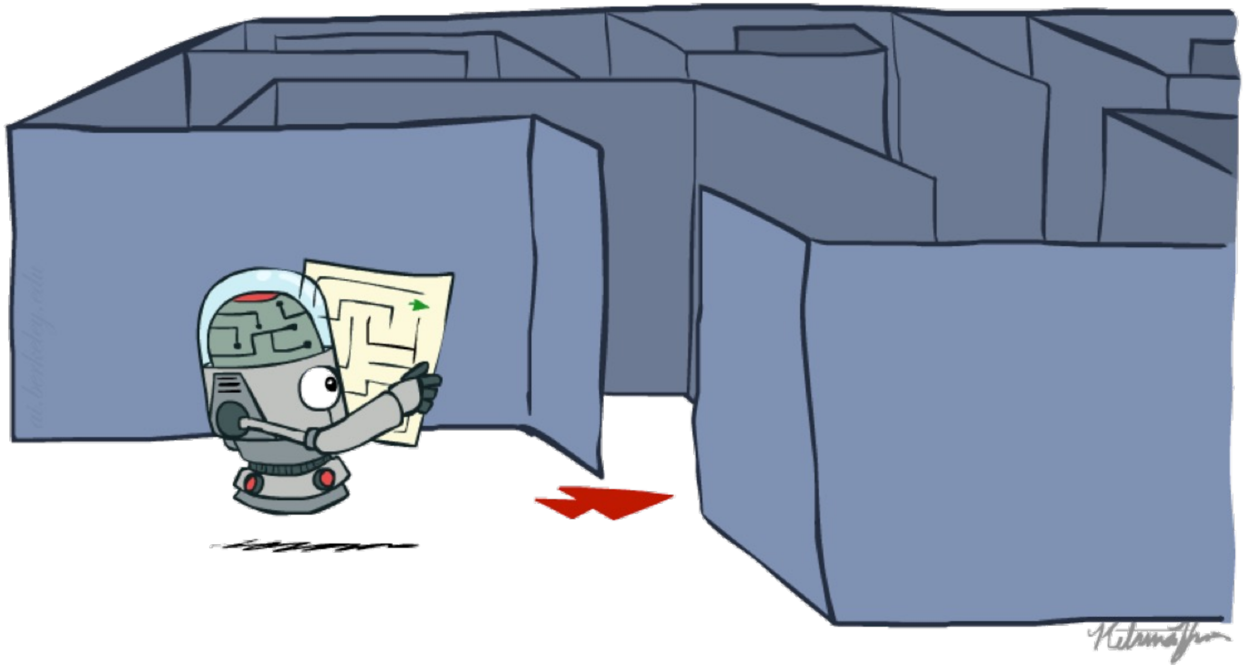
CSE 473: Artificial Intelligence

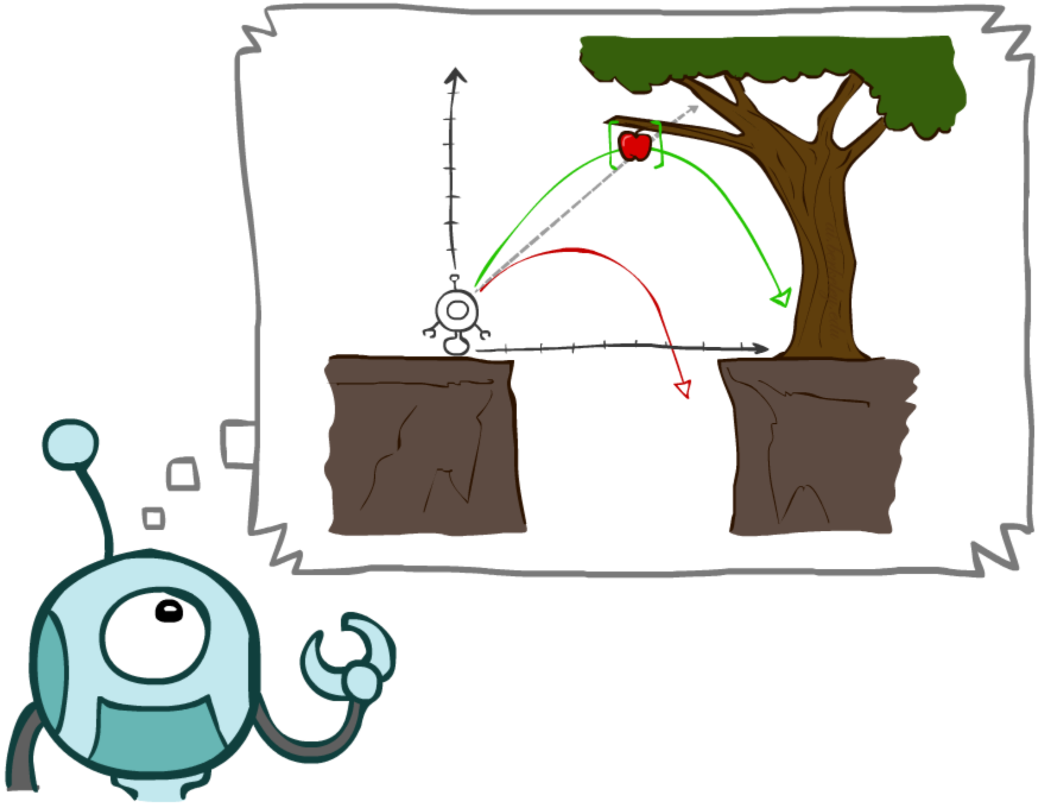
Search



slides adapted from
Stuart Russel, Dan Klein, Pieter Abbeel from ai.berkeley.edu
And Hanna Hajishirzi, Jared Moore, Dan Weld

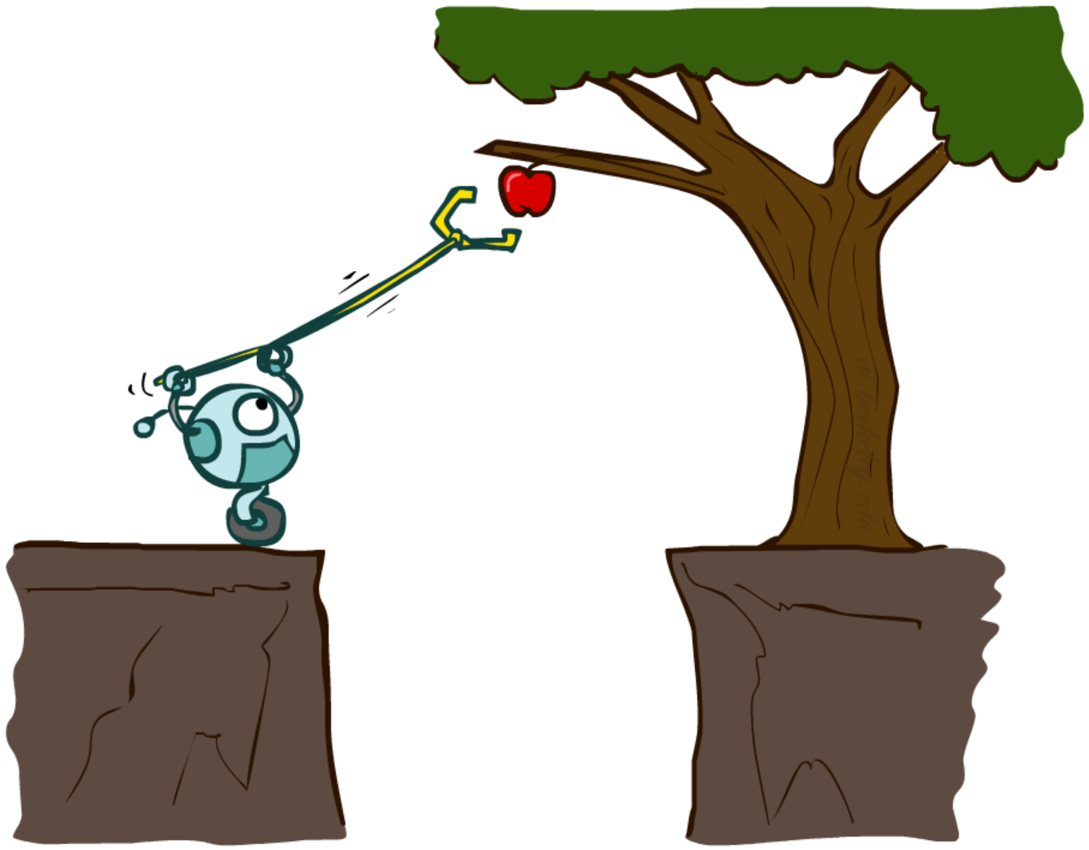
Today

- Agents that Plan Ahead
  - goal-based
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

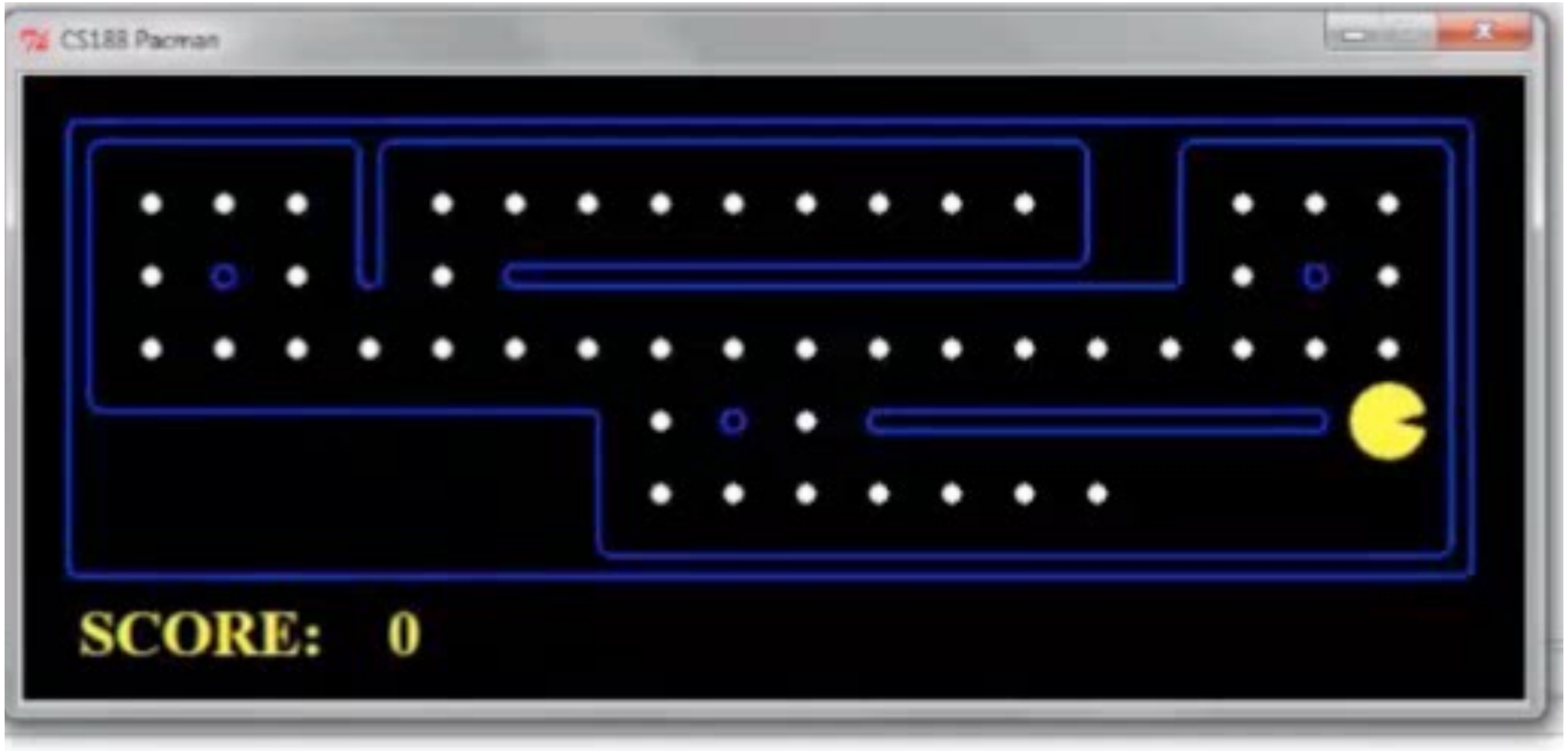


Planning Agents

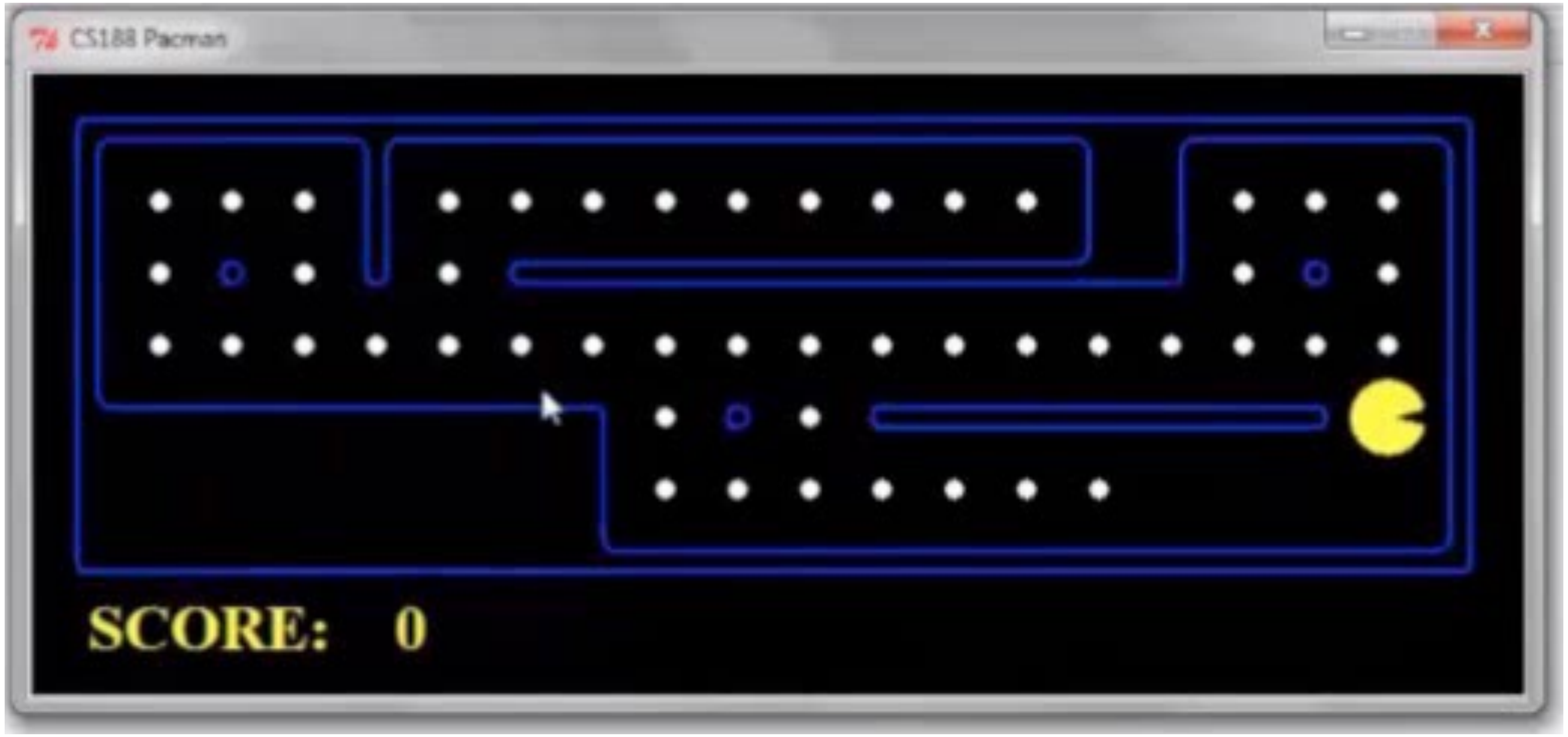
- Planning agents decide based on evaluating future action sequences
- Must have a model of how the world evolves in response to actions
- Usually have a definite goal
- Optimal: Achieve goal at least cost



Optimal?



