

Reinforcement Learning

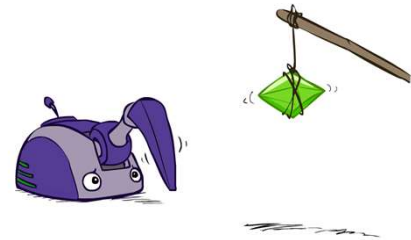


Steve Tanimoto

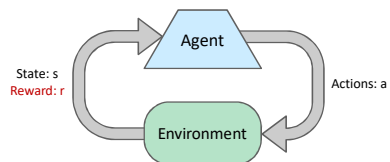
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[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

Reinforcement Learning



Reinforcement Learning



Basic idea:

- Receive feedback in the form of **rewards**
- Agent's utility is defined by the reward function
- Must (learn to) act so as to **maximize expected rewards**
- All learning is based on observed samples of outcomes!

Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

[Kohl and Stone, ICRA 2004]

Example: Learning to Walk



Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk



Training

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – training]

Example: Learning to Walk



Finished

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – finished]

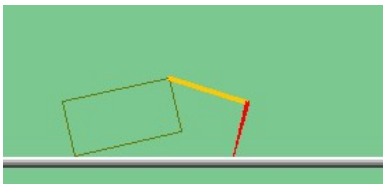
Example: Toddler Robot



[Tedrake, Zhang and Seung, 2005]

[Video: TODDLER – 40s]

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 3]

Video of Demo Crawler Bot



Reinforcement Learning

Still assume a Markov decision process (MDP):

- A set of states $s \in S$
- A set of actions (per state) A
- A model $T(s,a,s')$
- A reward function $R(s,a,s')$

Still looking for a policy $\pi(s)$

New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
- Must actually try actions and states out to learn



Offline (MDPs) vs. Online (RL)

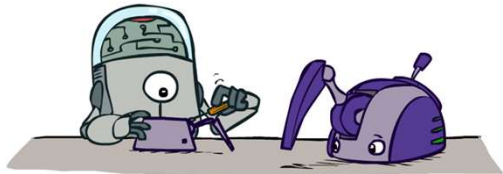


Offline Solution



Online Learning

Model-Based Learning



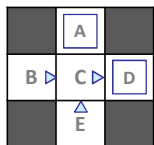
Model-Based Learning

- Model-Based Idea:
 - Learn an approximate model based on experiences
 - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
 - Count outcomes s' for each s, a
 - Normalize to give an estimate of $\hat{T}(s, a, s')$
 - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
- Step 2: Solve the learned MDP
 - For example, use value iteration, as before



Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

$T(B, \text{east}, C) = 1.00$
 $T(C, \text{east}, D) = 0.75$
 $T(C, \text{east}, A) = 0.25$
...

$\hat{R}(s, a, s')$

$R(B, \text{east}, C) = -1$
 $R(C, \text{east}, D) = -1$
 $R(D, \text{exit}, x) = +10$
...

Example: Expected Age

Goal: Compute expected age of CSE 473 students

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without $P(A)$, instead collect samples $[a_1, a_2, \dots, a_N]$

Unknown $P(A)$: "Model Based"

Why does this work? Because eventually you learn the right model.

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown $P(A)$: "Model Free"

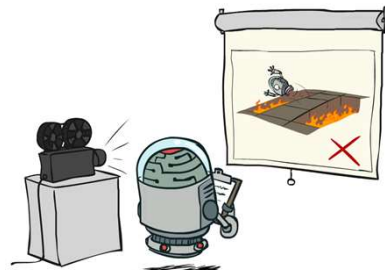
Why does this work? Because samples appear with the right frequencies.

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Model-Free Learning



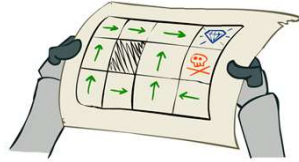
Passive Reinforcement Learning



Passive Reinforcement Learning

Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don't know the transitions $T(s,a,s')$
- You don't know the rewards $R(s,a,s')$
- Goal: learn the state values



In this case:

- Learner is "along for the ride"
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.

Direct Evaluation

- Goal: Compute values for each state under π

Idea: Average together observed sample values

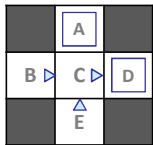
- Act according to π
- Every time you visit a state, write down what the sum of discounted rewards turned out to be
- Average those samples



- This is called direct evaluation

Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

		-10	
	A		
+8	B	+4	+10
	C		D
	E	-2	

Problems with Direct Evaluation

What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

Output Values

		-10	
	A		
+8	B	+4	+10
	C		D
	E	-2	

If B and E both go to C under this policy, how can their values be different?

What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

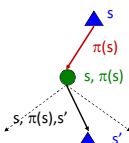
Why Not Use Policy Evaluation?

Simplified Bellman updates calculate V for a fixed policy:

- Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!

Key question: how can we do this update to V without knowing T and R ?

- In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

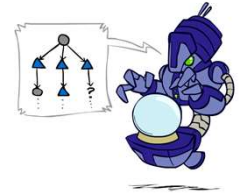
$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^\pi(s'_1)$$

$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^\pi(s'_2)$$

...

