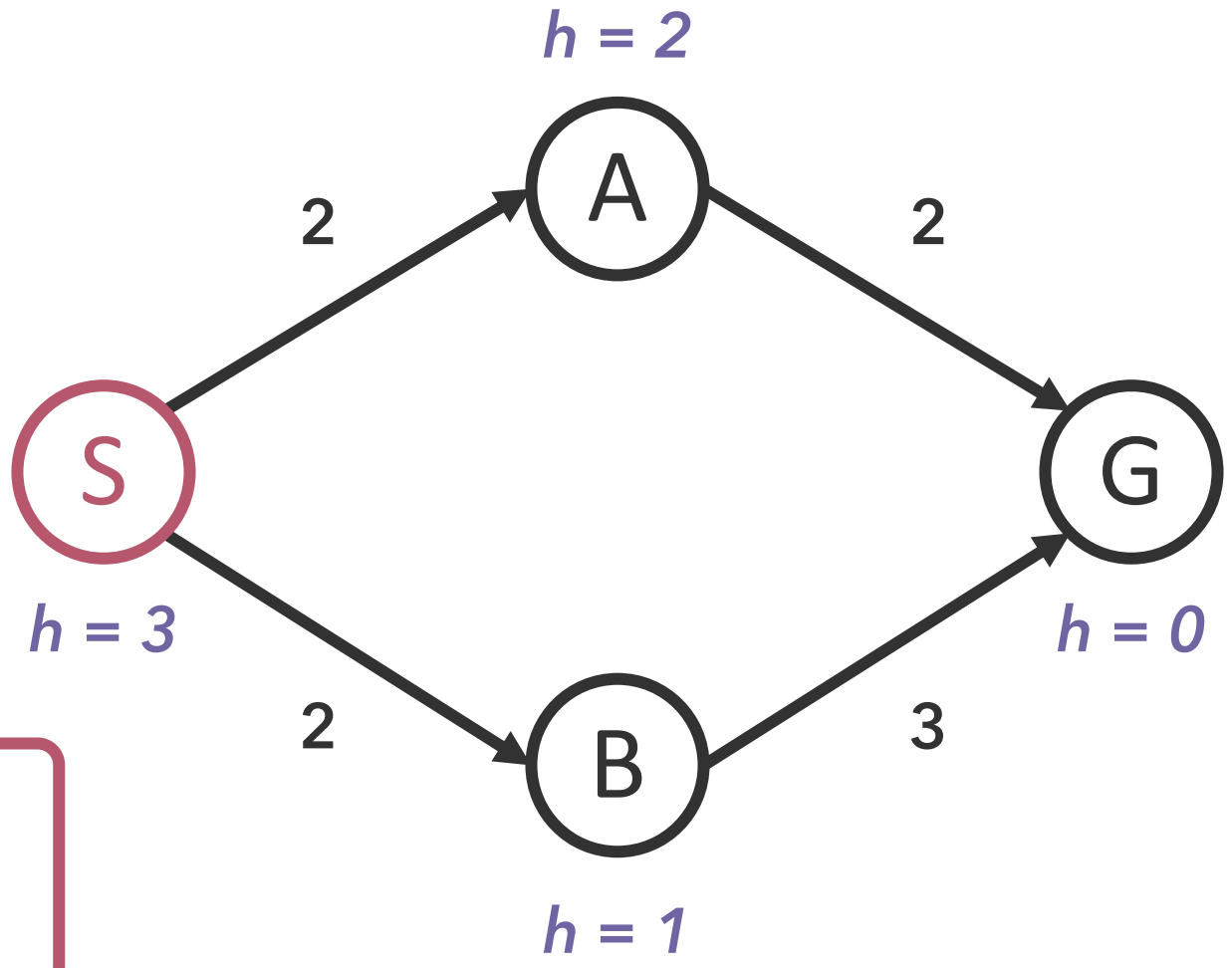






When to Dequeue Goal States? (1)

FOUNDATIONS

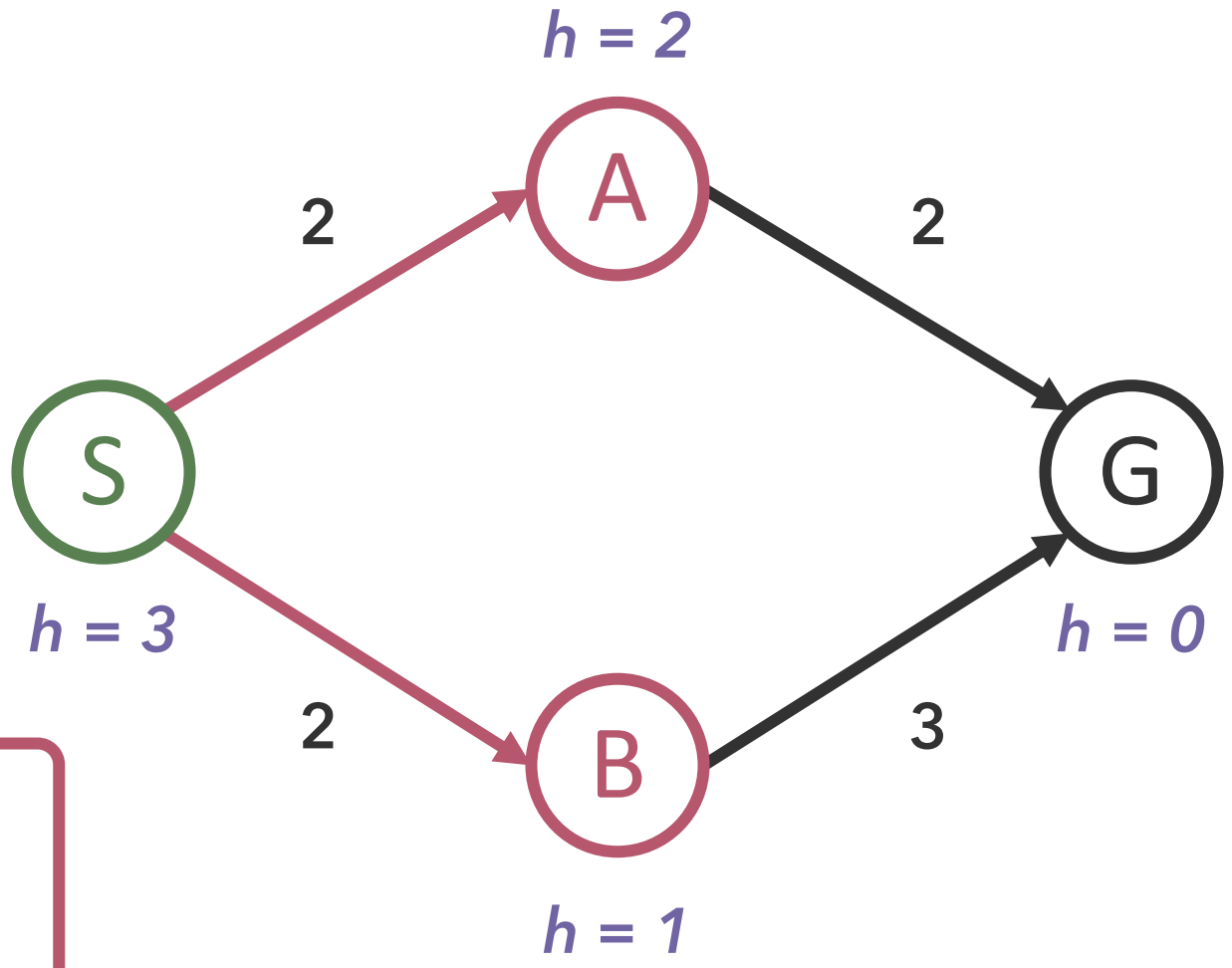


Fringe

S (0 + 3)