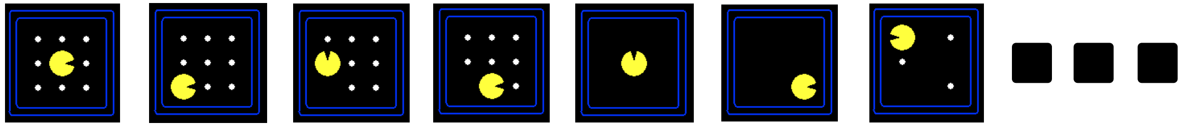
Precompute optimal plan, execute it



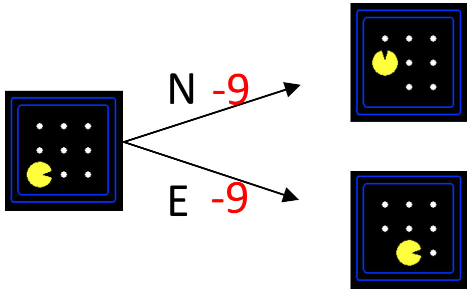
Search Problems

Search Problems

- A search problem consists of:
  - A state space $\mathcal{S}$

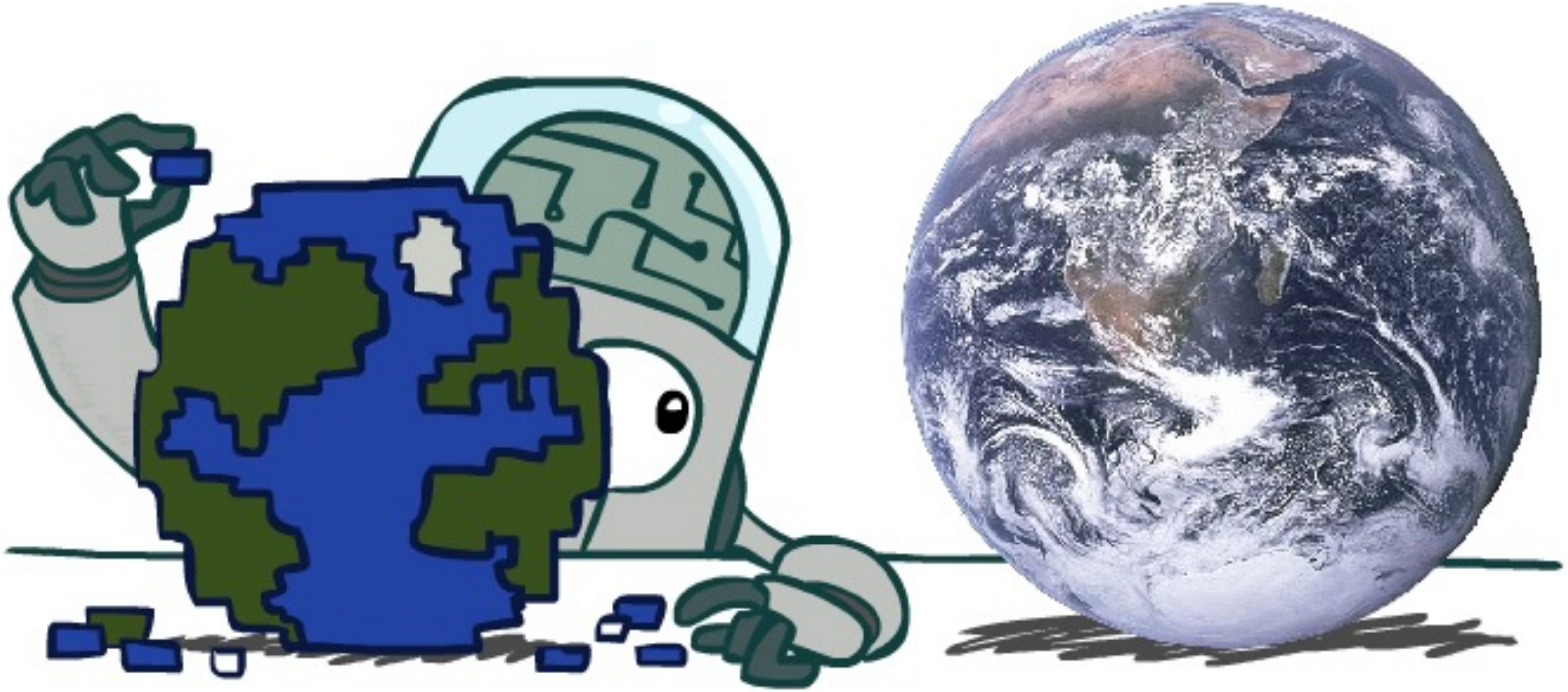


  - An initial state s_0
  - Actions $\mathcal{A}(s)$ in each state
  - Transition model *Result*(s,a)
  - A goal test $G(s)$
    - s has no dots left
  - Action cost $c(s,a,s')$
    - +1 per step; -10 food; -500 win; +500 die; -200 eat ghost



- A solution is an action sequence that reaches a goal state
- An optimal solution has least cost among all solutions

Search Problems Are Models



Example: Traveling in Romania

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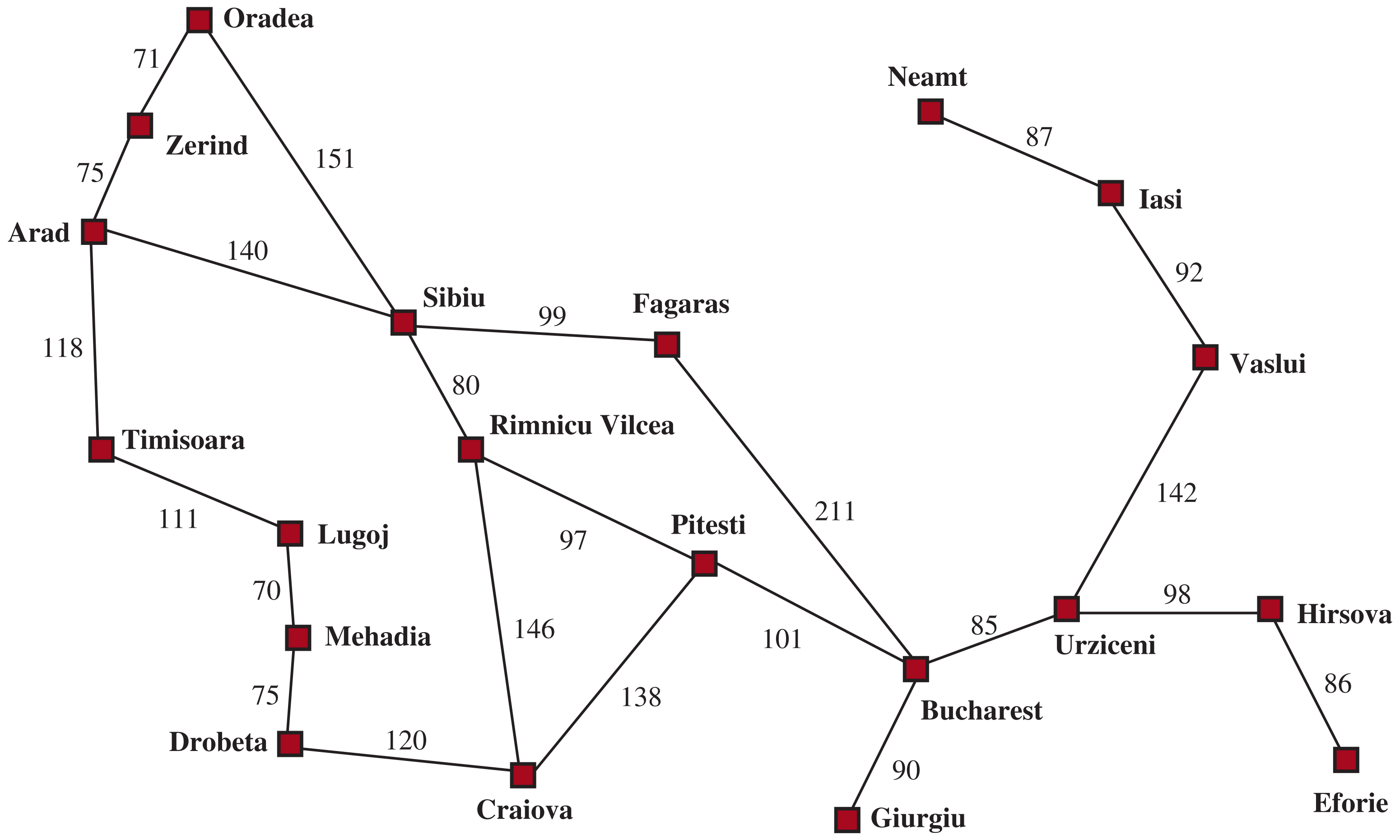


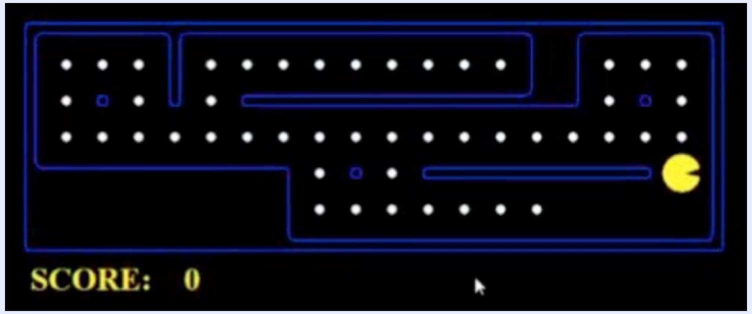
- State space:
  - Cities
- Initial state:
  - Arad
- Actions:
  - Go to adjacent city
- Transition model:
  - Reach adjacent city
- Goal test:
  - *s* = Bucharest?
- Action cost:
  - Road distance from *s* to *s'*
- Solution?

Models are almost always wrong

Start: Haugesund, Rogaland, Norway
End: Trondheim, Sør-Trøndelag, Norway
Total Distance: 2713.2 Kilometers
Estimated Total Time: 47 hours, 31 minutes

What's in a State Space?

The world state includes every last detail of the environment

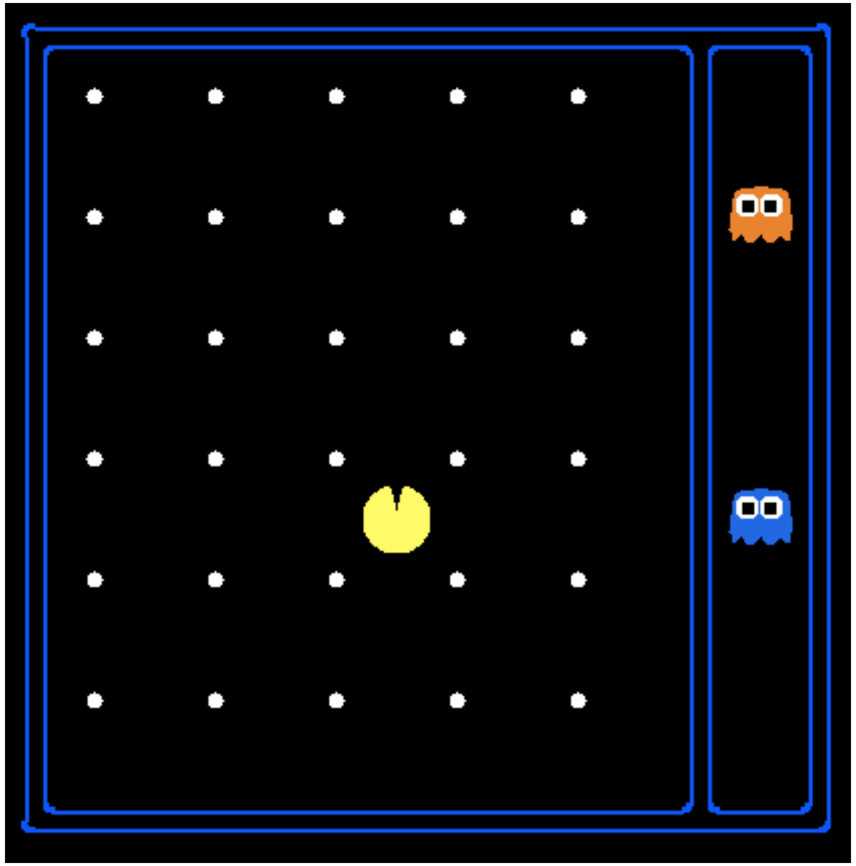


A search state keeps only the details needed for planning (abstraction)