$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^\pi(s'_n)$$

$$V_{k+1}^\pi(s) \leftarrow \frac{1}{n} \sum_i sample_i$$



Temporal Difference Learning

- Big idea: learn from every experience!
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
 - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
 - Policy still fixed, still doing evaluation!
 - Move values toward value of whatever successor occurs: running average

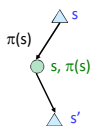
A diagram illustrating a state transition. A blue triangle labeled 's' is at the top. A green circle labeled 'a' is in the middle. A blue triangle labeled 's'' is at the bottom. Arrows point from 's' to 'a' and from 'a' to 's''. To the left of the 'a' node is the text $\pi(s)$. To the right of the 'a' node is the text $s, \pi(s)$.

Sample of $V(s)$: $sample = R(s, \pi(s), s') + \gamma V^n(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$

- **Big idea: learn from every experience!**
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
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- **Temporal difference learning of values**
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Sample of $V(s)$: $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)\text{sample}$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(\text{sample} - V^\pi(s))$

Exponential Moving Average

- Exponential moving average
 - The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
 - Makes recent samples more important:
$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$
 - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

- Exponential moving average
 - The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

	A	
B	C	D
	E	

Observed Transitions

B, east, C, -2

	0	
0	0	8
	0	

C, east, D, -2

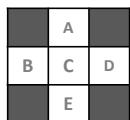
	0	
-1	0	8
	0	

Assume: $\gamma = 1$, $\alpha = 1/2$

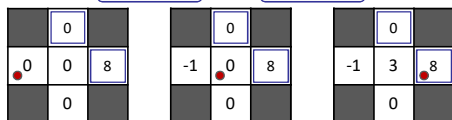
$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

States

Observed Transitions



Assume: $\gamma = 1, \alpha = 1/2$



$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

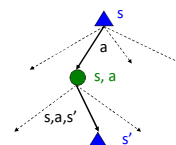
- Idea: learn Q-values, not values
- Makes action selection model-free too!

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
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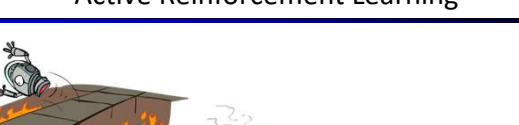
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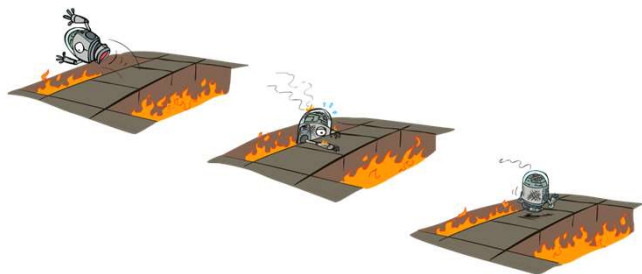
- Idea: learn Q-values, not values
- Makes action selection model-free too!



Active Reinforcement Learning





The image illustrates the concept of Active Reinforcement Learning through a sequence of three cartoon illustrations. Each illustration shows a 3x3 grid world with a robot (a small, round, grey vehicle with a single wheel and a small antenna) and a trail of fire representing the robot's path. The robot starts at the top-left cell (1,1) and moves to the top-middle cell (1,2). In the second frame, the robot is at the top-middle cell (1,2) and has moved to the bottom-middle cell (3,2). In the third frame, the robot is at the bottom-middle cell (3,2) and has moved to the bottom-right cell (3,3). The fire trail indicates the sequence of actions taken by the robot.



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - You choose the actions now
 - Goal: learn the optimal policy / values
- In this case:
 - Learner makes choices!
 - Fundamental tradeoff: exploration vs. exploitation
 - This is NOT offline planning! You actually take actions in the world and find out what happens...

A small, round, grey robot with a single eye and two antennae is standing on a grey grid. The grid is surrounded by bright orange and yellow flames, suggesting a dangerous environment. The robot appears to be exploring or navigating through the fire.

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - You choose the actions now
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 - In this case:
 - Learner makes choices!
 - Fundamental tradeoff: exploration vs. exploitation
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- 
- A small, grey, cylindrical robot with a single wheel and a sensor on top is positioned on a dark grey tiled floor. The floor is surrounded by bright orange and yellow flames, suggesting a hazardous environment. The robot appears to be navigating or exploring this environment.



Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values

- Start with $V_0(s) = 0$, which we know is right
- Given V_k , calculate the depth $k+1$ values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

- But Q-values are more useful, so compute them instead

- Start with $Q_0(s,a) = 0$, which we know is right
- Given Q_k , calculate the depth $k+1$ q-values for all q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma \max_{a'} Q_k(s', a')]$$

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma \max_{a'} Q_k(s', a')]$$

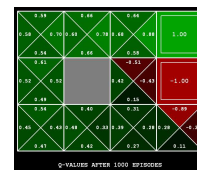
- Learn $Q(s,a)$ values as you go

- Receive a sample (s, a, s', r)
- Consider your old estimate: $Q(s, a)$
- Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [sample]$$



[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Video of Demo Q-Learning -- Gridworld



Video of Demo Q-Learning -- Crawler



Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!

- This is called **off-policy learning**

- Caveats:

- You have to explore enough
- You have to eventually make the learning rate small enough
- ... but not decrease it too quickly
- Basically, in the limit, it doesn't matter how you select actions (!)