When to Dequeue Goal States? (2)

FOUNDATIONS

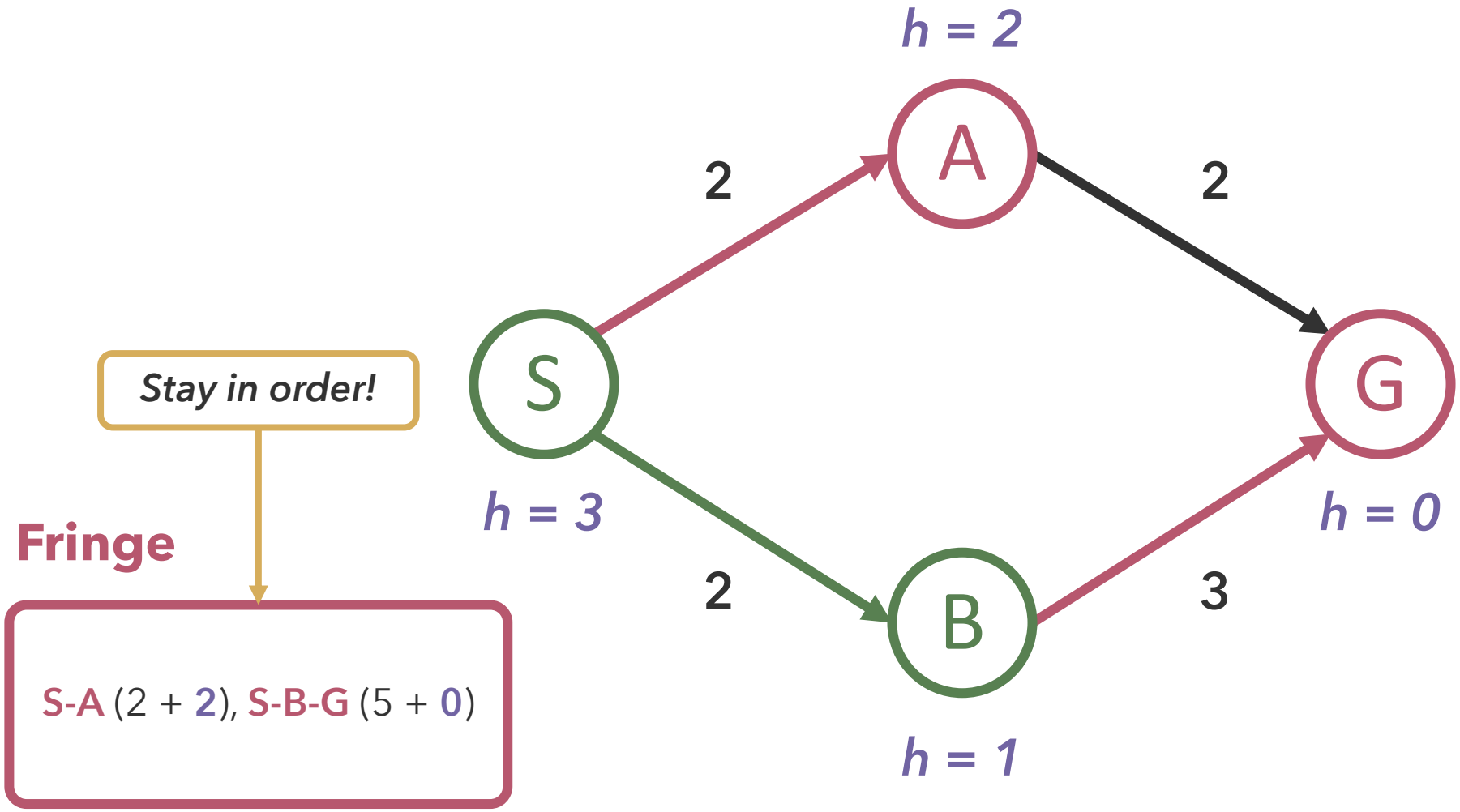


Fringe

S-A (2 + 2), S-B (2 + 1)

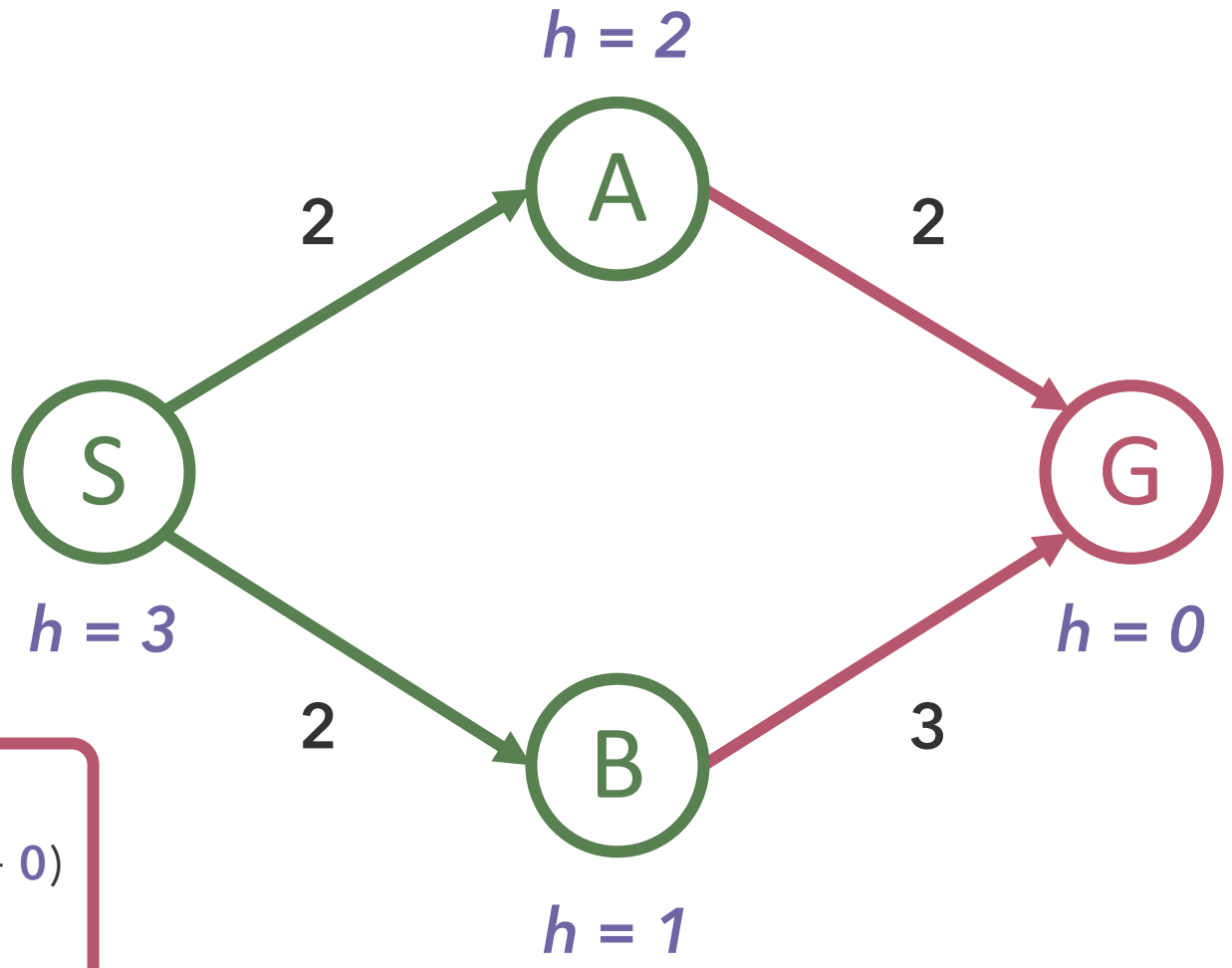
When to Dequeue Goal States? (3)