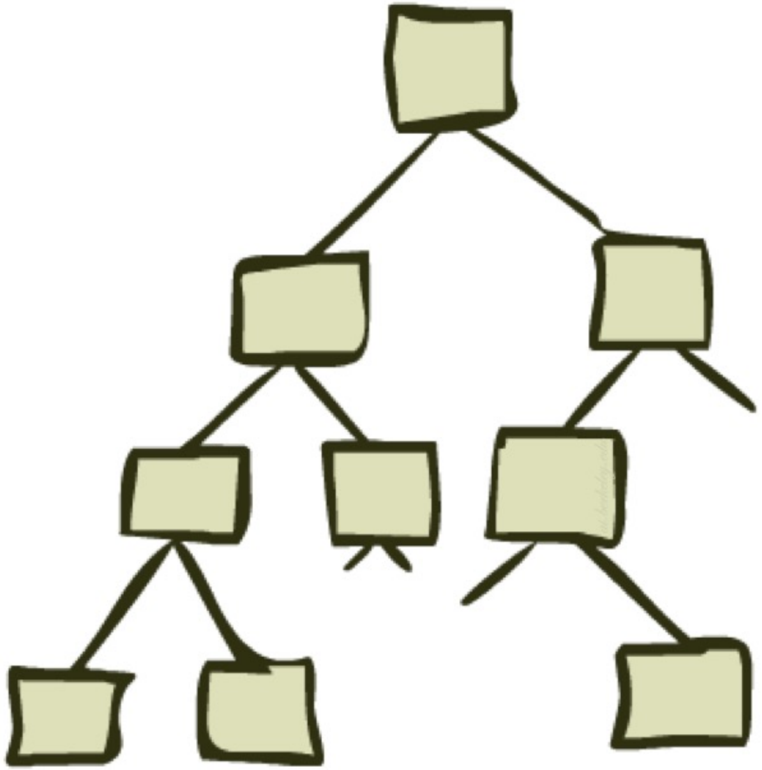
- Problem: Pathing (= path finding)
  - States: (x,y); location
  - Actions: NSEW
  - Transition: update x,y value
  - Goal test: is (x,y)=destination
- Problem: Eat-All-Dots
  - States: pacman location, boolean for each food
  - Actions: NSEW
  - Transition: update x,y and possibly a dot Boolean
  - Goal test: dots all false

State Space Sizes

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW
- How many
  - World states?
    $120 \times (2^{30}) \times (12^{2}) \times 4$
  - States for pathing (path finding)?
    120
  - States for eat-all-dots?
    $120 \times (2^{30})$

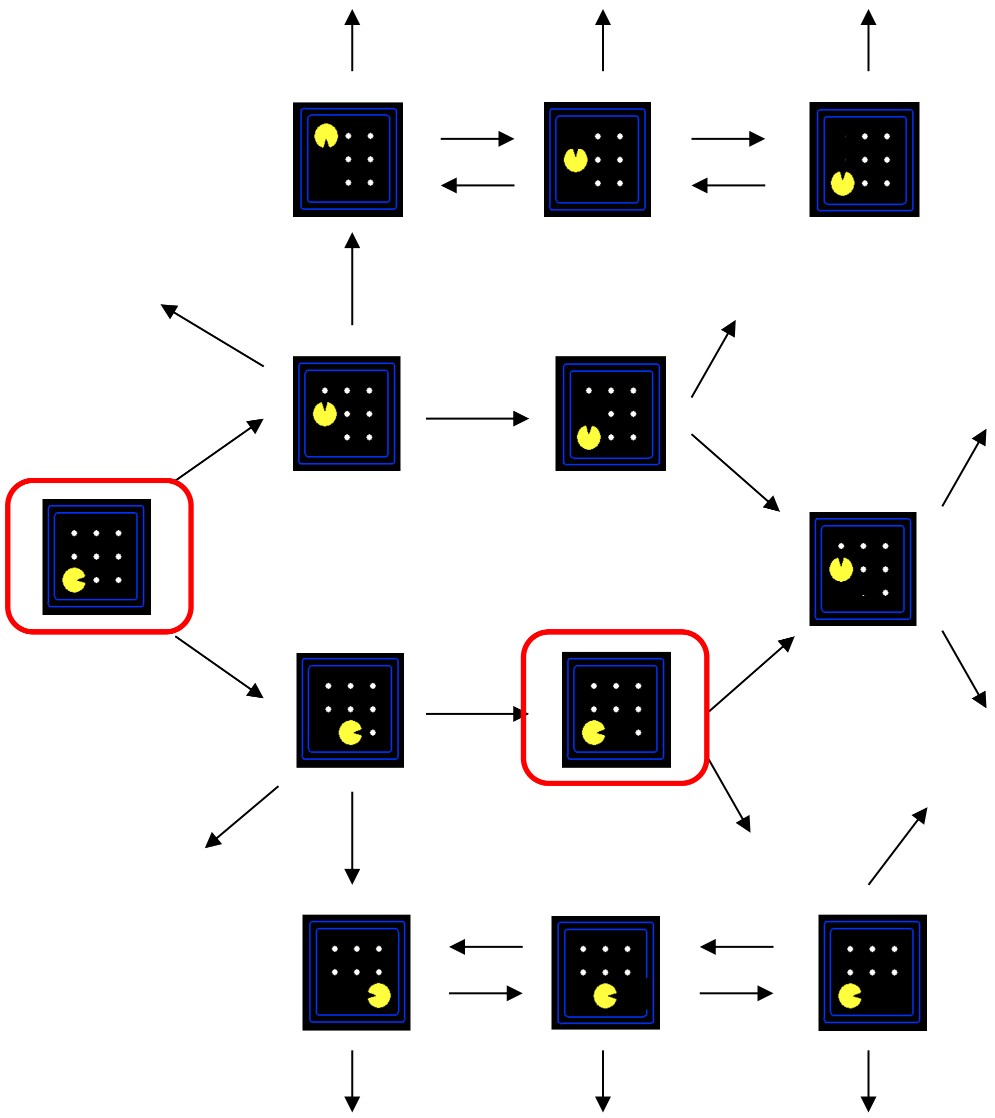


State Space Graphs and Search Trees



State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



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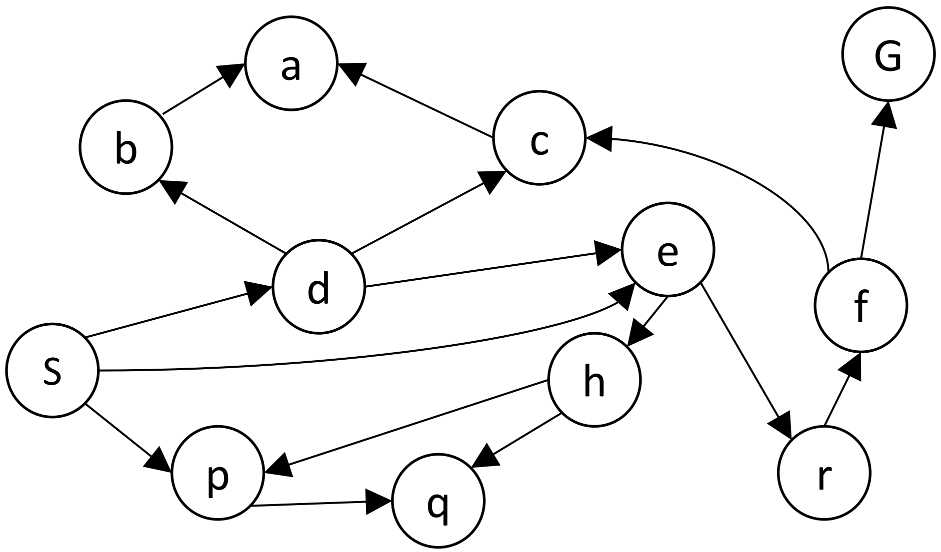
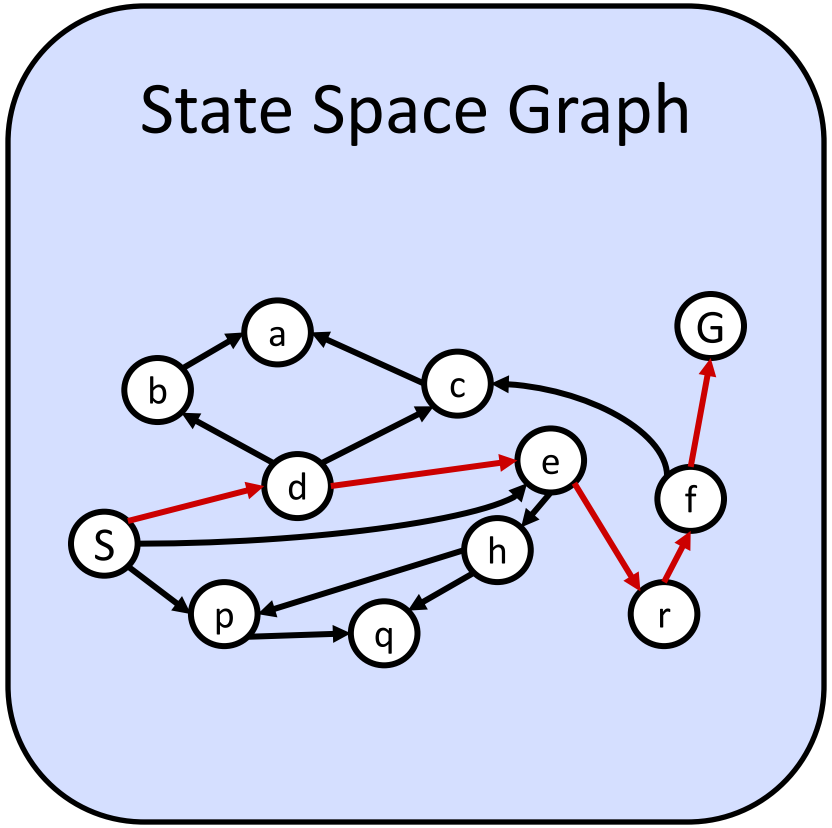


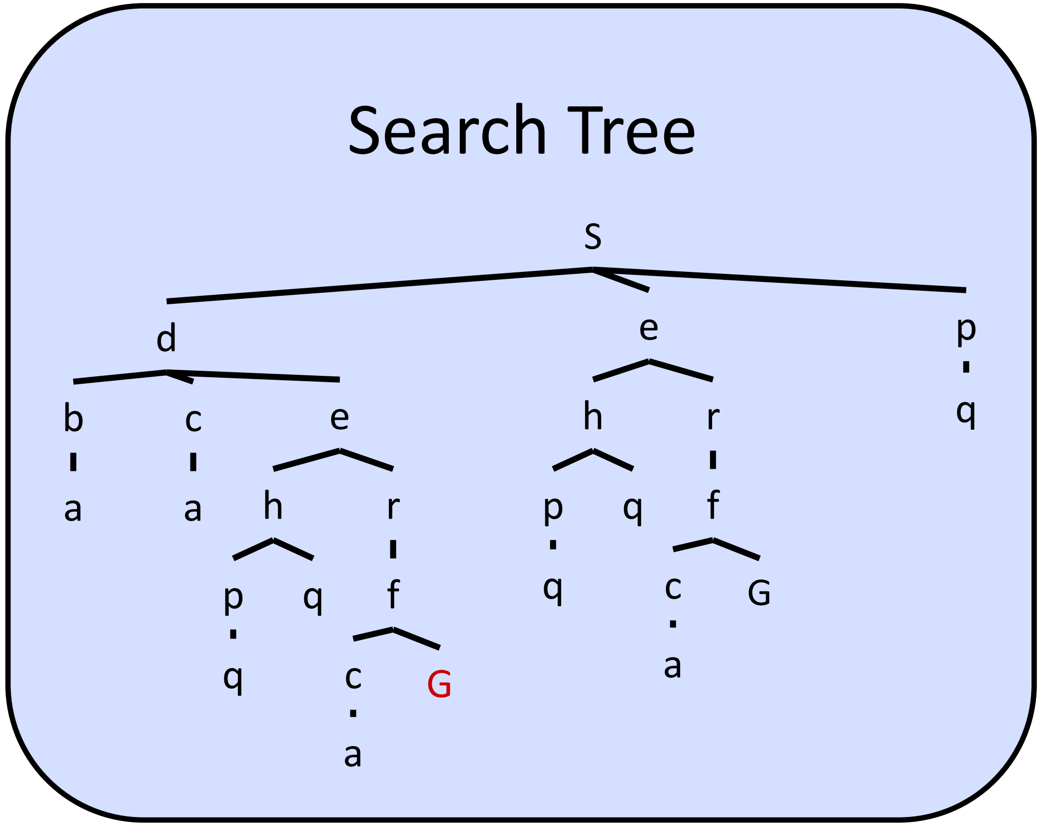
Tiny state space graph for a tiny search problem

State Space Graphs vs. Search Trees



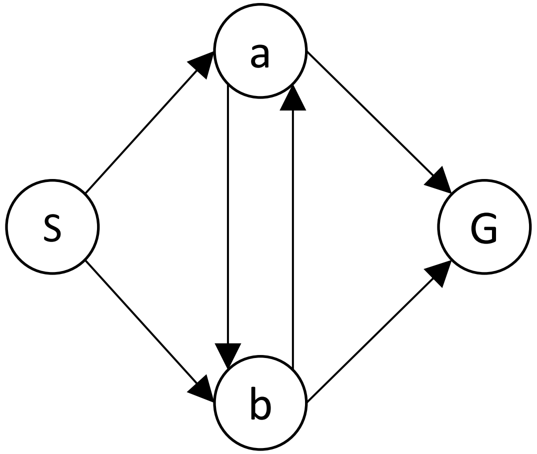
Each NODE in in the search tree is an entire PATH in the state space graph.

We construct the tree on demand – and we construct as little as possible.



