CSE 473: Artificial Intelligence

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Constraint Satisfaction Problems - Part 1 of 2

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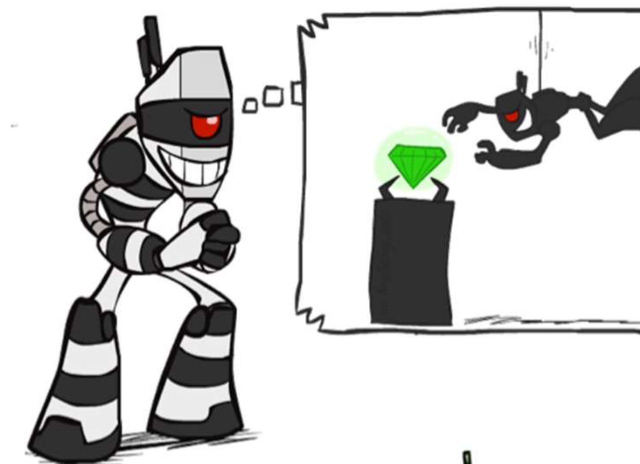
Previously

- Formulating problems as search
- Blind search algorithms
  - Depth first
  - Breadth first (uniform cost)
  - Iterative deepening
- Heuristic Search
  - Best first
    - Beam (Hill climbing)
    - A*
    - IDA*
- Heuristic generation
  - Exact soln to a relaxed problem
  - Pattern databases
- Local Search
  - Hill climbing, random moves, random restarts, simulated annealing

What is Search For?

lanning: sequences of actions

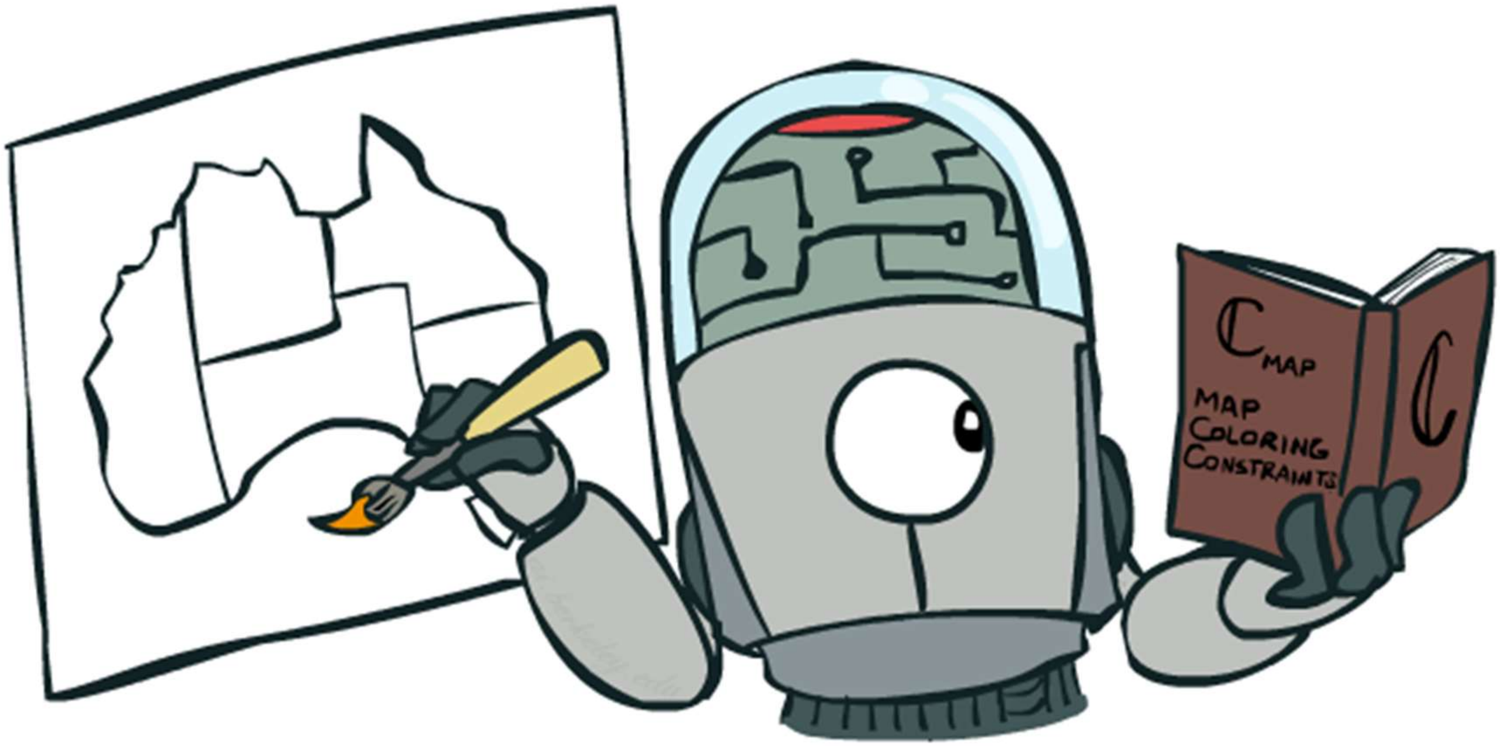
- The ***path to the goal*** is the important thing
- Paths have various costs, depths
- Assume little about problem structure