FOUNDATIONS



When to Dequeue Goal States? (4)

FOUNDATIONS

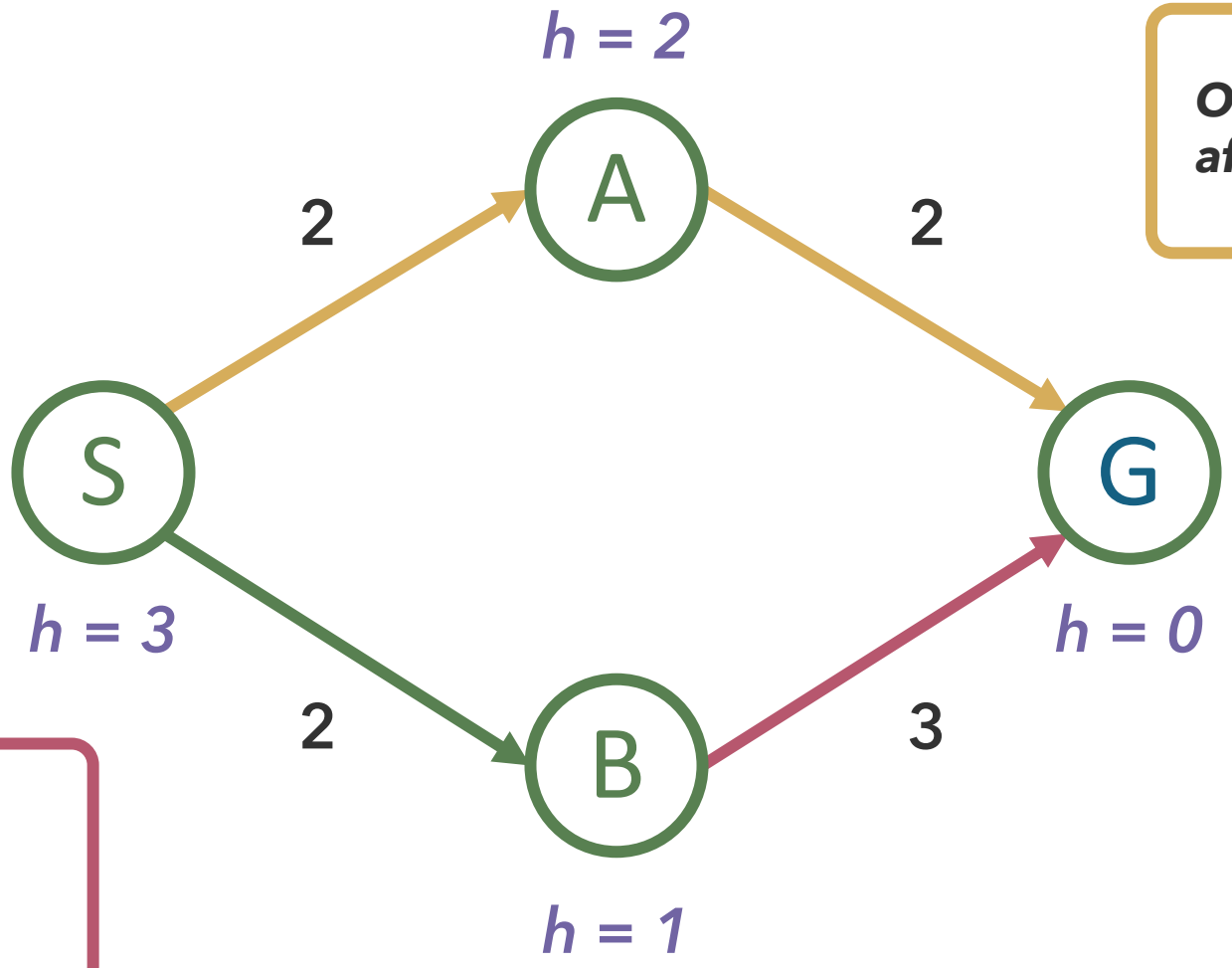


Fringe

S-B-G (5 + 0), S-A-G (4 + 0)

When to Dequeue Goal States? (5)

FOUNDATIONS



Only perform a goal test after <u>dequeuing</u> a node!

Fringe

S-B-G (5 + **0**)

Questions?

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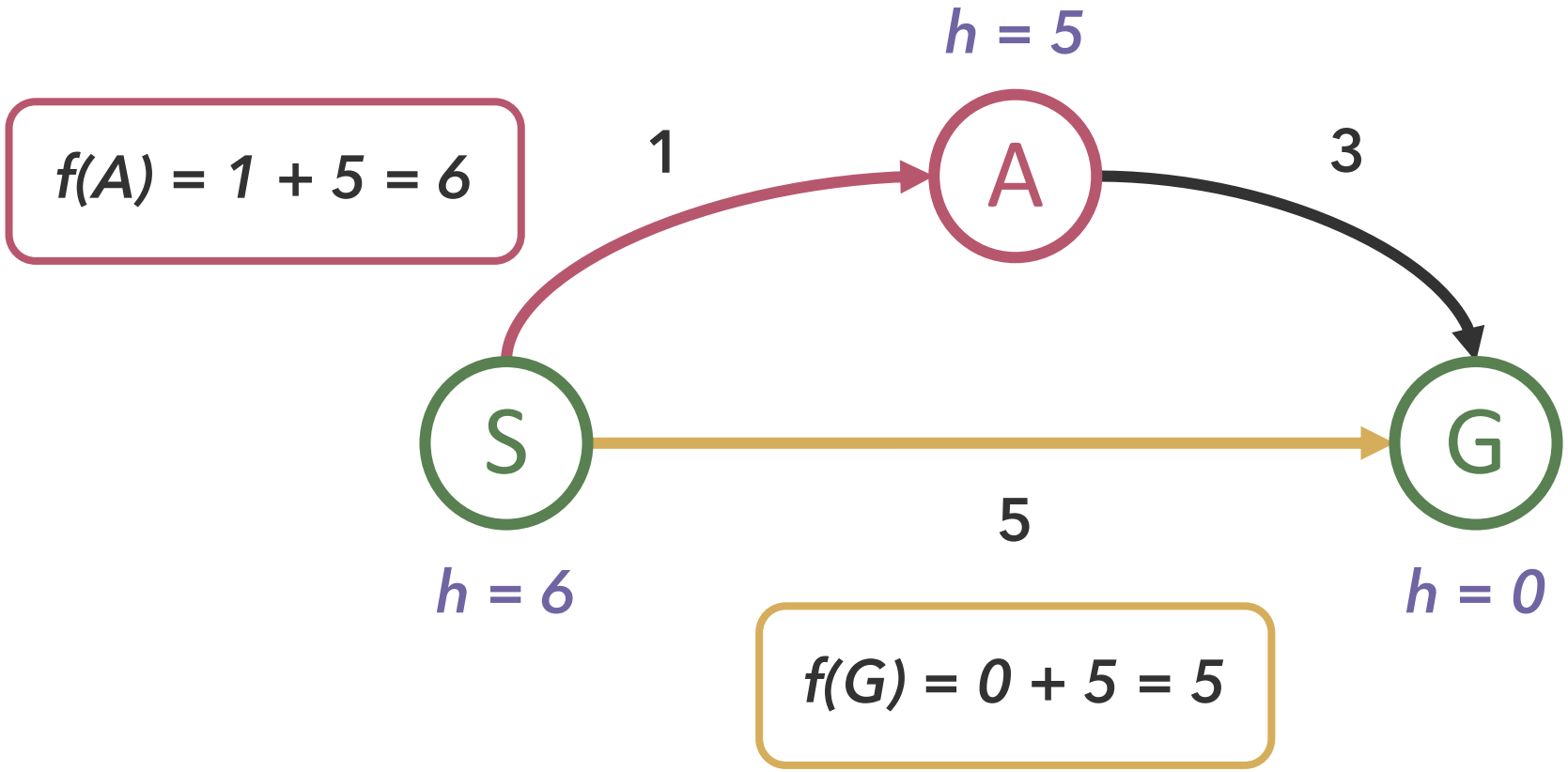
Horrible Heuristics