Quiz: State Space Graphs vs. Search Trees

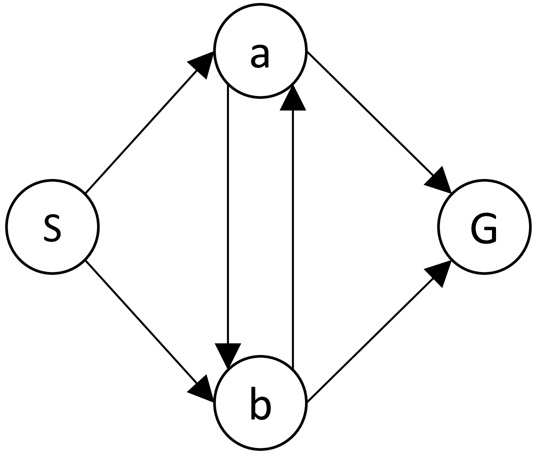
Consider this 4-state graph:



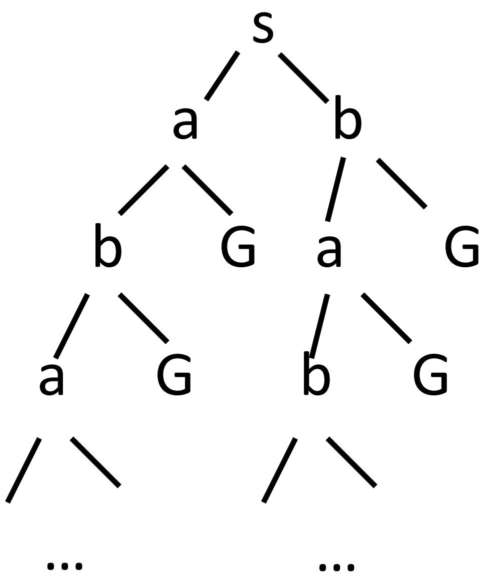
How big is its search tree (from S)?

Quiz: State Space Graphs vs. Search Trees

Consider this 4-state graph:

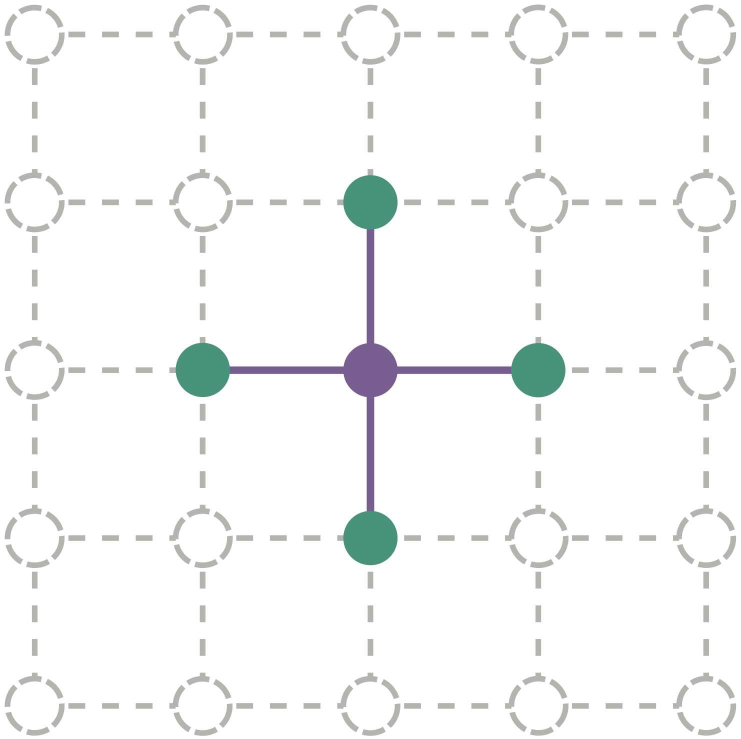


How big is its search tree (from S)?



Quiz: State Space Graphs vs. Search Trees

Consider a rectangular grid:

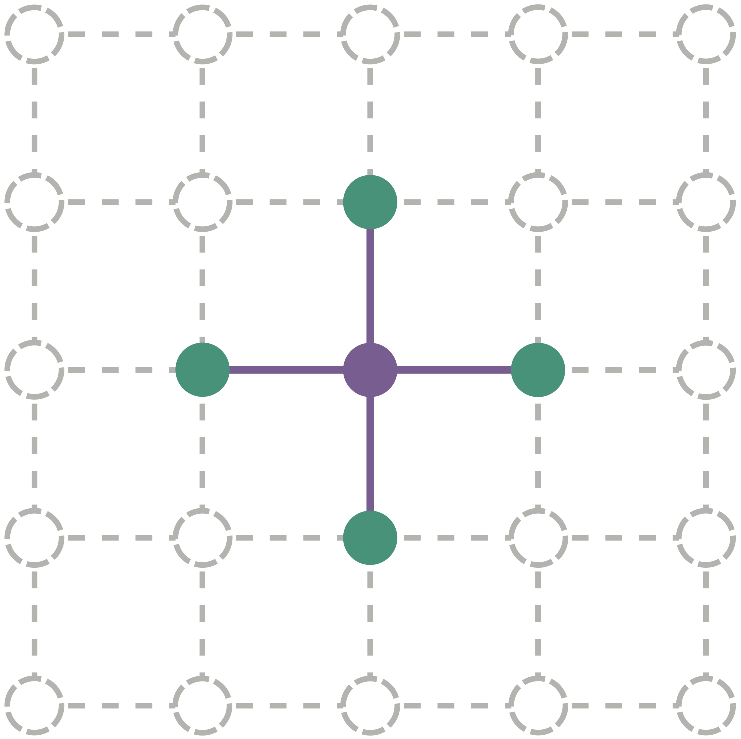


How many unique states within d steps of start?

How many states in search tree of depth d?

Quiz: State Space Graphs vs. Search Trees

Consider a rectangular grid:



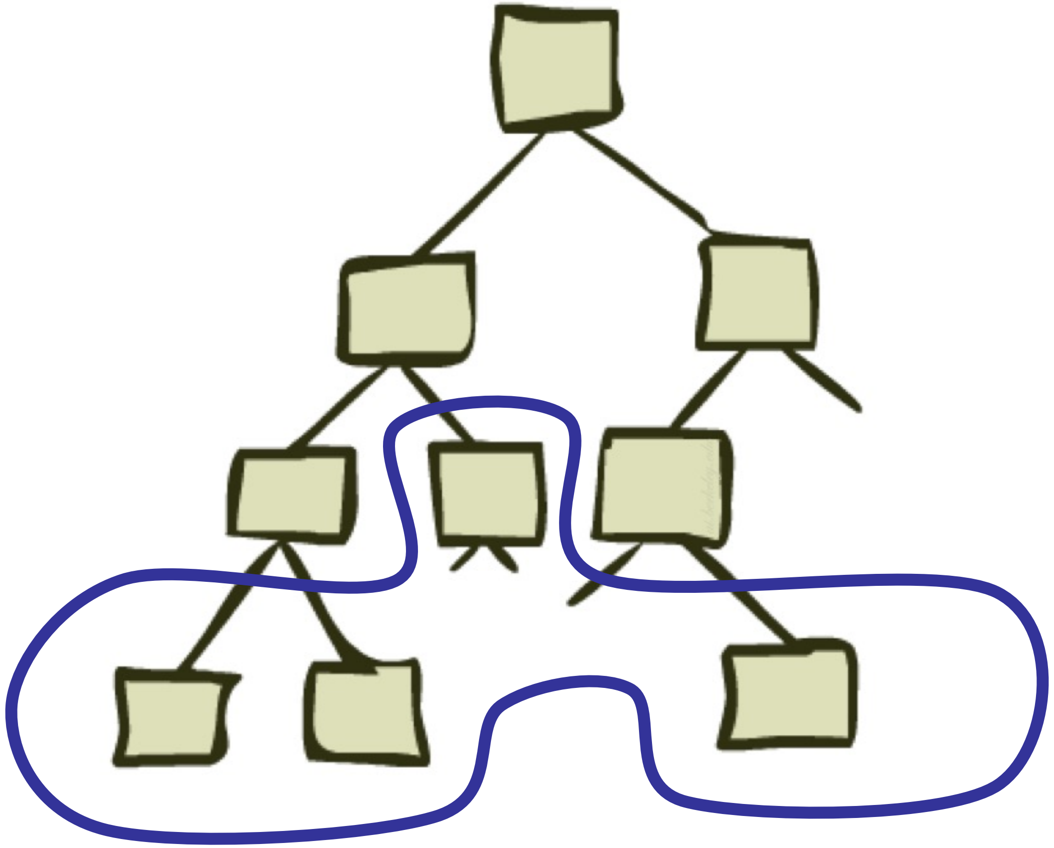
How many unique states within d steps of start?

Enumerate after step 1: {4, 4 + 8, 4 + 8 + 12, …}
*= Min(5*5 – 1, \sum{i=1}^{d} 4i)*

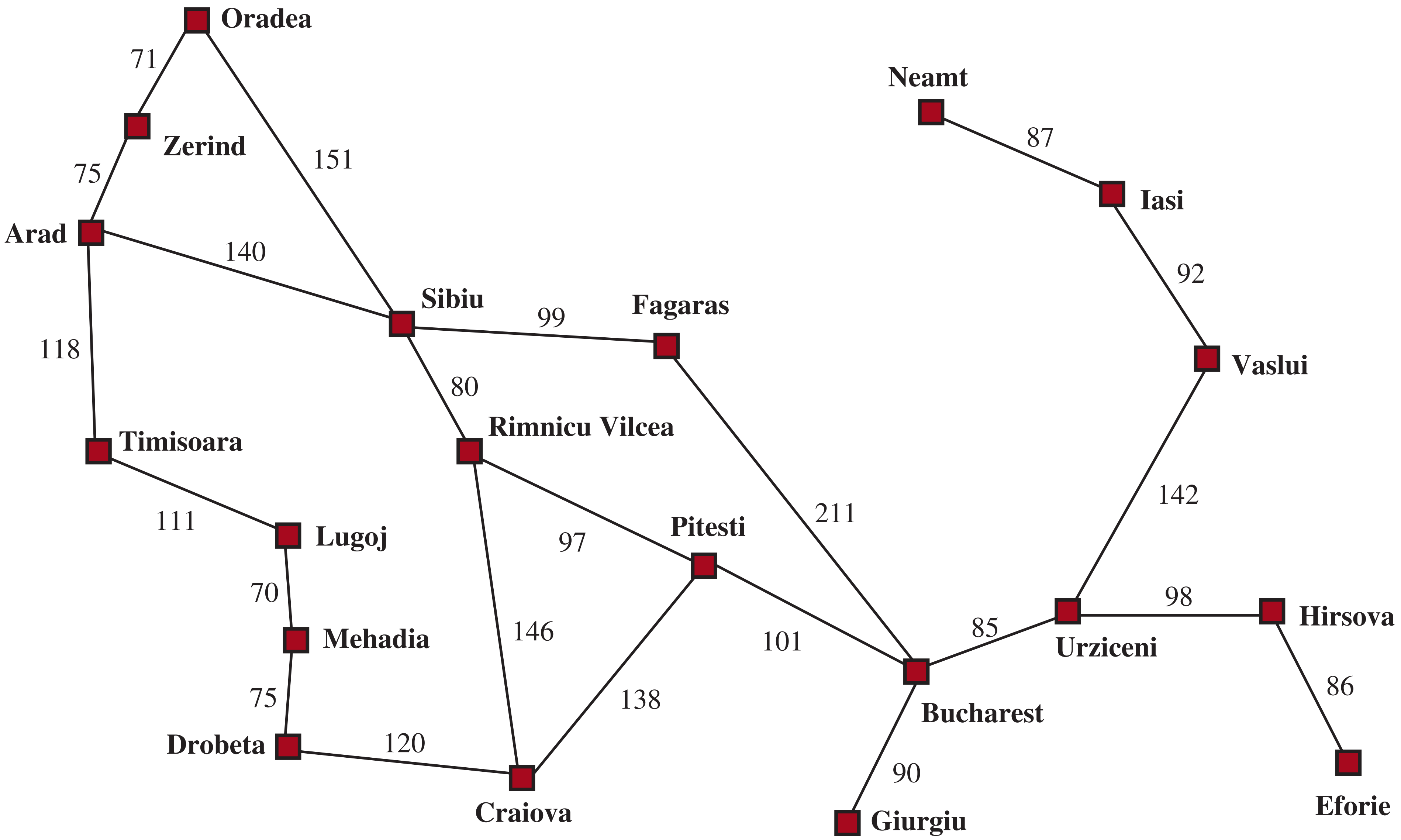
How many states in search tree of depth d?

= O(4^d)