lentification: assignments to variables

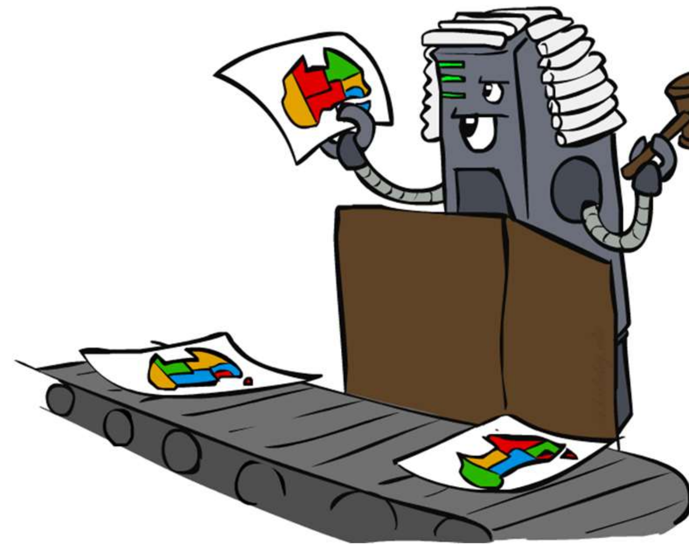
- The ***goal itself*** is important, ***not the path***
- All paths at the same depth (for some formulations)

Constraint Satisfaction Problems



CSPs are *structured* (factored) identification problems

Constraint Satisfaction Problems

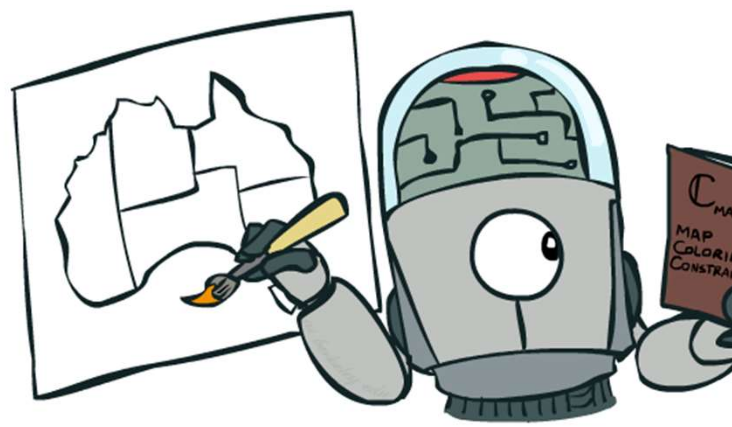


Standard search problems:

- State is a "black box": arbitrary data structure
- Goal test can be any function over states
- Successor function can also be anything

Constraint satisfaction problems (CSPs):