CONTEXT



What went wrong?

- Heuristic made bad path seem better than good path

➔ Need $h(n) \leq h^*(n)$

Admissibility

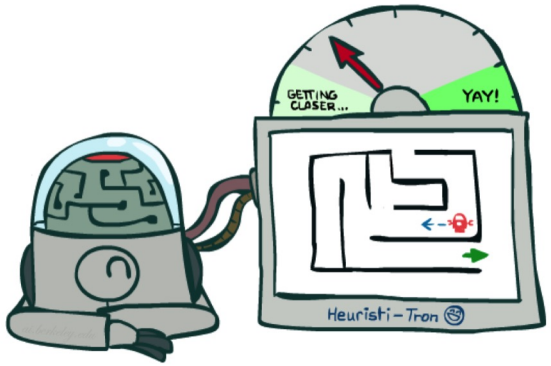
DEFINITION

Heuristics must be *underestimates* of the true cost for A* to be optimal

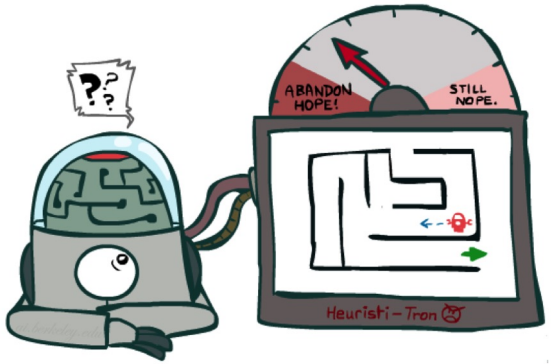
- An **admissible heuristic** is optimistic about cost to goal

$$0 \leq h(n) \leq h^*(n)$$

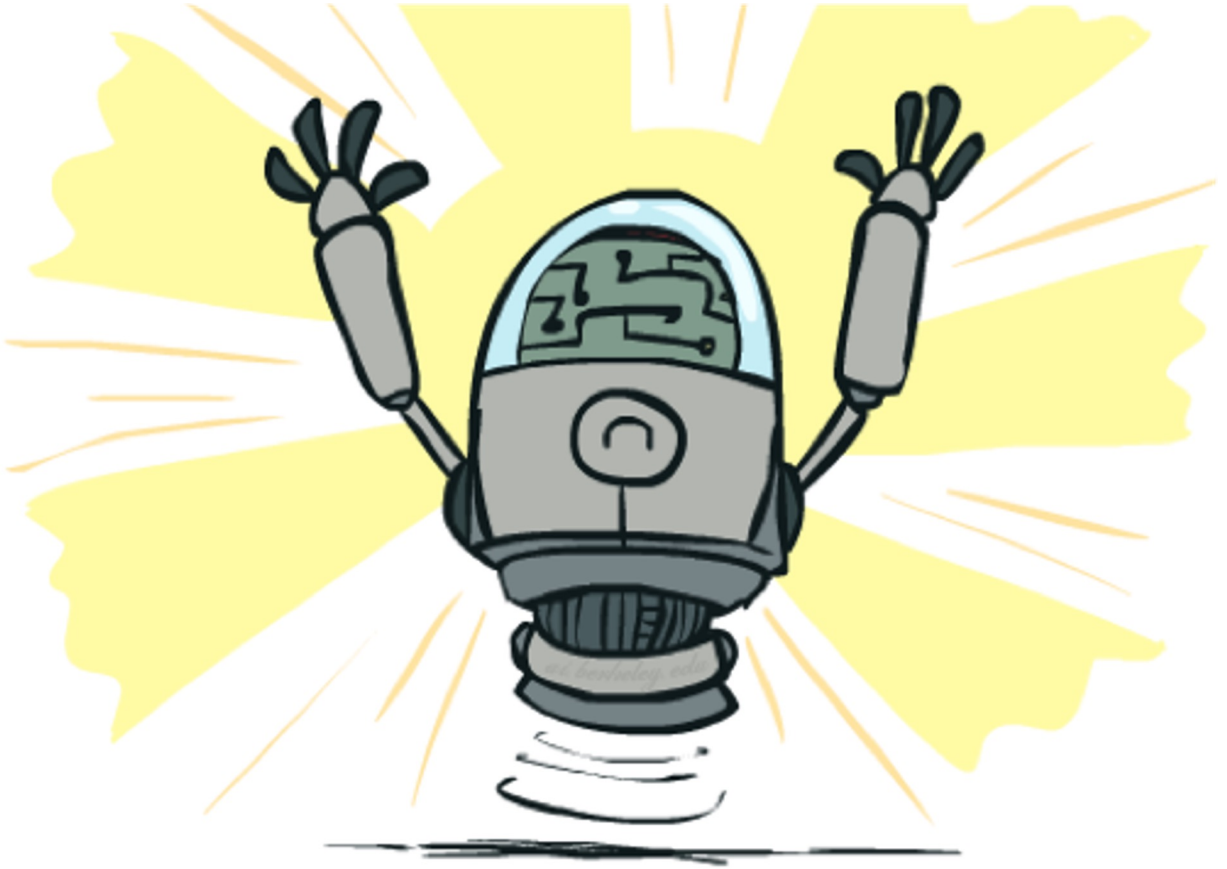
$h^*(n)$ is the **true cost** from *n* to the goal