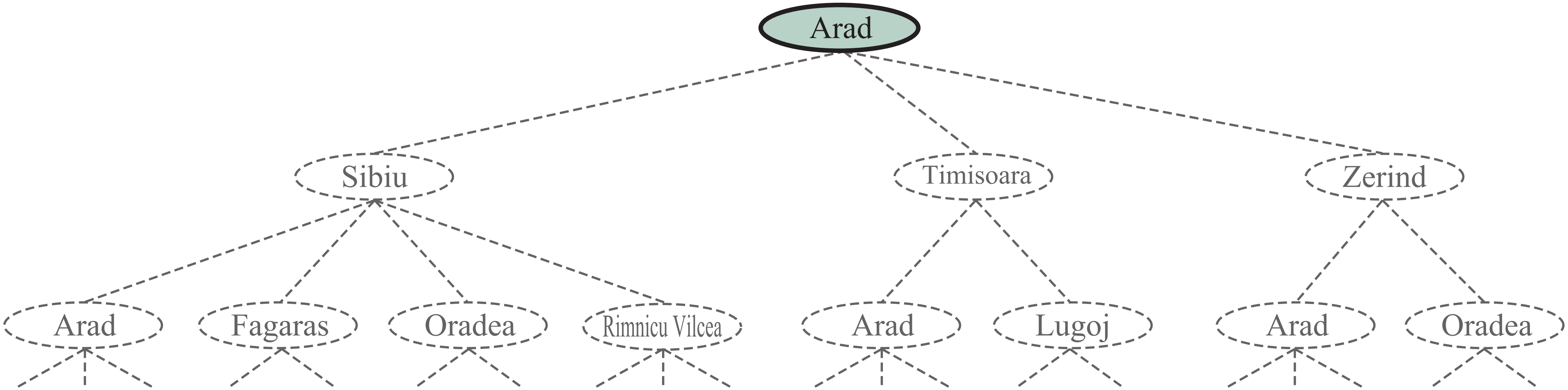
Tree Search



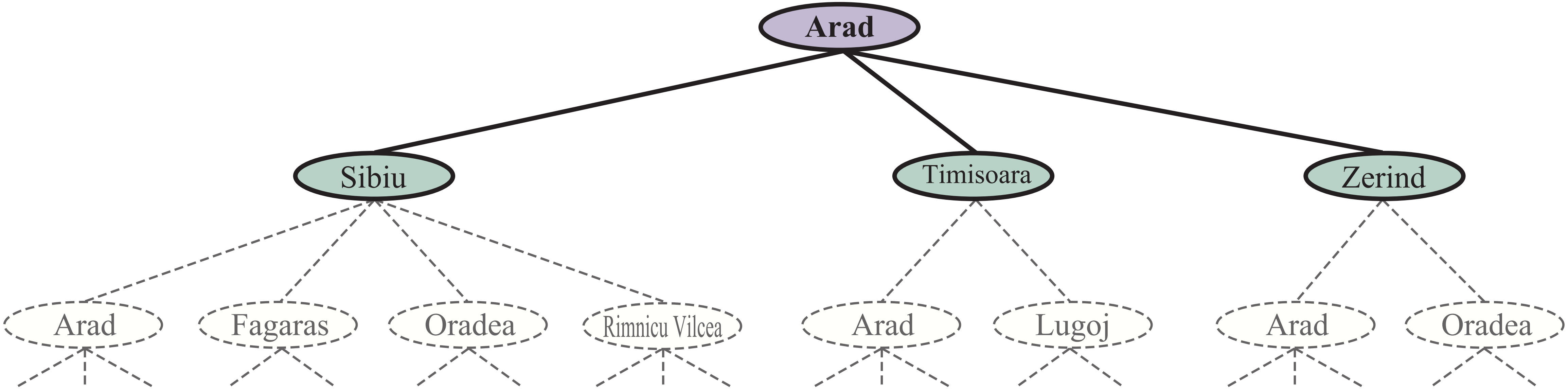
Search Example: Romania



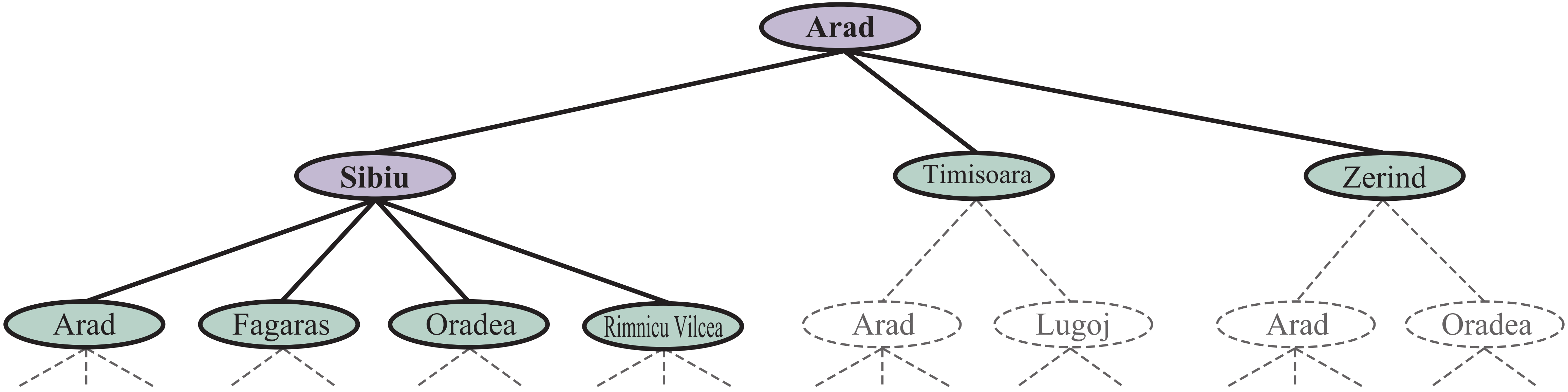
Creating the search tree



Creating the search tree



Creating the search tree

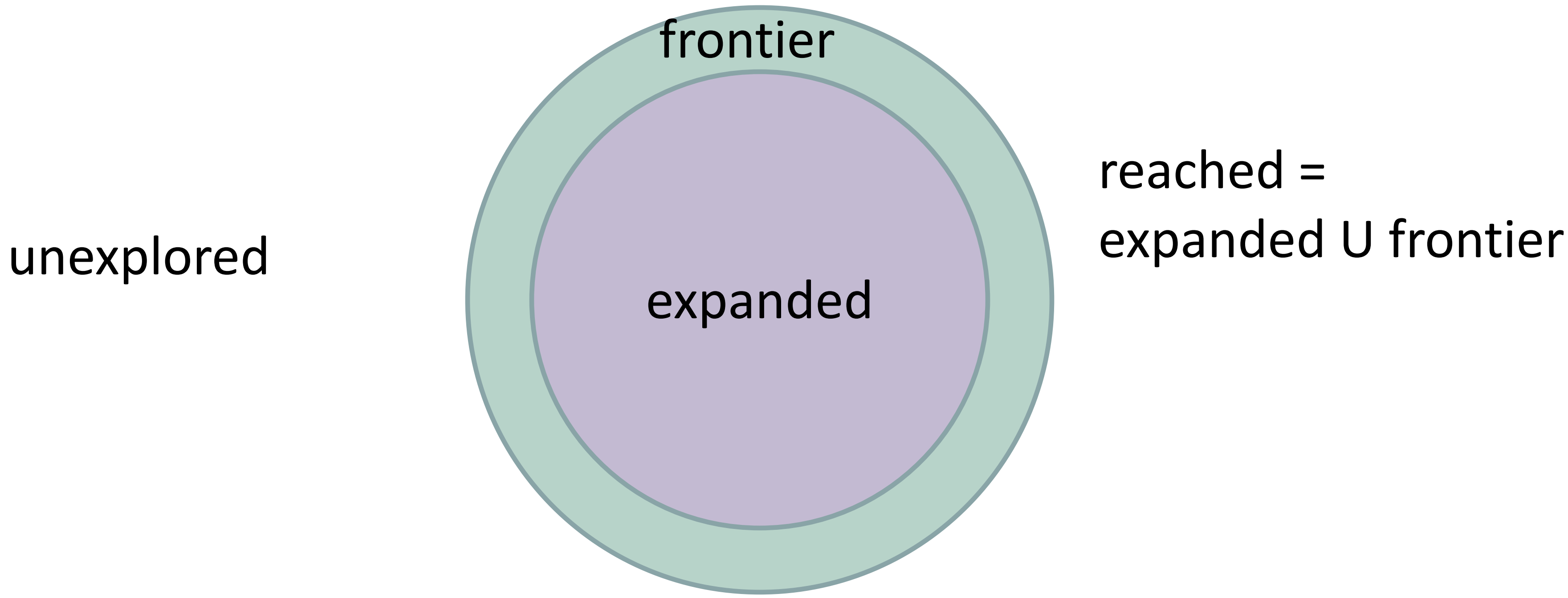


General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Main variations:
  - Which leaf node to expand next
  - Whether to check for repeated states
  - Data structures for frontier, expanded nodes

Systematic search



1. Frontier separates expanded from unexplored region of state-space graph
2. Expanding a frontier node:
   a. Moves a node from frontier into expanded
   b. Adds nodes from unexplored into frontier, maintaining property 1

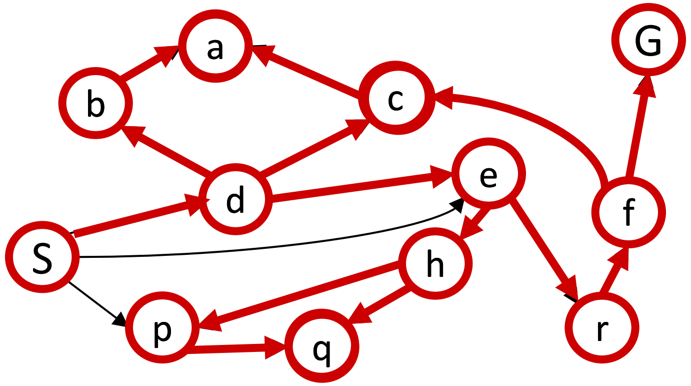
Depth-First Search

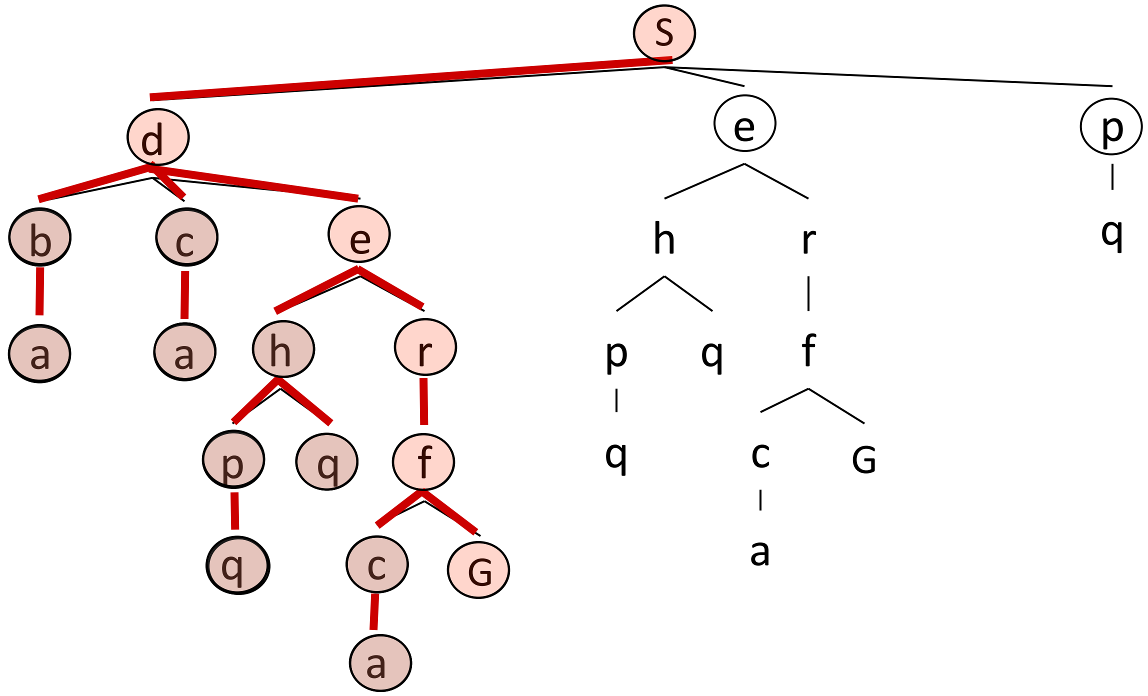
Depth-First Search

Strategy: expand a deepest node first

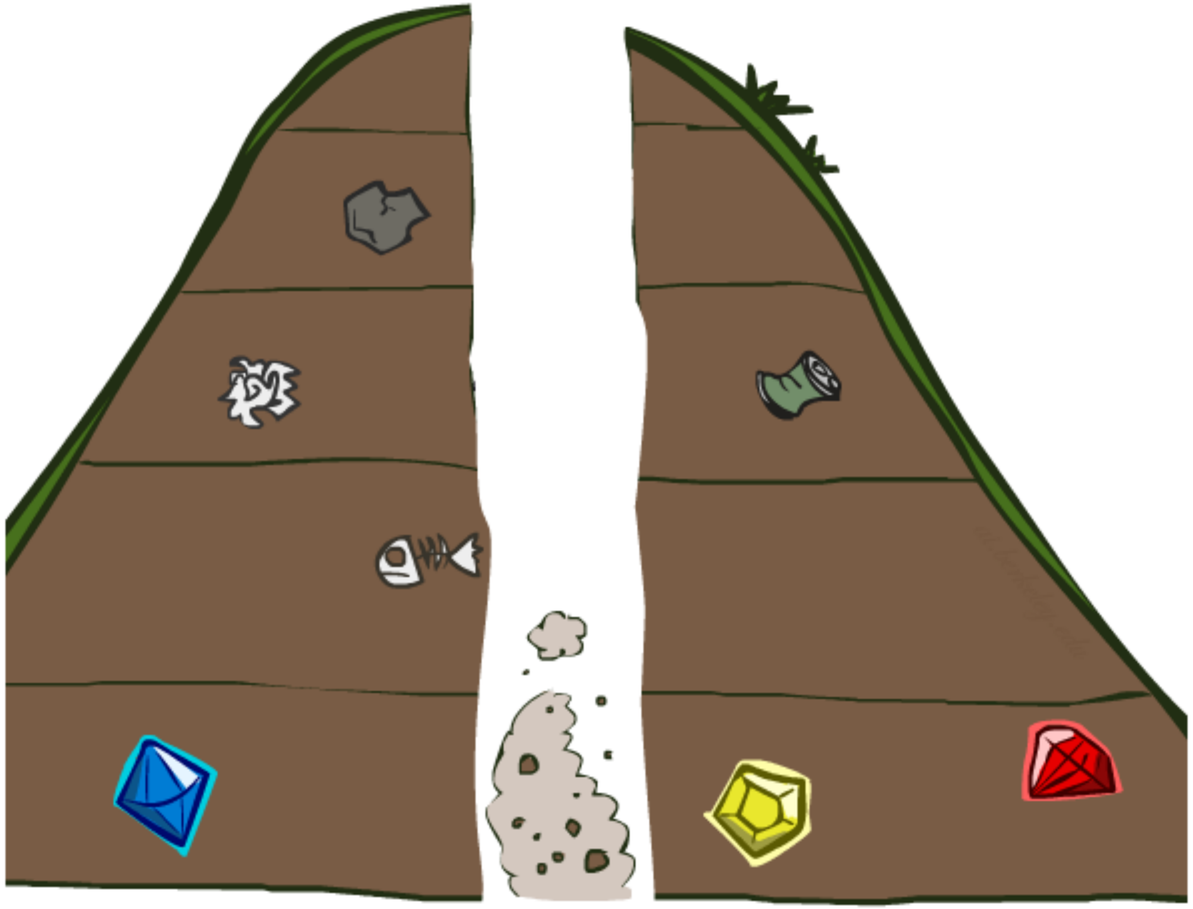
Implementation: Frontier is a LIFO stack

(last in first out)





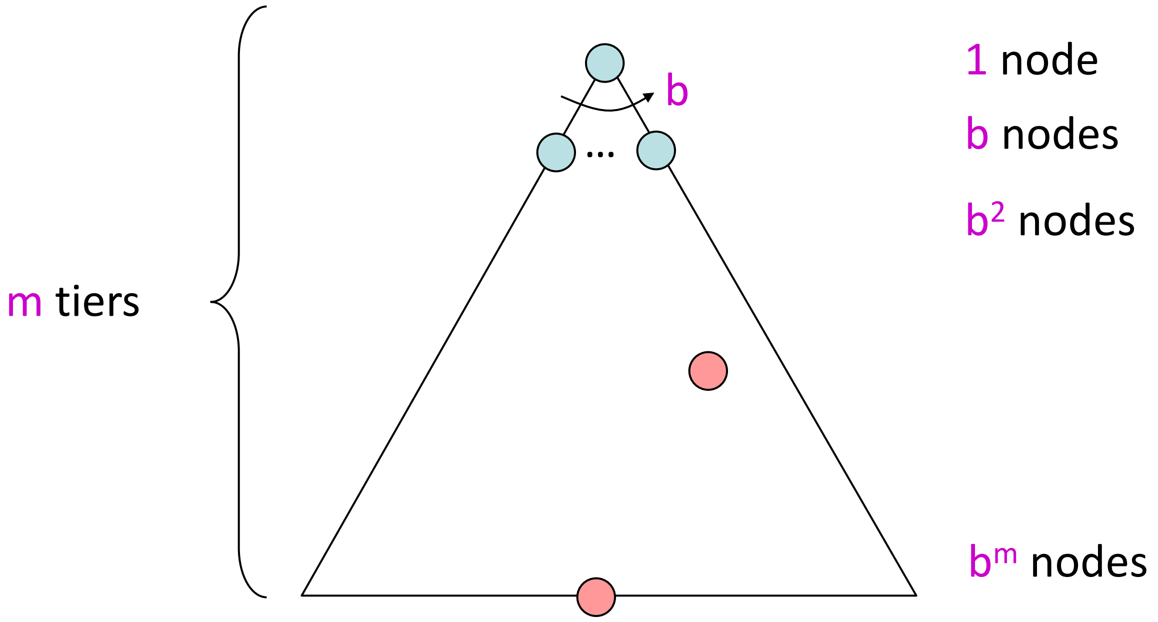
Search Algorithm Properties



Search Algorithm Properties

- **Complete:** Guaranteed to find a solution if one exists?
- **Optimal:** Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?