- A special subset of search problems
- State is defined by variables X_i with values from a domain D (sometimes D depends on i)
- Goal test is a set of constraints specifying allowable combinations of values for subsets of variables



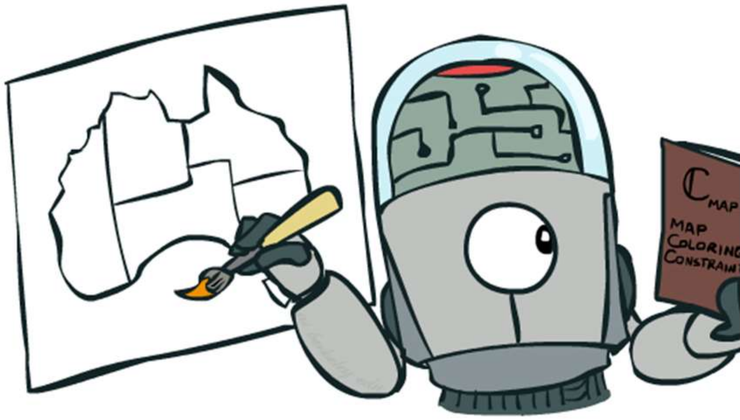
Making use of CSP formulation allows for optimized algorithms

- Typical example of trading generality for utility (in this case, speed)

Constraint Satisfaction Problems

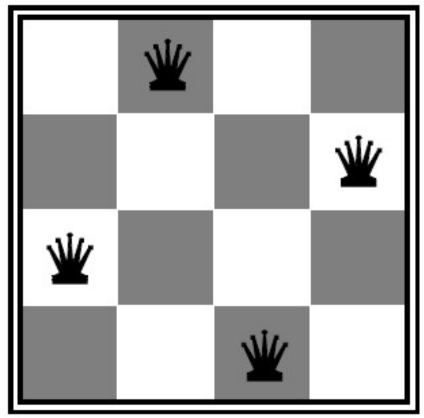
- "Factoring" the state space
- Representing the state space in a knowledge representation

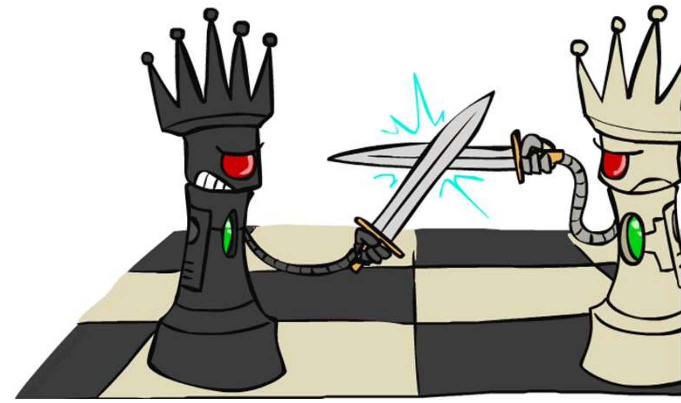
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables



CSP Example: N-Queens

Is there a queen at X_{ij}?





Formulation 1:

- Variables: X_{ij}
- Domains: $\{0, 1\}$
- Constraints

$$\forall i, j, k \;\; (X_{ij}, X_{ik}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{kj}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i, j, k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0),(0,1),(1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

CSP Example: N-Queens

What column is the queen on for row k?

ormulation 2:

Variables: Q_k

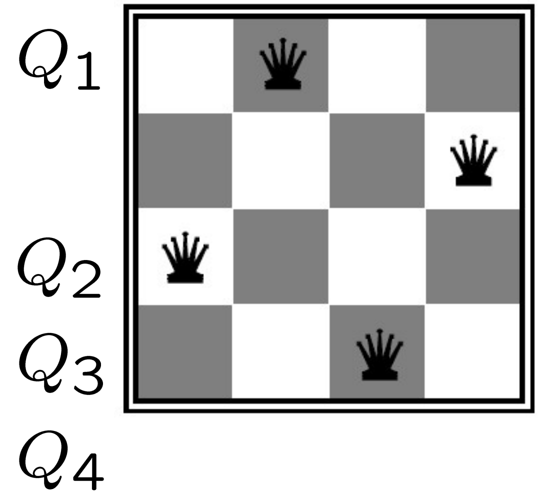
Domains: $\{1, 2, 3, \ldots N\}$

Constraints:

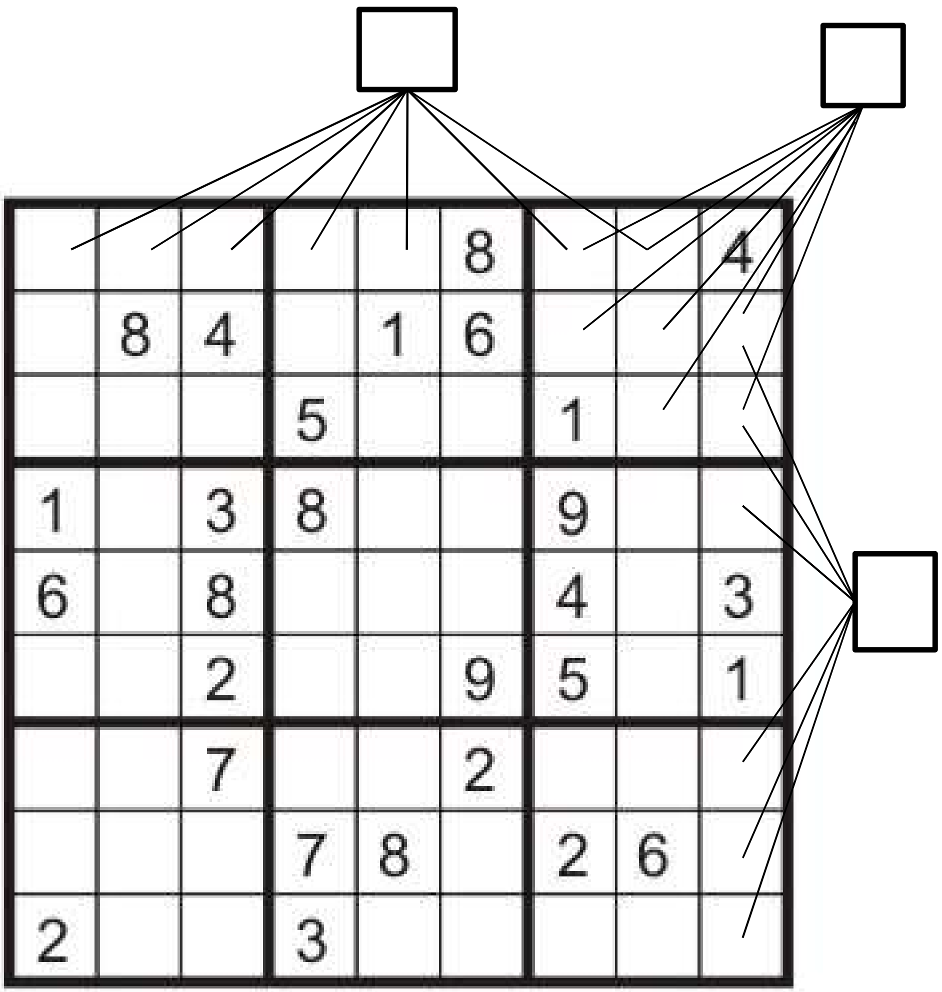
Implicit: $\forall i, j \ \text{non-threatening}(Q_i, Q_j)$

Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

$\cdots$



CSP Example: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

  9-way alldiff for each column

  9-way alldiff for each row

  9-way alldiff for each region

  (or can have a bunch of pairwise inequality constraints)