***Admissible* heuristics** slow down bad paths but never outweigh the true cost



***Inadmissible* heuristics** trap good plans on the fringe, breaking optimality

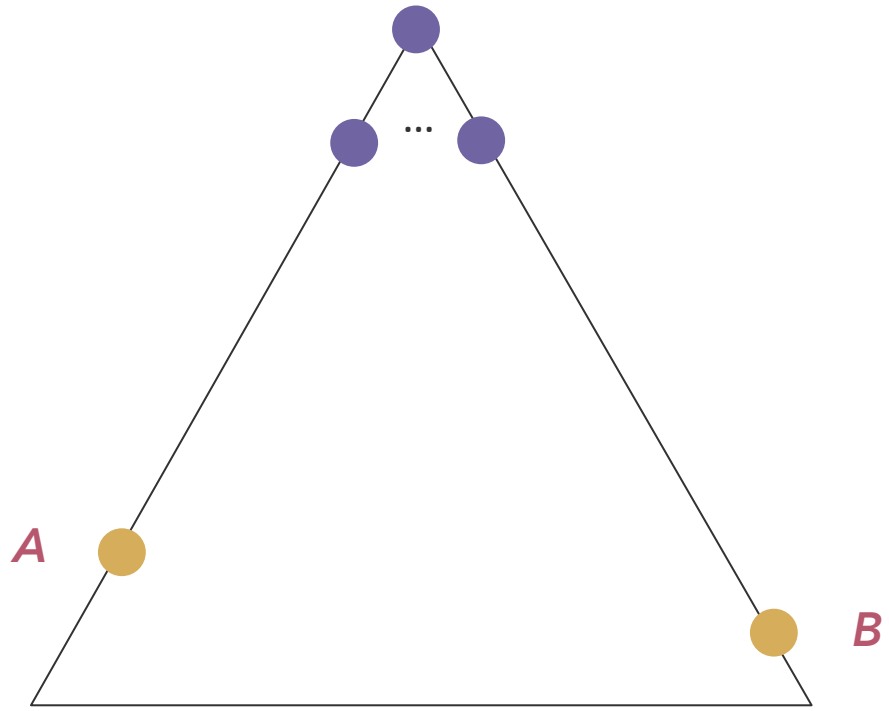


Claim: A* Search is Optimal!

Optimality of A* Tree Search

PROOF

- Assume
  - *A* is an optimal goal node
  - *B* is a suboptimal goal node
  - *h* is admissible
- Claim
  - *A* will be chosen for expansion before *B*

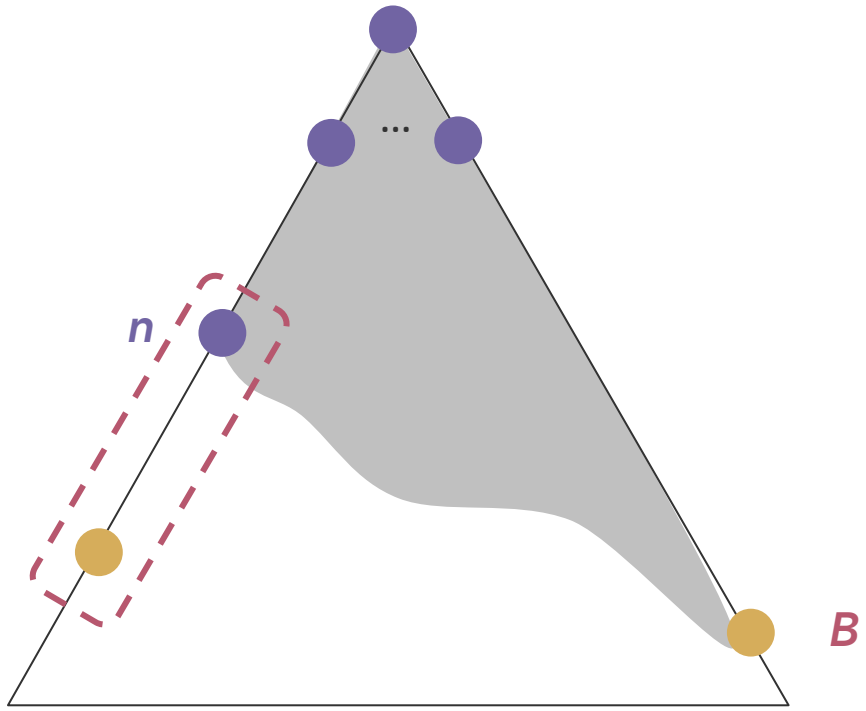


Optimality of A* Tree Search (1)

PROOF

- Proof
  - Let ***B*** be on the fringe
  - Some ancestor ***n*** of ***A*** is on the fringe, too
    - (maybe ***A*** itself)
- Claim
  - ***n*** will be expanded before ***B***
- Proof
  - $f(n) \leq f(A)$

$f(n) = g(n) + h(n)$
$f(n) \leq g(A)$ because ***h*** is admissible
$g(A) = f(A)$
$\rightarrow f(n) \leq f(A)$

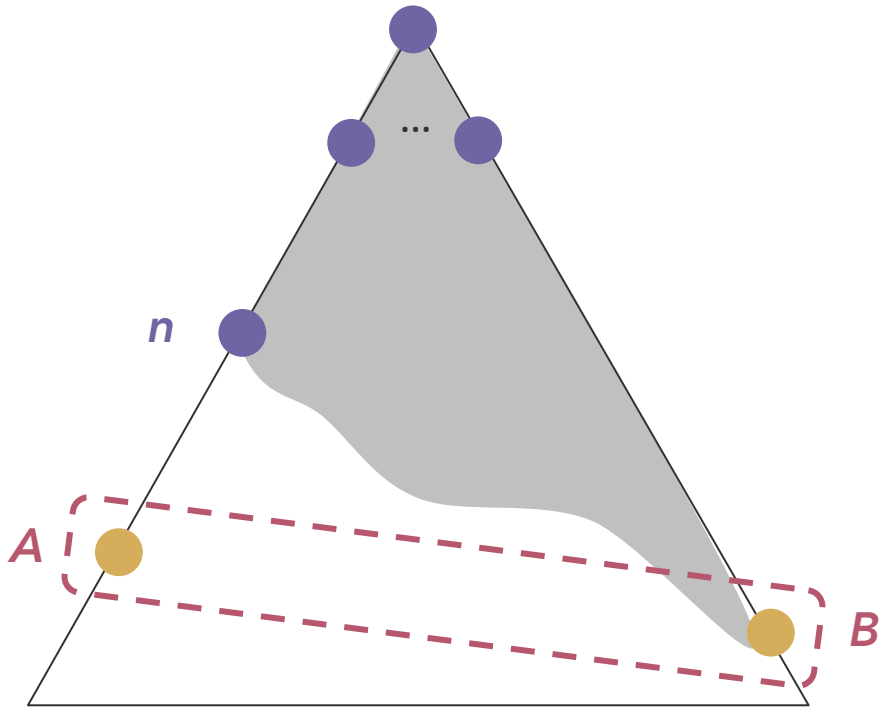


Optimality of A* Tree Search (2)

PROOF

- Proof
  - Let ***B*** be on the fringe
  - Some ancestor ***n*** of ***A*** is on the fringe, too
    - (maybe ***A*** itself)
- Claim
  - ***n*** will be expanded before ***B***
- Proof
  - $f(n) \leq f(A)$
  - $f(A) < f(B)$

