CSE 473: Artificial Intelligence  
Spring 2018  
Heuristics & Pattern Databases for Search  
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With thanks to Dan Weld, Dan Klein, Richard Korf, Stuart Russell, Andrew Moore, and Luke Zettlemoyer

Recap: Search Problem

- States
  - configurations of the world
- Successor function:
  - function from states to lists of (state, action, cost) triples
- Start state
- Goal test

N-Queens as Search?

- Given N x N chess board
- Can you place N queens so they don’t fight?

States are Board Positions

Search Methods

- Depth first search (DFS)
- Breadth first search (BFS)
- Iterative deepening depth-first search (IDS)
- Best first search
- Uniform cost search (UCS)
- Greedy search
- A*
- Iterative Deepening A* (IDA*)
- Beam search, hill climbing
- Stochastic Search
- Constraint Satisfaction

IDA* for N-Queens?

- Given N x N chess board
- Can you place N queens so they don’t fight?
Best-First Search
- Generalization of breadth-first search
- Fringe = Priority queue of nodes to be explored
- Cost function $f(n)$ applied to each node

Add initial state to priority queue
While queue not empty
  Node = head(queue)
  If goal?(node) then return node
  Add children of node to queue

"expanding the node"

Iterative-Deepening A*
- Like iterative-deepening depth-first, but...
- Depth bound modified to be an $f$-limit
  - Start with $f$-limit = $h$(start)
  - Prune any node if $f$(node) > $f$-limit
  - Next $f$-limit = min-cost of any node pruned

IDA* Analysis
- Complete & Optimal (a la A*)
- Space usage $\propto$ depth of solution
- Each iteration is DFS - no priority queue!
- # nodes expanded relative to A*
  - Depends on # unique values of heuristic function
  - In 8 puzzle: few values $\Rightarrow$ close to # A* expands
  - In eastern-europe travel: each $f$ value is unique
    $\Rightarrow 1+2+\ldots+n = O(n^2)$ where $n$ nodes A* expands
    if $n$ is too big for main memory, $n^2$ is too long to wait!
  - Generates duplicate nodes in cyclic graphs

Beam Search
- Idea
  - Best first
  - But discard all but $N$ best items on priority queue
- Evaluation
  - Complete? No
  - Time Complexity? $O(b^d)$
  - Space Complexity? $O(b + N)$

Hill Climbing
- Idea
  - Always choose best child; no backtracking
  - Beam search with $|\text{queue}| = 1$
- Problems?
  - Local maxima
  - Plateaus
  - Diagonal ridges

Heuristics
It's what makes search actually work
Admissible Heuristics

- $f(x) = g(x) + h(x)$
- $g$: cost so far
- $h$: underestimate of remaining costs

Where do heuristics come from?

Relaxed Problems

- Derive admissible heuristic from exact cost of a solution to a relaxed version of problem
- For blocks world, distance = # move operations
- heuristic = number of misplaced blocks
- What is relaxed problem?

What's being relaxed?
Heuristic = Euclidean distance

Traveling Salesman Problem
Objective: shortest path visiting every city

What can be Relaxed?

Heuristics for eight puzzle

- What can we relax?

$h1 = \text{number of tiles in wrong place}$

$h2 = \sum \text{distances of tiles from correct loc}$

Importance of Heuristics

h1 = number of tiles in wrong place
Importance of Heuristics

\[ h_1 = \text{number of tiles in wrong place} \]
\[ h_2 = \sum \text{distances of tiles from correct loc} \]

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Decrease effective branching factor

Need More Power!

Performance of Manhattan Distance Heuristic

- 8 Puzzle < 1 second
- 15 Puzzle 1 minute
- 24 Puzzle 65000 years

Need even better heuristics!

Subgoal Interactions

- Manhattan distance assumes
  - Each tile can be moved independently of others
- Underestimates because
  - Doesn’t consider interactions between tiles

Pattern Databases

- Pick any subset of tiles
  - E.g., 3, 7, 11, 12, 13, 14, 15
  - (or as drawn)
- Precompute a table
  - Optimal cost of solving just these tiles
  - For all possible configurations
    - 57 Million in this case
  - Use A* or IDA*
  - State = position of just these tiles (& blank)

Using a Pattern Database

- As each state is generated
  - Use position of chosen tiles as index into DB
  - Use lookup value as heuristic, h(n)
  - Admissible?

Combining Multiple Databases

- Can choose another set of tiles
- Precompute multiple tables
- How combine table values?
  - E.g. Optimal solutions to Rubik’s cube
    - First found w/ IDA* using pattern DB heuristics
    - Multiple DBs were used (dif cubie subsets)
    - Most problems solved optimally in 1 day
    - Compare with 574,000 years for IDDFS
Drawbacks of Standard Pattern DBs

- Since we can only take max
- Diminishing returns on additional DBs
- Would like to be able to add values

Disjoint Pattern DBs

- Partition tiles into disjoint sets
  - For each set, precompute table
  - E.g. 8 tile DB has 519 million entries
  - And 7 tile DB has 58 million
- During search
  - Look up heuristic values for each set
  - Can add values without overestimating!
- Manhattan distance is a special case of this idea where each set is a single tile

Performance

- 15 Puzzle: 2000x speedup vs Manhattan dist
  - IDA* with the two DBs shown previously solves 15 Puzzles optimally in 30 milliseconds
- 24 Puzzle: 12 million x speedup vs Manhattan
  - IDA* can solve random instances in 2 days.
  - Requires 4 DBs as shown
    - Each DB has 128 million entries
    - Without PDBs: 65,000 years