CSE 573: Introduction to Artificial Intelligence

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Search
(Un-informed, Informed Search)

slides adapted from
Dan Klein, Pieter Abbeel ai.berkeley.edu
And Dan Weld, Luke Zettlemoyer
Recap: Search

- **Search problem:**
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test

- **Search tree:**
  - Nodes: represent plans for reaching states

- **Search algorithm:**
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans
Example: Pancake Problem

Cost: Number of pancakes flipped
Example: Pancake Problem

BOUND FOR SORTING BY PREFIX REVERSAL

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For a permutation $\sigma$ of the integers from 1 to $n$, let $f(\sigma)$ be the smallest number of prefix reversals that will transform $\sigma$ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all $\sigma$ in (the symmetric group) $S_n$. We show that $f(n) \leq (5n + 5)/3$, and that $f(n) \geq 17n/16$ for $n$ a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leq g(n) \leq 2n + 3$. 
Example: Pancake Problem

State space graph with costs as weights
General Tree Search

function **Tree-Search**( problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
end

Action: flip top two
Cost: 2

Path to reach goal:
Flip four, flip three
Total cost: 7
Uniform Cost Issues

- Remember: UCS explores increasing cost contours

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location

- We’ll fix that soon!
Up next: Informed Search

- Uninformed Search
  - DFS
  - BFS
  - UCS

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
  - Graph Search
A heuristic is:

- A function that estimates how close a state is to a goal
- Designed for a particular search problem
- Pathing?
- Examples: Manhattan distance, Euclidean distance for pathing.
Example: Heuristic Function

$h(x)$

Straight-line distance to Bucharest:
- Arad: 366
- Bucharest: 0
- Craiova: 160
- Dobresta: 242
- Eforie: 161
- Fagaras: 178
- Giurgiu: 77
- Hirsova: 151
- Iasi: 226
- Lugoj: 244
- Mehadia: 241
- Neamt: 234
- Oradea: 380
- Pitești: 98
- Râmnicu Vâlcea: 193
- Sibiu: 253
- Timisoara: 329
- Urziceni: 80
- Vaslui: 199
- Zerind: 374
Example: Heuristic Function

Heuristic: the number of the largest pancake that is still out of place

\[ h(x) \]
Greedy Search
Greedy Search

- Expand the node that seems closest...

- Is it optimal?
  - No. Resulting path to Bucharest is not the shortest!
Greedy Search

- **Strategy**: expand a node that you think is closest to a goal state
  - **Heuristic**: estimate of distance to nearest goal for each state

- A common case:
  - Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS
Video of Demo Contours Greedy (Empty)
Video of Demo Contours Greedy (Pacman Small Maze)
A* Search
A* Search
Combining UCS and Greedy

- **Uniform-cost** orders by path cost, or *backward cost* $g(n)$
- **Greedy** orders by goal proximity, or *forward cost* $h(n)$

- **A* Search** orders by the sum: $f(n) = g(n) + h(n)$

Example: Teg Grenager
When should A* terminate?

- Should we stop when we enqueue a goal?
  - No: only stop when we dequeue a goal

```
<table>
<thead>
<tr>
<th></th>
<th>S</th>
<th>A</th>
<th>B</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>g</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>h +</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>
```

- S -> A: 2 2 4
- S -> B: 2 1 3
- S -> B -> G: 5 0 5
- S -> A -> G: 4 0 4
What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!
Idea: Admissibility

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs
Admissible Heuristics

○ A heuristic \( h \) is *admissible* (optimistic) if:

\[
0 \leq h(n) \leq h^*(n)
\]

where \( h^*(n) \) is the true cost to a nearest goal.

○ Examples:

○ Coming up with admissible heuristics is most of what’s involved in using A* in practice.
Properties of A*

Uniform-Cost

A*
UCS vs A* Contours

- Uniform-cost expands equally in all “directions”

- A* expands mainly toward the goal, but does hedge its bets to ensure optimality
Comparison

Greedy  Uniform Cost  A*
Video of Demo Contours (Empty) -- UCS
Video of Demo Contours (Empty) -- Greedy
Video of Demo Contours (Empty) – A*
Video of Demo Contours (Pacman Small Maze) – A*
Which algorithm?
Which algorithm?
Optimality of A* Tree Search
Optimality of A* Tree Search