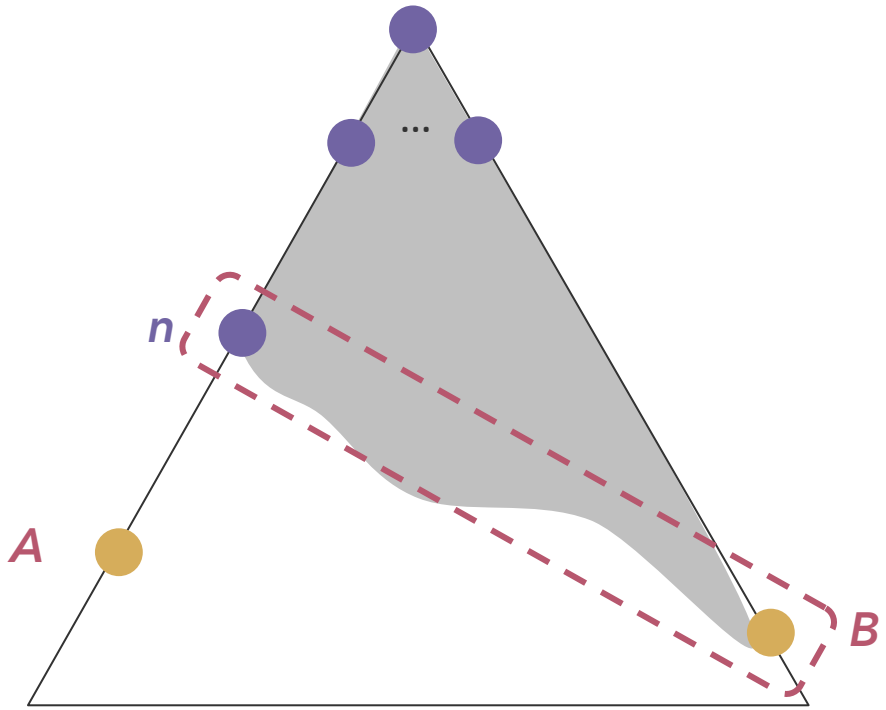
$g(A) < g(B)$ ***B*** is suboptimal
$\rightarrow f(A) < f(B)$ ***h*** = 0 at goal



Optimality of A* Tree Search (3)

PROOF

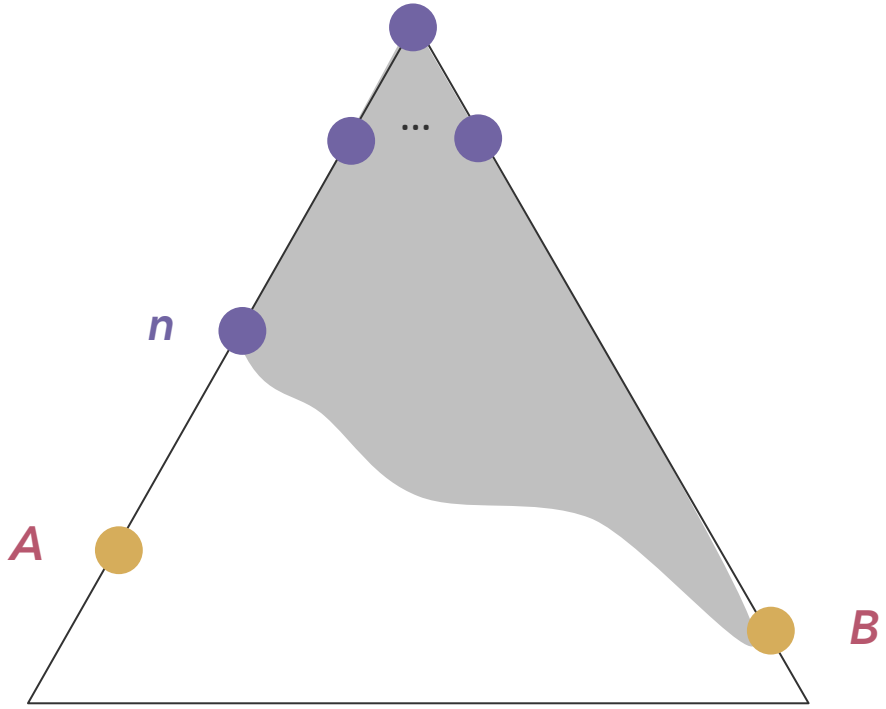
- Proof
  - Let ***B*** be on the fringe
  - Some ancestor ***n*** of ***A*** is on the fringe, too
    - (maybe ***A*** itself)
- Claim
  - ***n*** will be expanded before ***B***
- Proof
  - $f(n) \leq f(A)$
  - $f(A) \leq f(B)$
  - $\rightarrow f(n) \leq f(A) < f(B)$



Optimality of A* Tree Search (4)

PROOF

- Proof
  - Let ***B*** be on the fringe
  - Some ancestor ***n*** of ***A*** is on the fringe, too
    - (maybe ***A*** itself)
- Proof
  - $\rightarrow f(n) \leq f(A) < f(B)$
- Conclusion
  - All ancestors of ***A*** are expanded before ***B***
  - Therefore ***A*** itself is expanded before ***B***
  - Therefore **A* tree search is optimal**



Questions?

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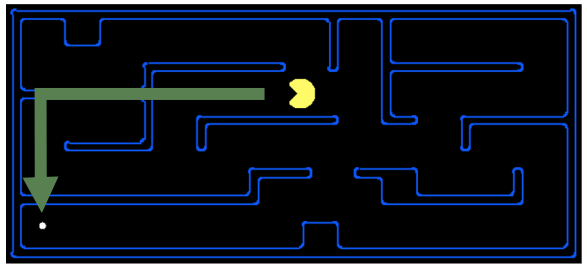
Creating Heuristics

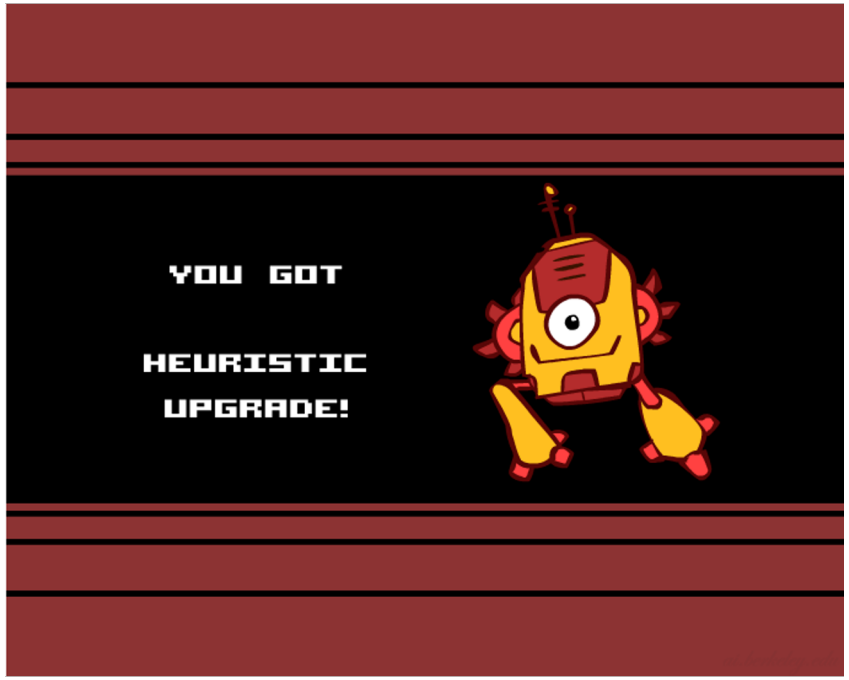
CONTEXT

- Most of the work in solving hard search problems involves finding **admissible heuristics**

- Often, admissible heuristics are solutions to *relaxed problems* where new actions are available