- Cartoon of search tree:
  - *b* is the branching factor
  - *m* is the maximum depth
  - solutions at various depths

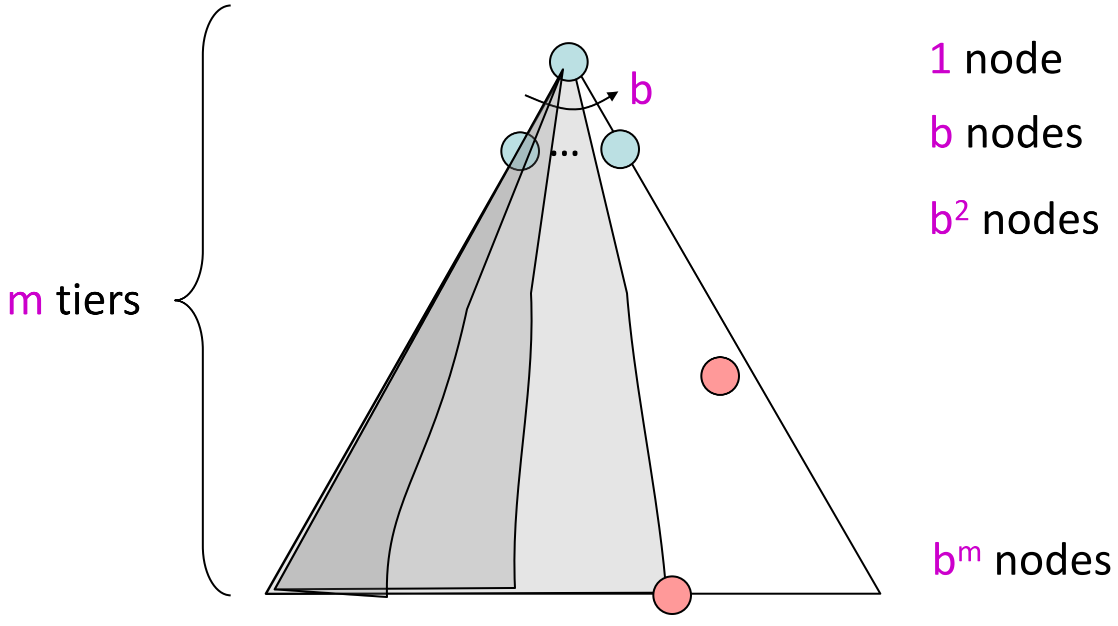


- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots . b^m = O(b^m)$

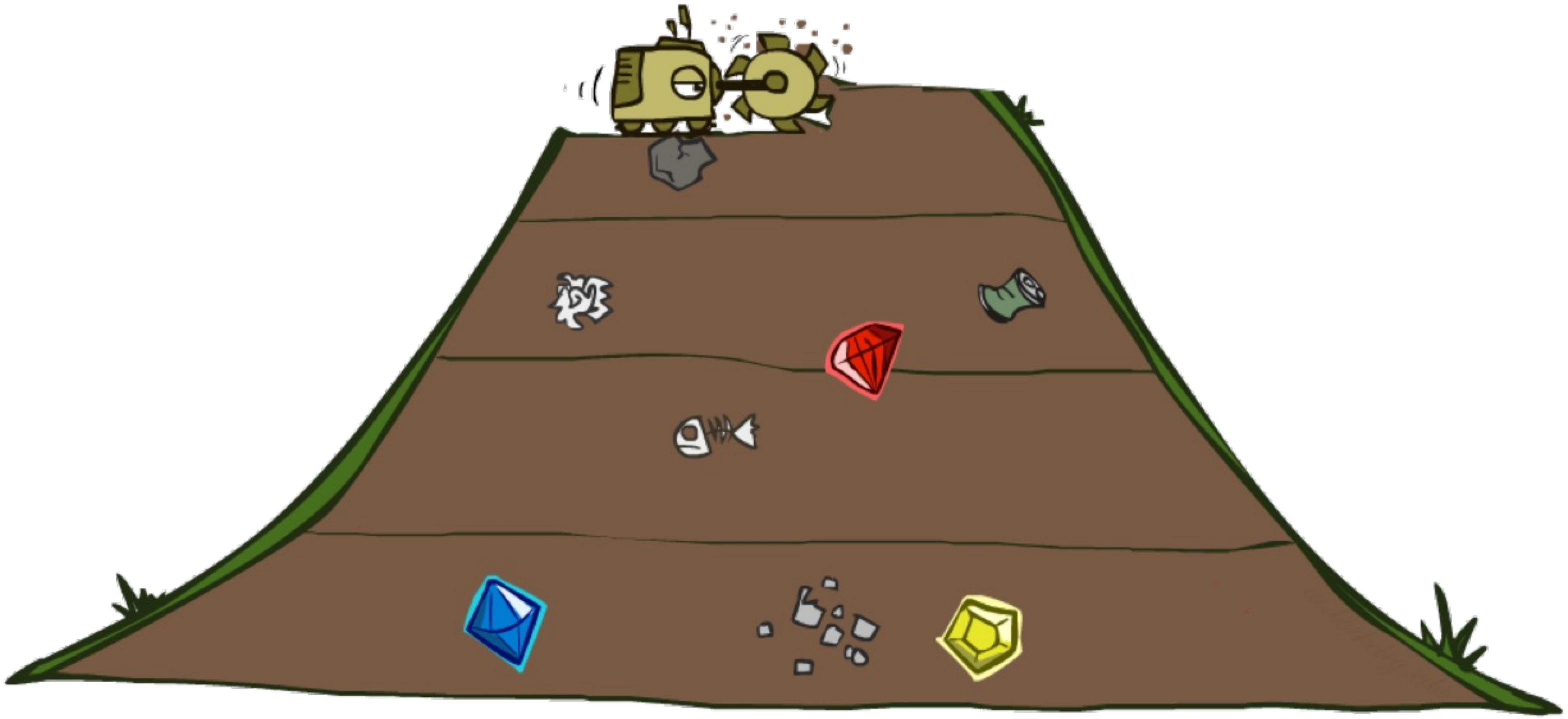
Remember O(..) is the upper bound of the function

Depth-First Search (DFS) Properties

- What nodes does DFS expand?
  - Some left prefix of the tree down to depth *m*.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the frontier take?
  - Only has siblings on path to root, so $O(bm)$
- Is it complete?
  - *m* could be infinite
  - preventing cycles may help (more later)
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost



Breadth-First Search

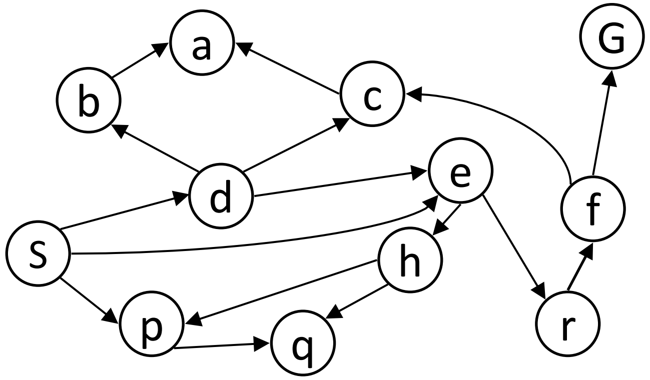


Breadth-First Search

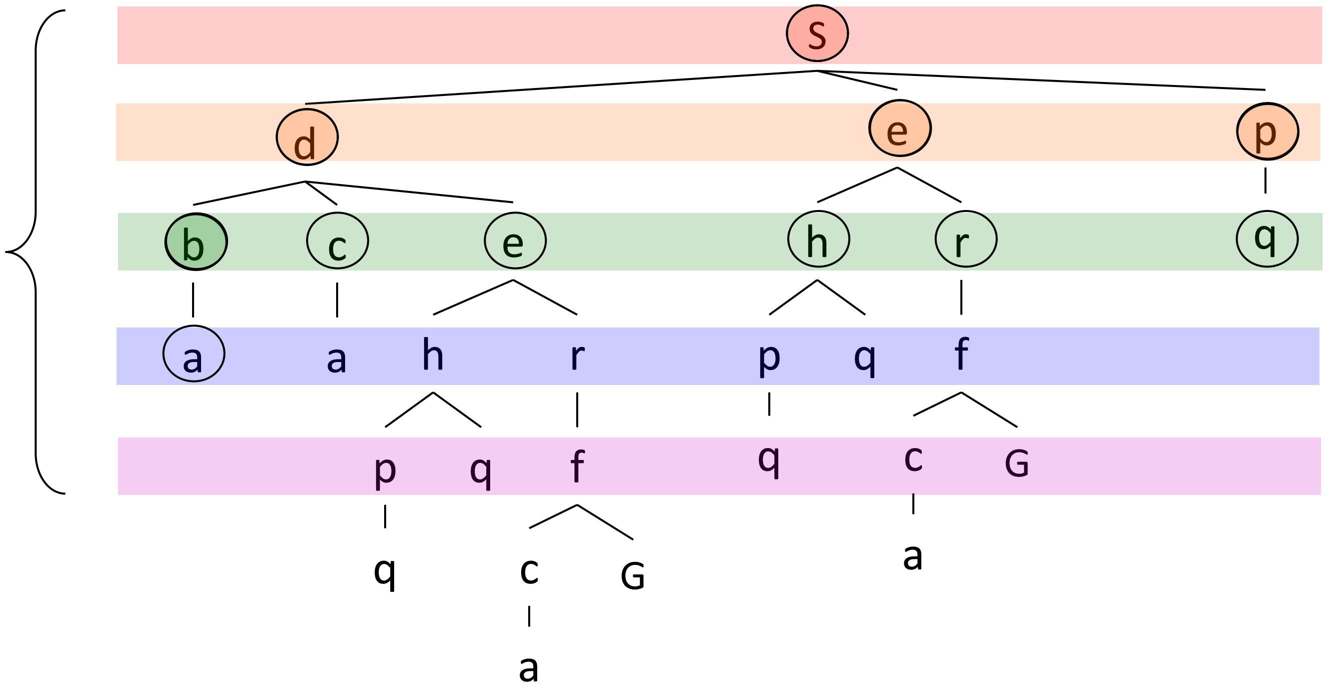
Strategy: expand a shallowest node first

Implementation: Frontier is a FIFO queue

(first in first out)

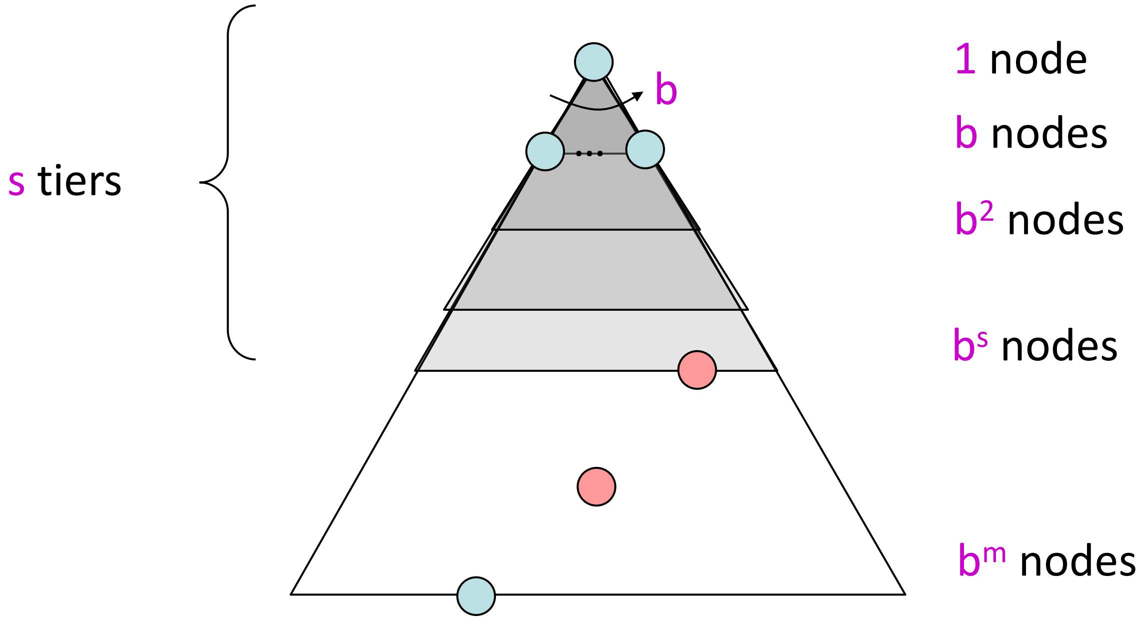


Search Tiers

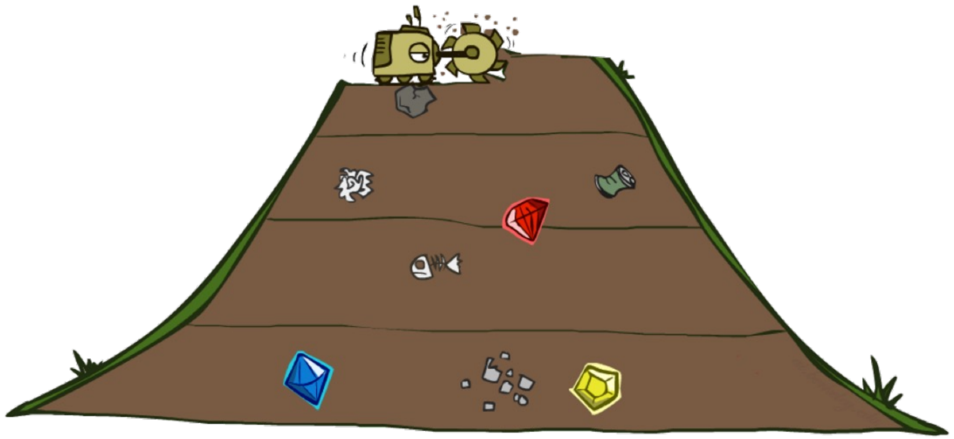


Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be *s*
  - Search takes time $O(b^s)$
- How much space does the frontier take?
  - Has roughly the last tier, so $O(b^s)$
- Is it complete?
  - *s* must be finite if a solution exists, so yes!
- Is it optimal?
  - If costs are equal (e.g., 1)



Quiz: DFS vs BFS



Quiz: DFS vs BFS

(In terms of S, the depth of the shallowest solution and M, the maximum depth)

- When will BFS outperform DFS?