Assume:
- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:
- A will exit the fringe before B
Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$

\[
\begin{align*}
  f(n) &= g(n) + h(n) & \text{Definition of f-cost} \\
  f(n) &\leq g(A) & \text{Admissibility of } h \\
  g(A) &= f(A) & h = 0 \text{ at a goal}
\end{align*}
\]
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$

\[
g(A) < g(B) \quad \text{B is suboptimal} \\
f(A) < f(B) \quad h = 0 \text{ at a goal}
\]
Optimality of A* Tree Search: Blocking

Proof:
- Imagine B is on the fringe
- Some ancestor $n$ of A is on the fringe, too (maybe A!)
- Claim: $n$ will be expanded before B
  1. $f(n)$ is less or equal to $f(A)$
  2. $f(A)$ is less than $f(B)$
  3. $n$ expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

$f(n) \leq f(A) < f(B)$
A*: Summary
A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible (optimistic) heuristics
- Heuristic design is key: often use relaxed problems
Video of Demo Empty Water Shallow/Deep – Guess Algorithm
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics.

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available.

- Inadmissible heuristics are often useful too.
Example: 8 Puzzle

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

Admissible heuristics?
8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- $h(\text{start}) = 8$
- This is a relaxed-problem heuristic

Start State

Goal State

Average nodes expanded when the optimal path has...

<table>
<thead>
<tr>
<th></th>
<th>...4 steps</th>
<th>...8 steps</th>
<th>...12 steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>112</td>
<td>6,300</td>
<td>$3.6 \times 10^6$</td>
</tr>
<tr>
<td>TILES</td>
<td>13</td>
<td>39</td>
<td>227</td>
</tr>
</tbody>
</table>

Statistics from Andrew Moore
What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

Total Manhattan distance

Why is it admissible?

\[ 3 + 1 + 2 + \ldots = 18 \]

\( h(\text{start}) = \)

| Average nodes expanded when the optimal path has... |
|-----------------------------------------------|---|---|---|
| ...4 steps                                    | ...8 steps | ...12 steps |
| TILES                                         | 13 | 39 | 227 |
| MANHATTAN                                     | 12 | 25 | 73  |
How about using the *actual cost* as a heuristic?
- Would it be admissible?
- Would we save on nodes expanded?
- What’s wrong with it?

With A*: a trade-off between quality of estimate and work per node
- As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself
Example: Pancake Problem

- **Action:** Flip over top $n$ pancakes

- **Cost:** Number of pancakes
Semi-Lattice of Heuristics
Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

Heuristics form a semi-lattice:
- Max of admissible heuristics is admissible
  $$h(n) = \max(h_a(n), h_b(n))$$

Trivial heuristics
- Bottom of lattice is the zero heuristic (what does this give us?)
- Top of lattice is the exact heuristic
Graph Search
Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.
Graph Search

- In BFS, for example, we shouldn’t bother expanding the circled nodes (why?)
Graph Search

- Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but...
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

- Important: store the closed set as a set, not a list

- Can graph search wreck completeness? Why/why not?

- How about optimality?
A* Graph Search Gone Wrong?

State space graph

Search tree

Closed Set: S B C A
Consistency of Heuristics

- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal
    \[ h(A) \leq \text{actual cost from A to G} \]
  - Consistency: heuristic “arc” cost ≤ actual cost for each arc
    \[ h(A) - h(C) \leq \text{cost(A to C)} \]

- Consequences of consistency:
  - The f value along a path never decreases
    \[ h(A) \leq \text{cost(A to C)} + h(C) \]
  - A* graph search is optimal
A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal
Optimality of A* Search

- With a admissible heuristic, Tree A* is optimal.
- With a consistent heuristic, Graph A* is optimal.
- With h=0, the same proof shows that UCS is optimal.
Pseudo-Code

function TREE-SEARCH(problem, fringe) return a solution, or failure
  fringe ← INSERT(make-node(initial-state[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, state[node]) then return node
    for child-node in EXPAND(state[node], problem) do
      fringe ← INSERT(child-node, fringe)
    end
  end
end

function GRAPH-SEARCH(problem, fringe) return a solution, or failure
  closed ← an empty set
  fringe ← INSERT(make-node(initial-state[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, state[node]) then return node
    if state[node] is not in closed then
      add state[node] to closed
      for child-node in EXPAND(state[node], problem) do
        fringe ← INSERT(child-node, fringe)
      end
    end
  end
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …
A* in Recent Literature

- Joint A* CCG Parsing and Semantic Role Labeling (EMLN’15)

- Diagram Understanding (ECCV’17)
Search and Models

- Search operates over models of the world
  - The agent doesn’t actually try all the plans out in the real world!
  - Planning is all “in simulation”
  - Your search is only as good as your models…
Search Gone Wrong?