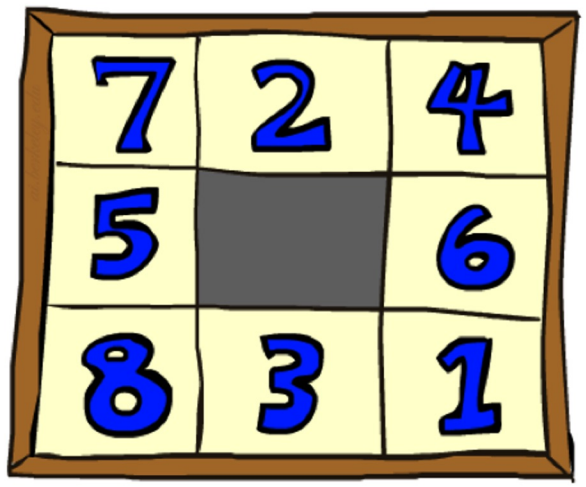


- Inadmissible heuristics can be useful too

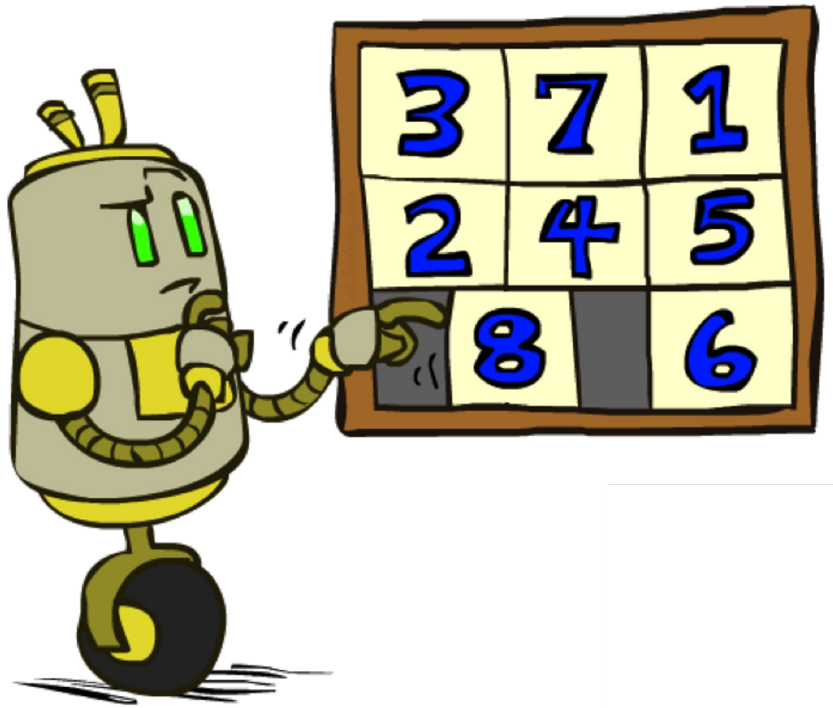


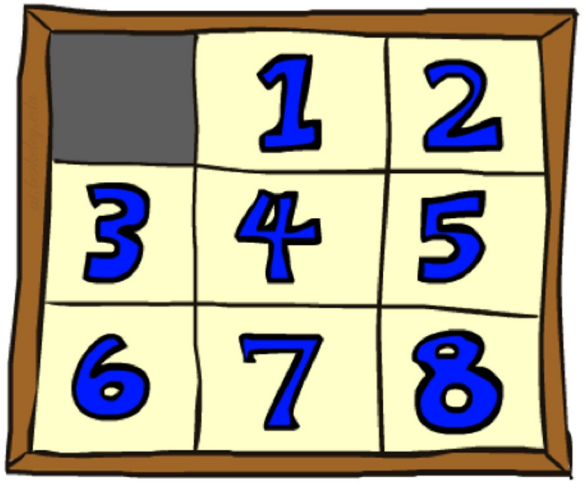
8 Puzzle

EXAMPLE



Start State



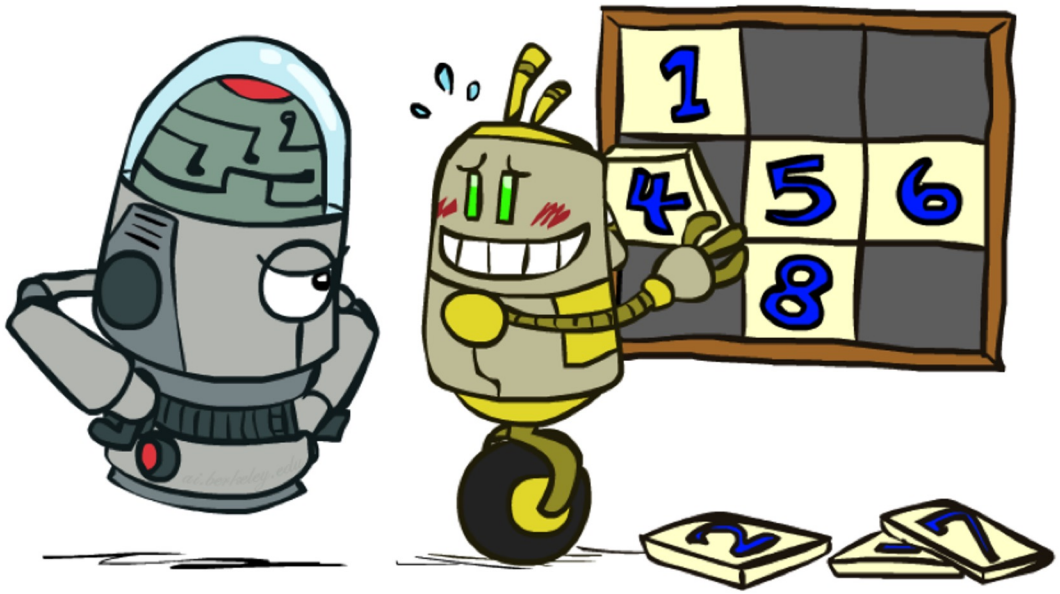


Goal State

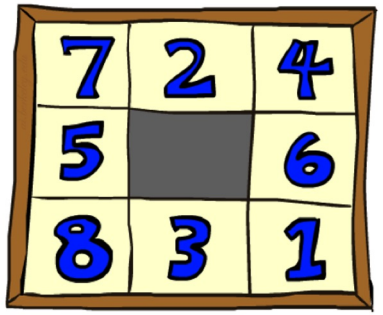
8 Puzzle: Heuristic

EXAMPLE

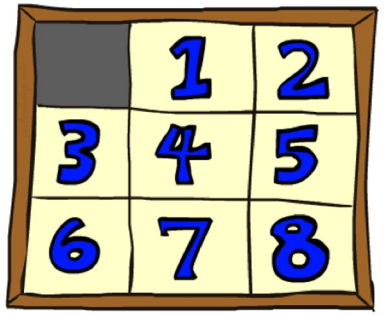
- **Heuristic**: number of misplaced tiles
  - *Admissible?* Yes, because putting tiles in correct position requires at least as many moves