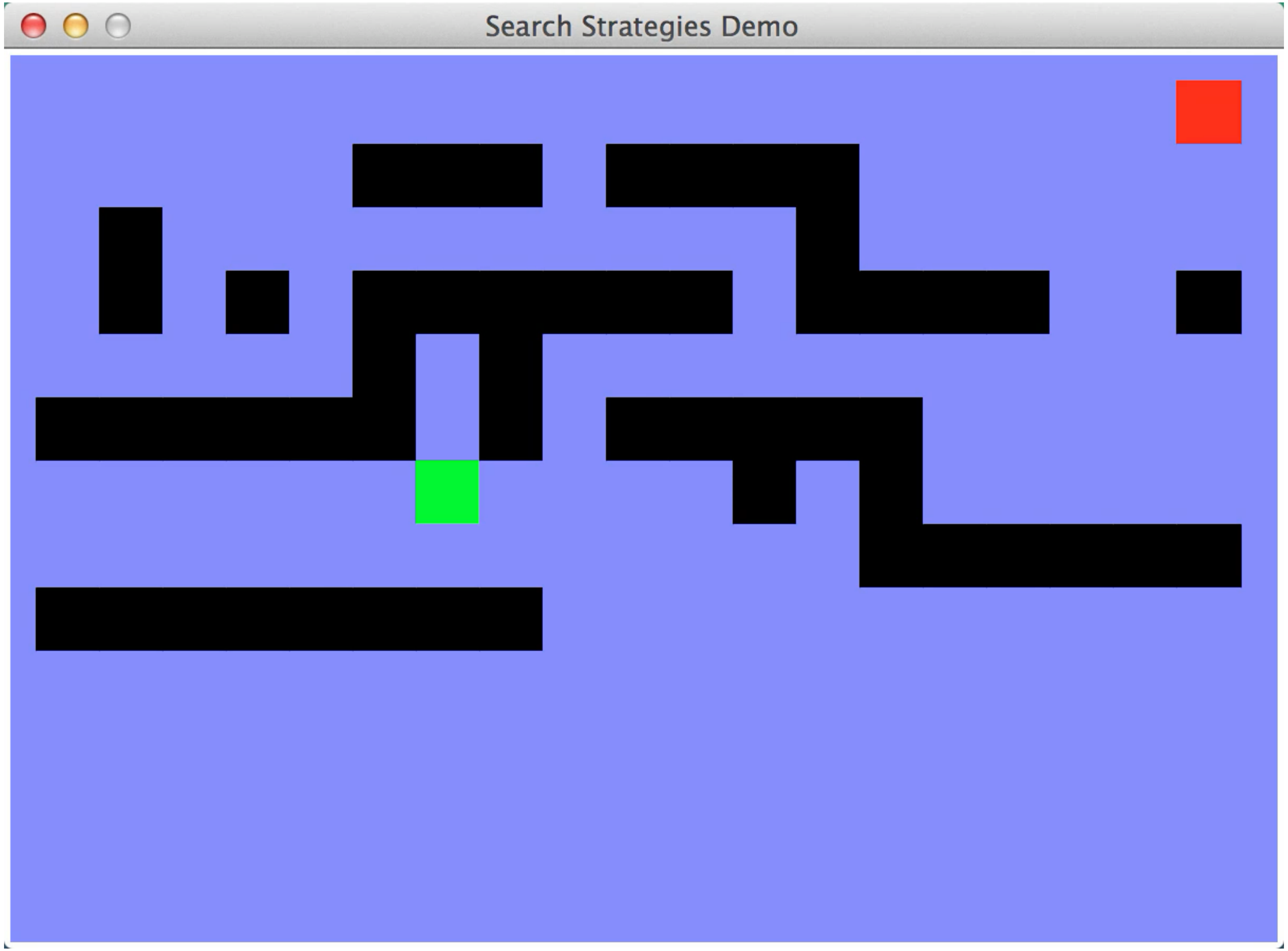
- When will DFS outperform BFS?

Quiz: DFS vs BFS

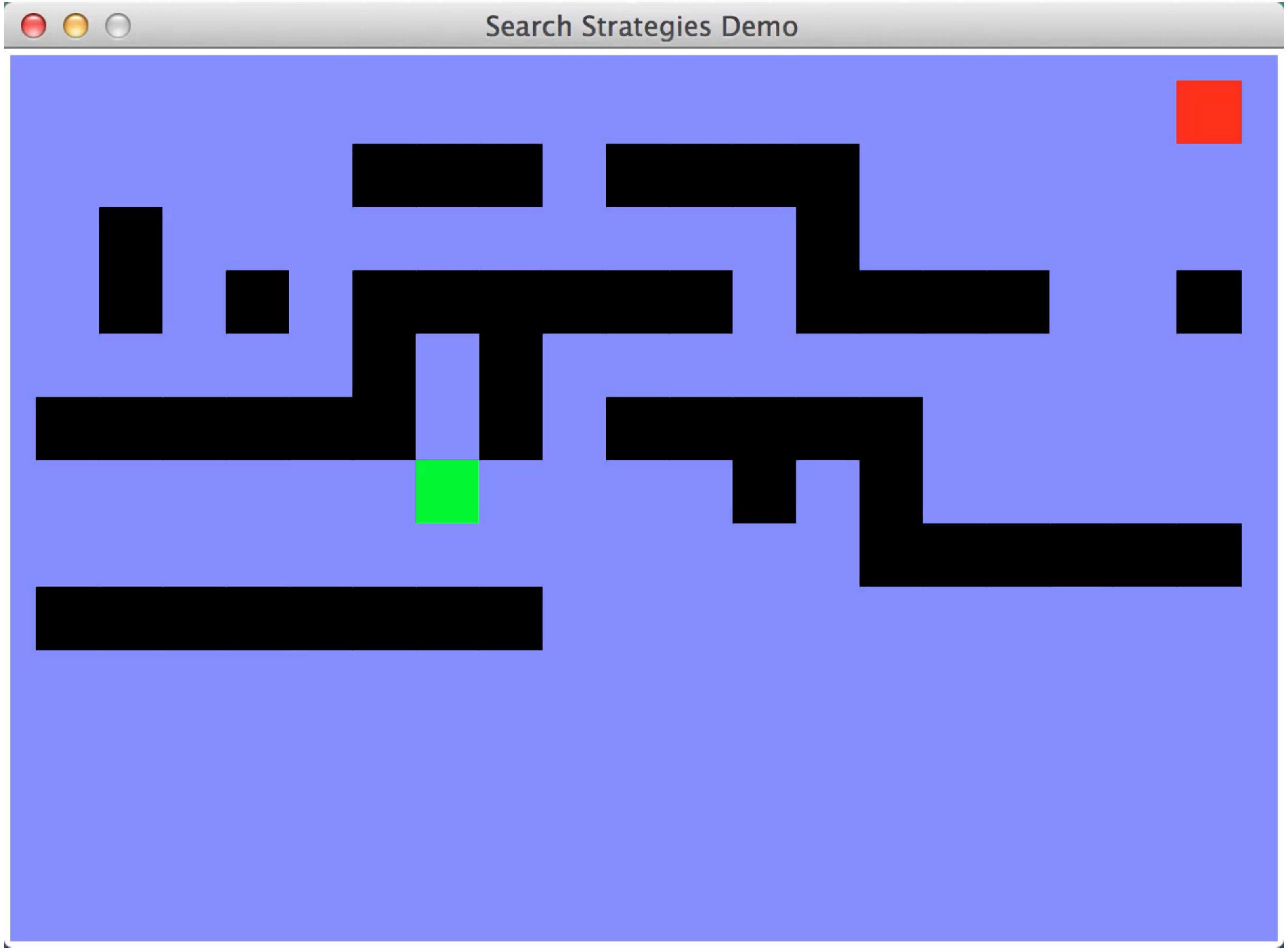
(In terms of S, the depth of the shallowest solution and M, the maximum depth)

- When will BFS outperform DFS?
  - S << M
- When will DFS outperform BFS?
  - S ~= M

Example: Maze Water DFS/BFS (part 1)

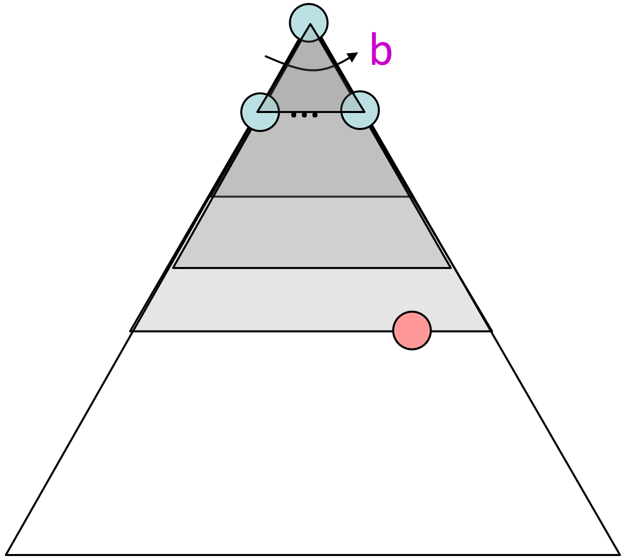


Example: Maze Water DFS/BFS (part 2)



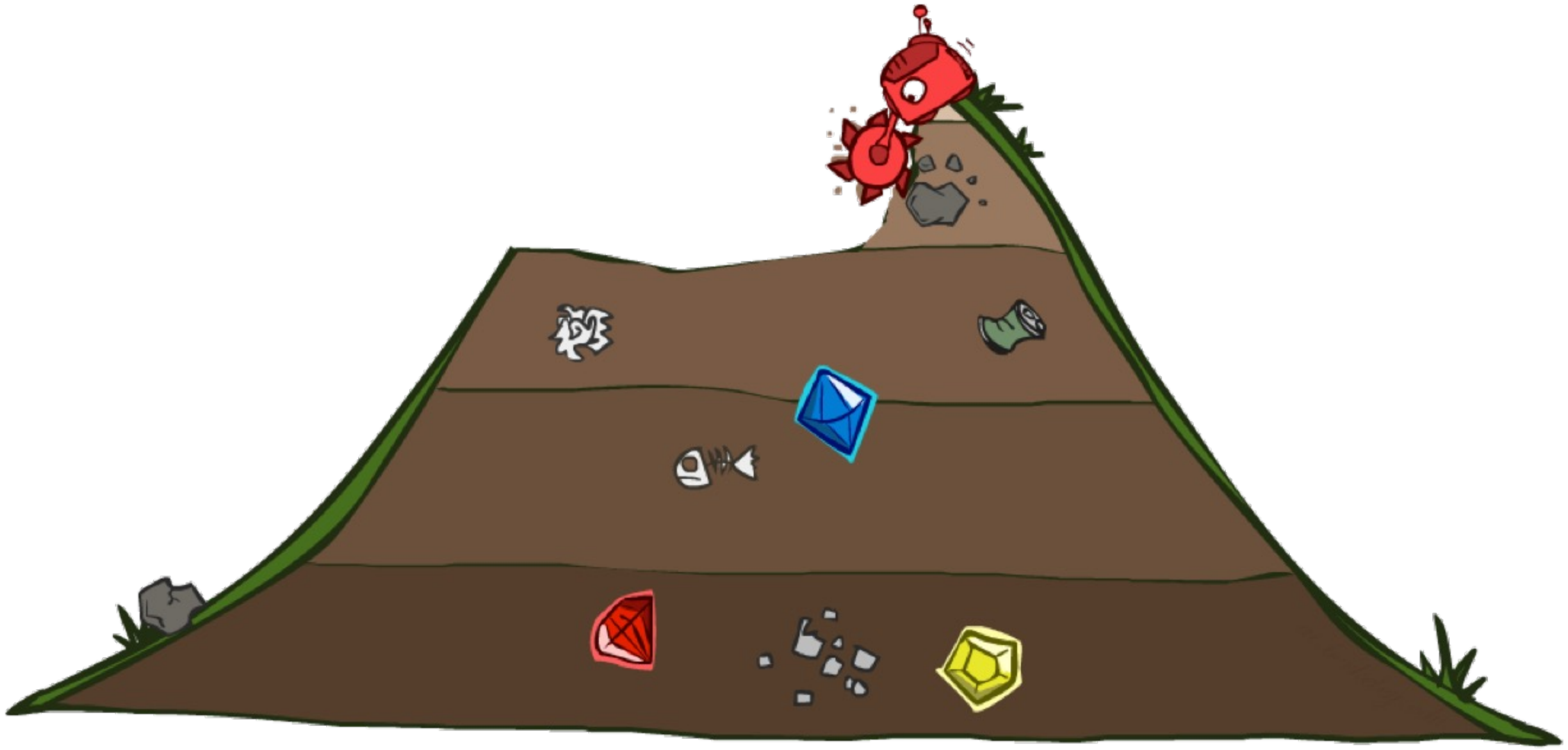
Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with **depth limit** 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..



- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!
  - Also useful for the meta data

Uniform Cost Search

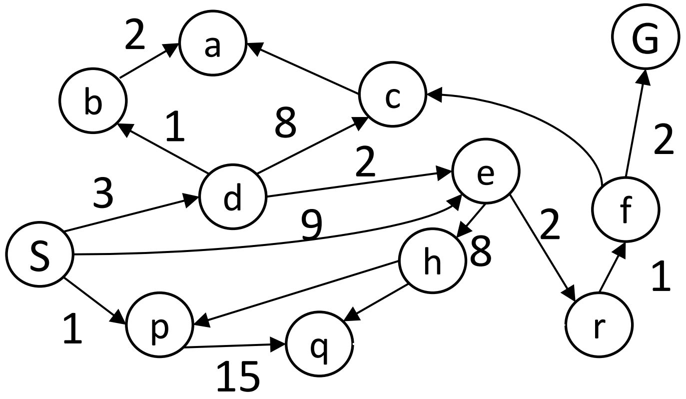


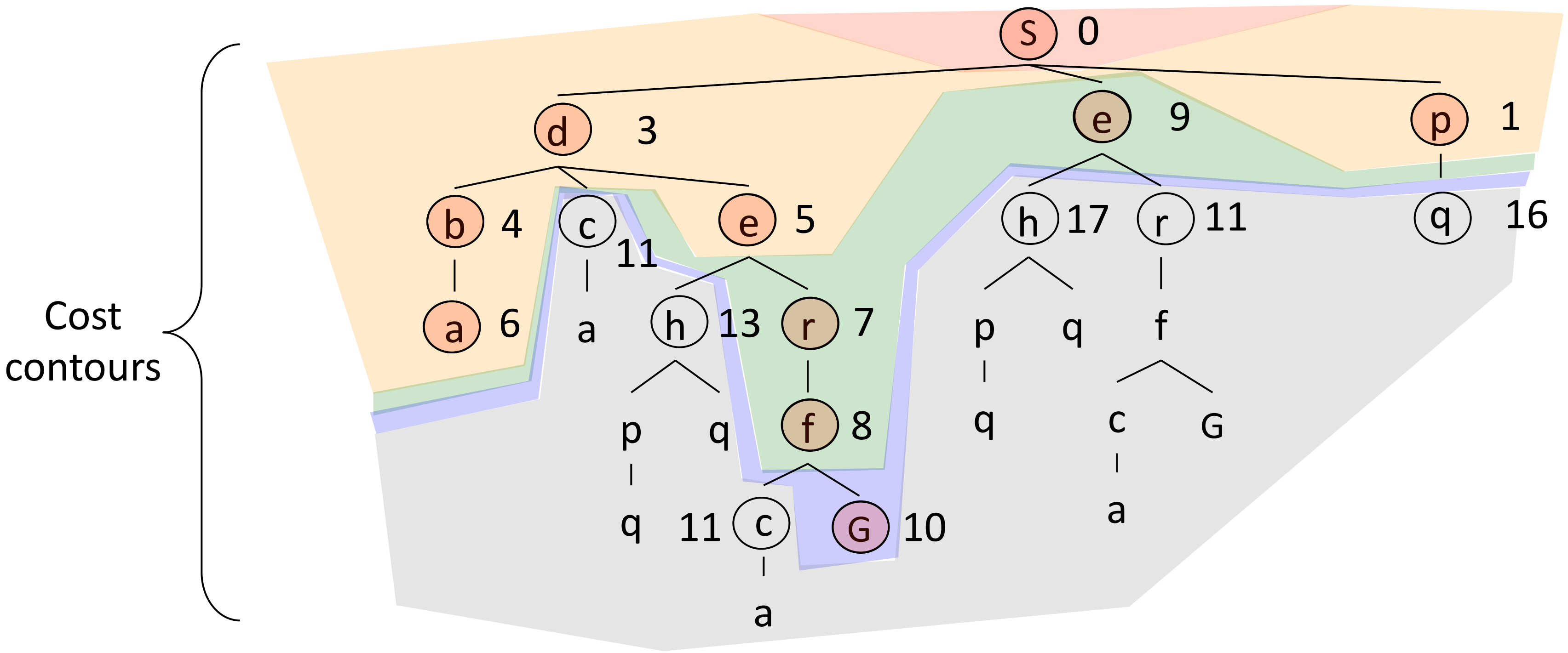
Uniform Cost Search

g(n) = cost from root to n

Strategy: expand lowest g(n)

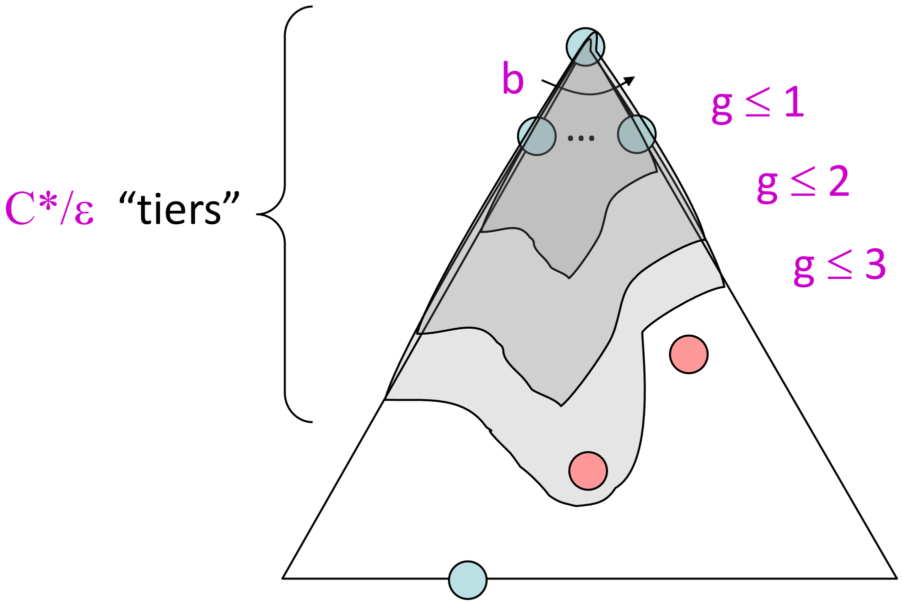
Frontier is a priority queue sorted by g(n)



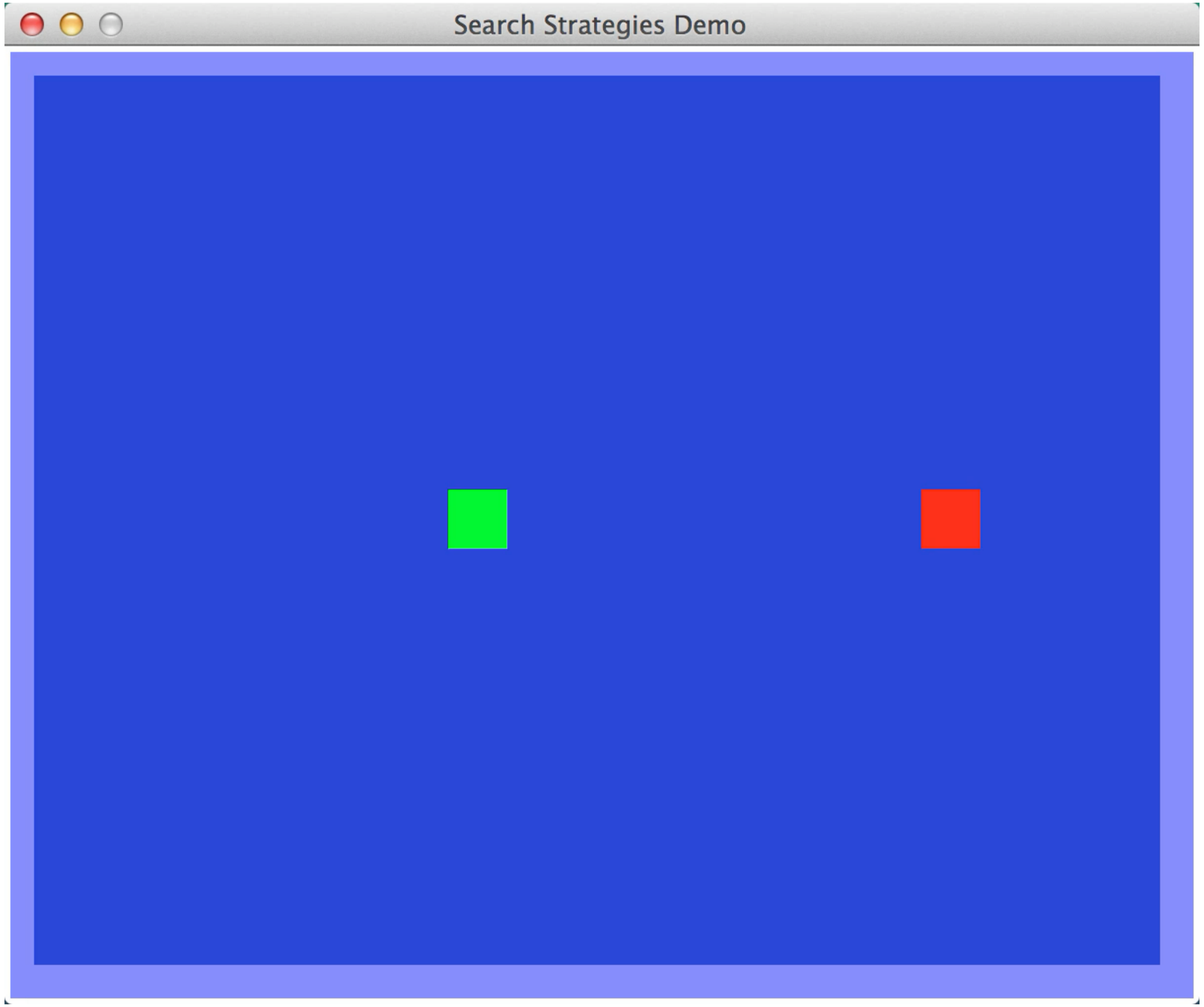


Uniform Cost Search (UCS) Properties

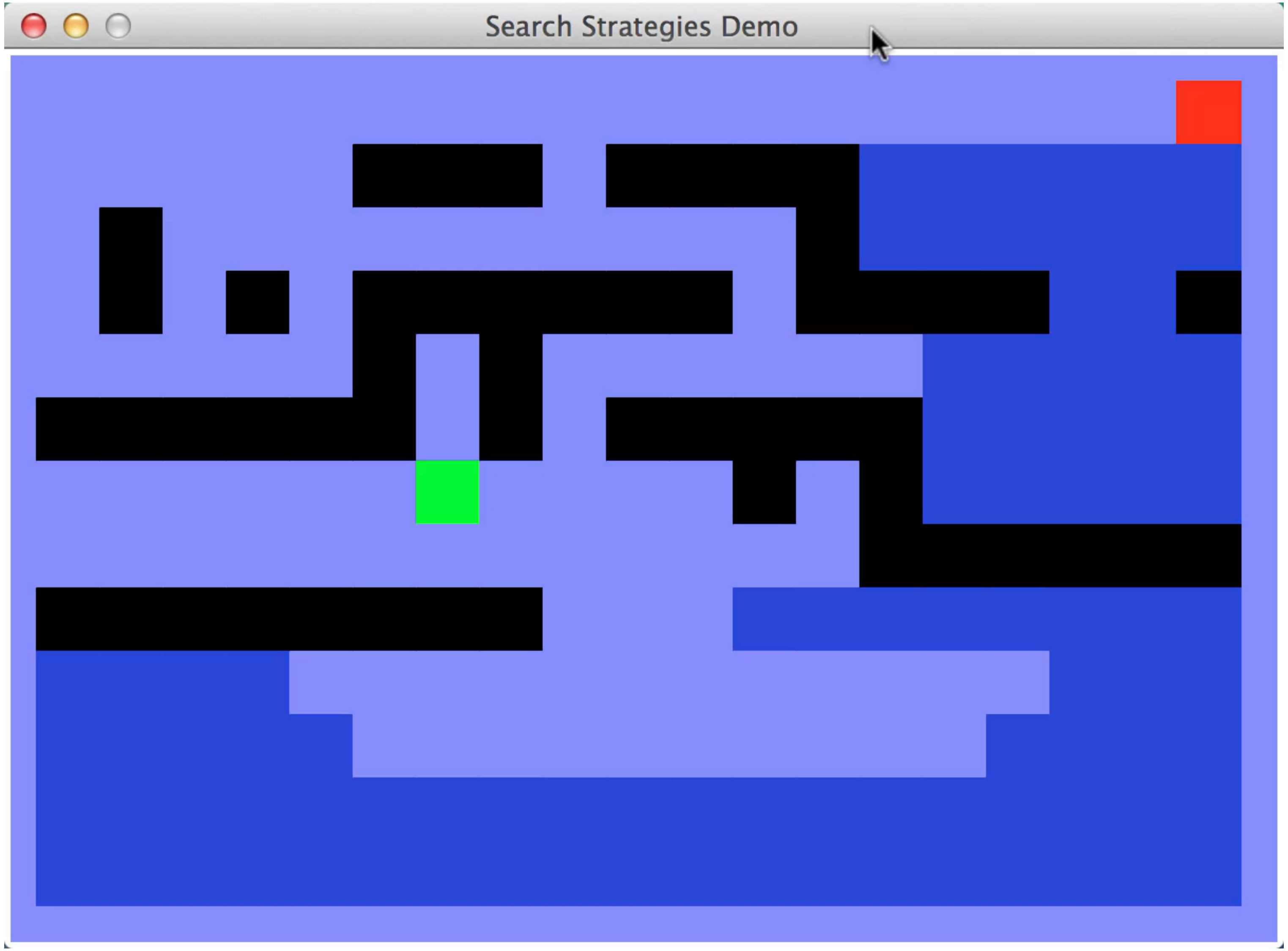
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C* and arcs cost at least ε , then the "effective depth" is roughly C*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the frontier take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming C* is finite and $\varepsilon > 0$, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



Video of Demo Empty UCS



Video of Demo Maze with Deep/Shallow Water --- BFS or UCS? (part 1)



Video of Demo Maze with Deep/Shallow Water --- BFS or UCS? (part 2)