Start State



Goal State

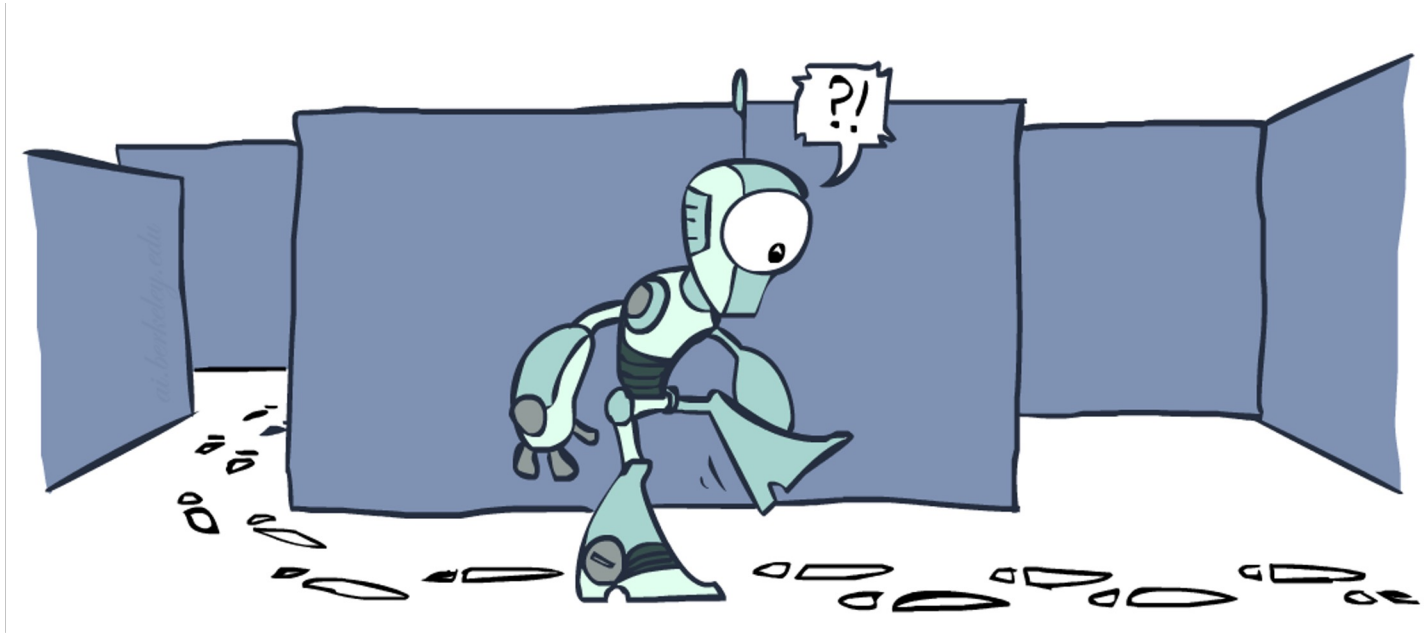


General Graph Search

ALGORITHM

```
function GRAPH-SEARCH(problem, strategy) returns solution or failure
  closed ← empty set
  initialize fringe using initial state of problem
  loop do
        if fringe empty then return failure
        node ← remove node from fringe
        if node is goal state then return corresponding solution
        if node not in closed then
                expand node and add resulting nodes to fringe
                add node to closed
  end
```

Is A Graph Search Optimal?*



Consistency

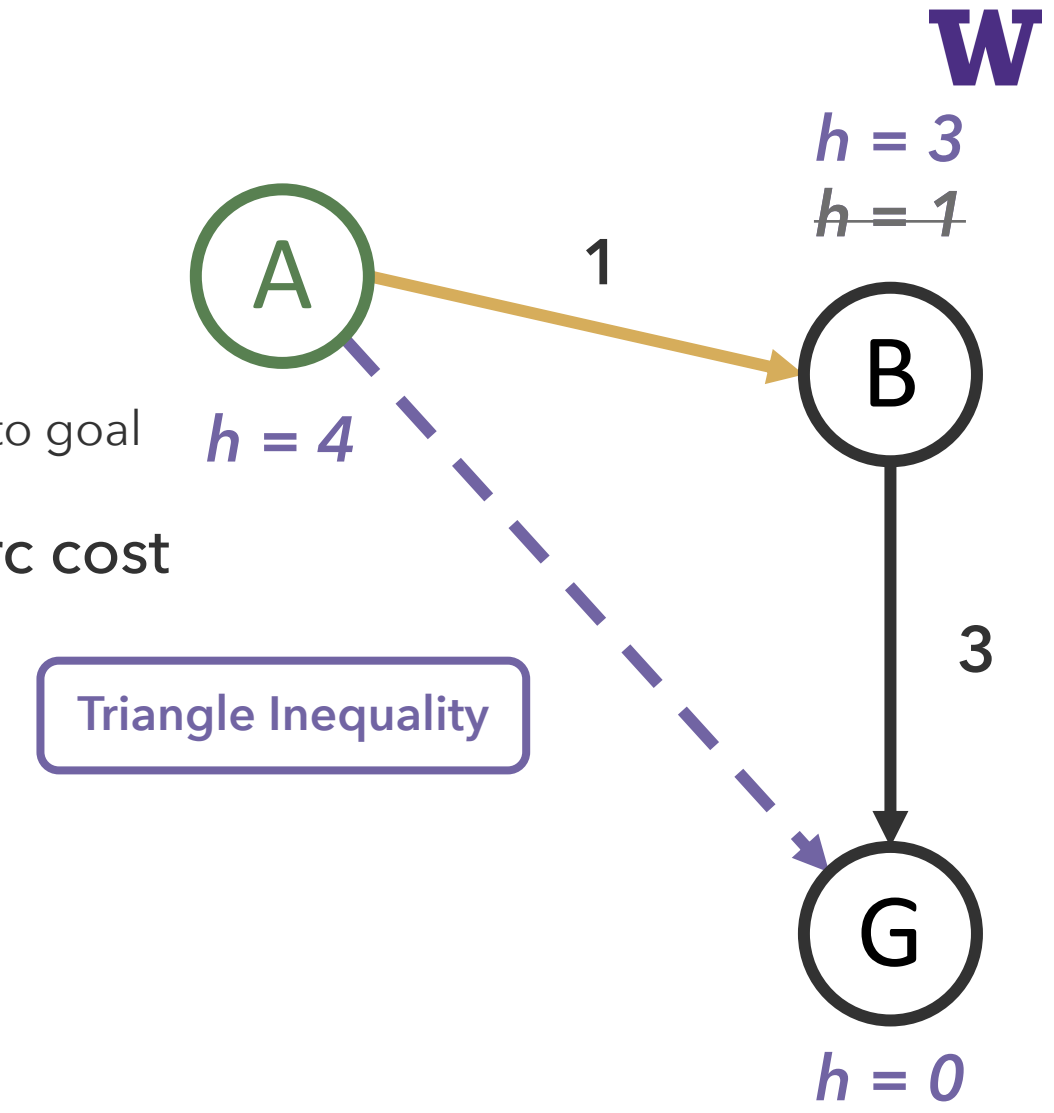
DEFINITION

- We want heuristic costs ≤ actual costs
  - **Admissibility** guarantees heuristic cost ≤ actual cost to goal
- We also want heuristic "arc" cost ≤ actual arc cost
  - **Consistency** guarantees this

$$h(A) - h(B) \leq \text{cost}(A, B)$$

$$h(A) \leq \text{cost}(A, C) + h(B)$$

- Consequences
  - ***f*** value along a path never decreases
  - A* graph search is optimal
  - **Consistency implies admissibility**



Questions?

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that's it for today!

SUMMARY

- Informing the direction of our search with heuristics
- Consequences of heuristic admissibility and consistency

UPCOMING

- Multi-Agent Search

REMINDERS

- Complete **Practice Problem 4**
- Do **Homework 1** (due 7.3)
- Do **Project 1** (due 7.9)