Propositional Logic

$$((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$$

Variables: propositional variables

Domains: {T, F}

Constraints: logical formula

CSP Example: Map Coloring

- Variables: WA, NT, Q, NSW, V, SA, T

- Domains: $D = \{red, green, blue\}$

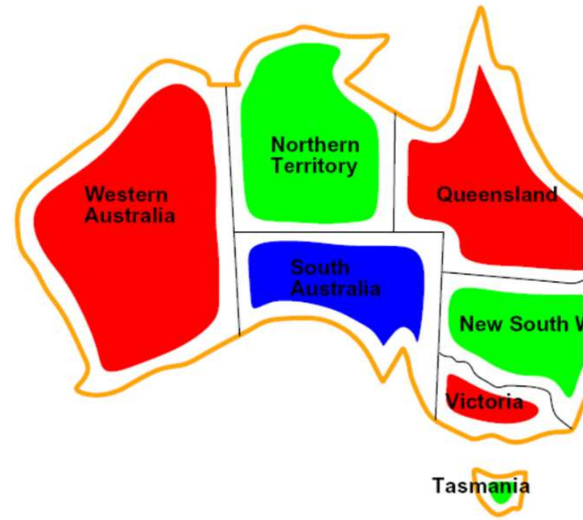
- Constraints: adjacent regions must have different colors

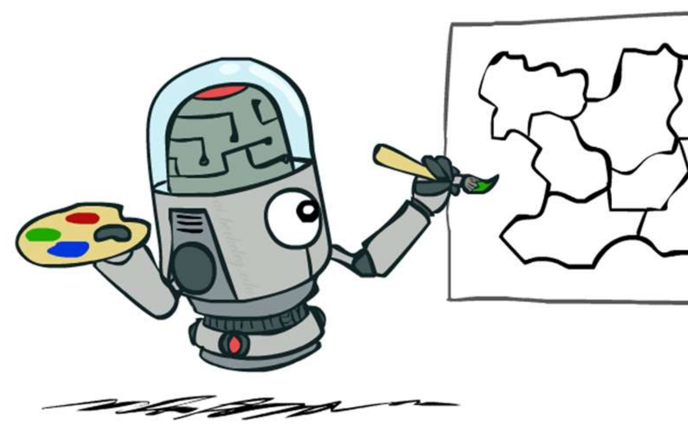
  Implicit: $WA \neq NT$

  Explicit: $(WA, NT) \in \{(red, green), (red, blue), \ldots\}$

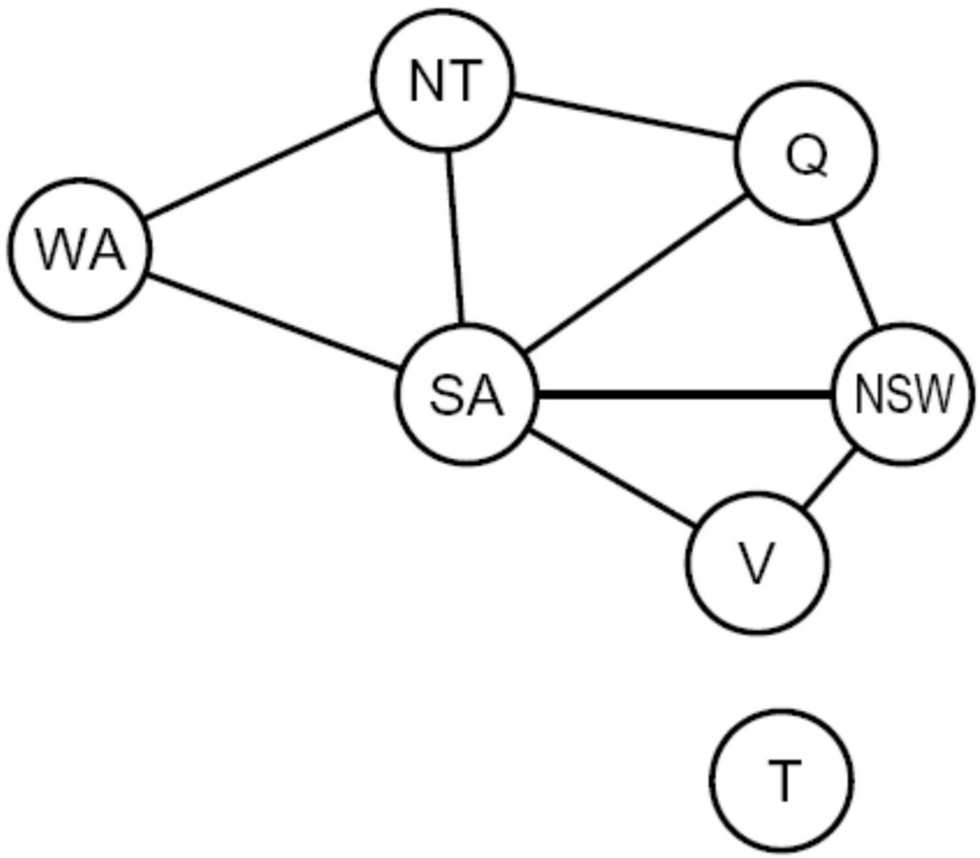
- Solutions are assignments satisfying all constraints, e.g.:

  {WA=red, NT=green, Q=red, NSW=green,
  V=red, SA=blue, T=green}



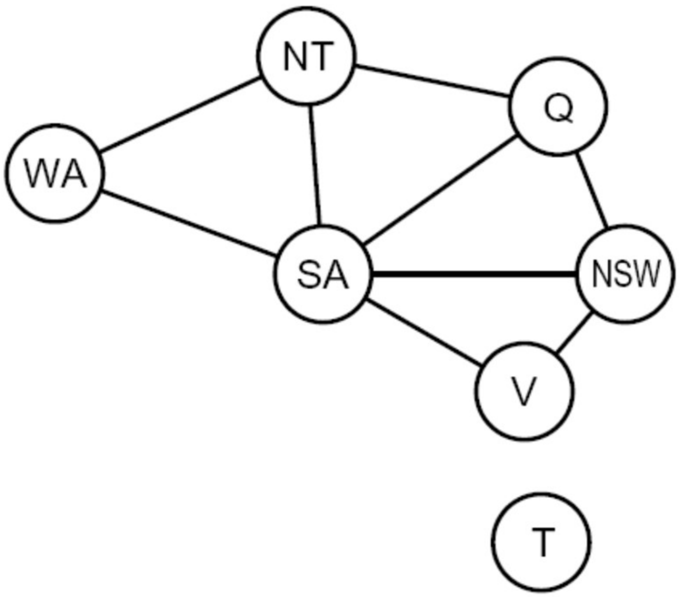


Constraint Graphs



Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



Example: Cryptarithmetic

Variables:

$F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$

Domains:

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

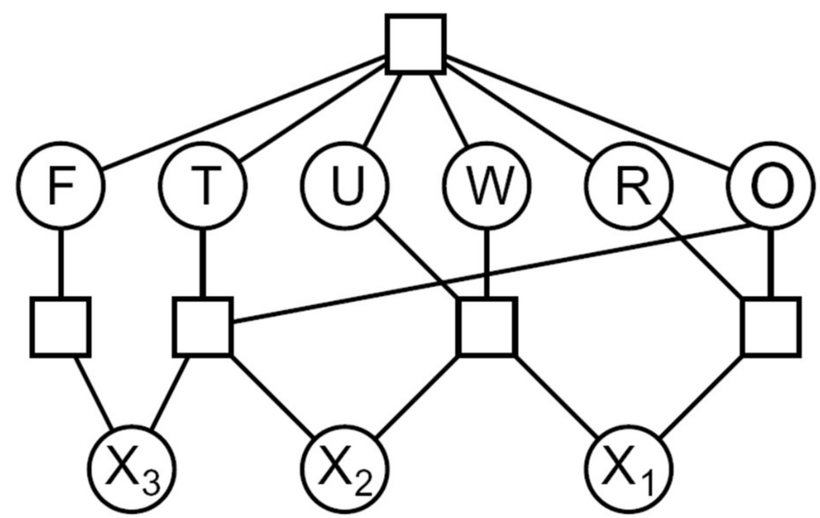
Constraints:

$\text{alldiff}(F, T, U, W, R, O)$

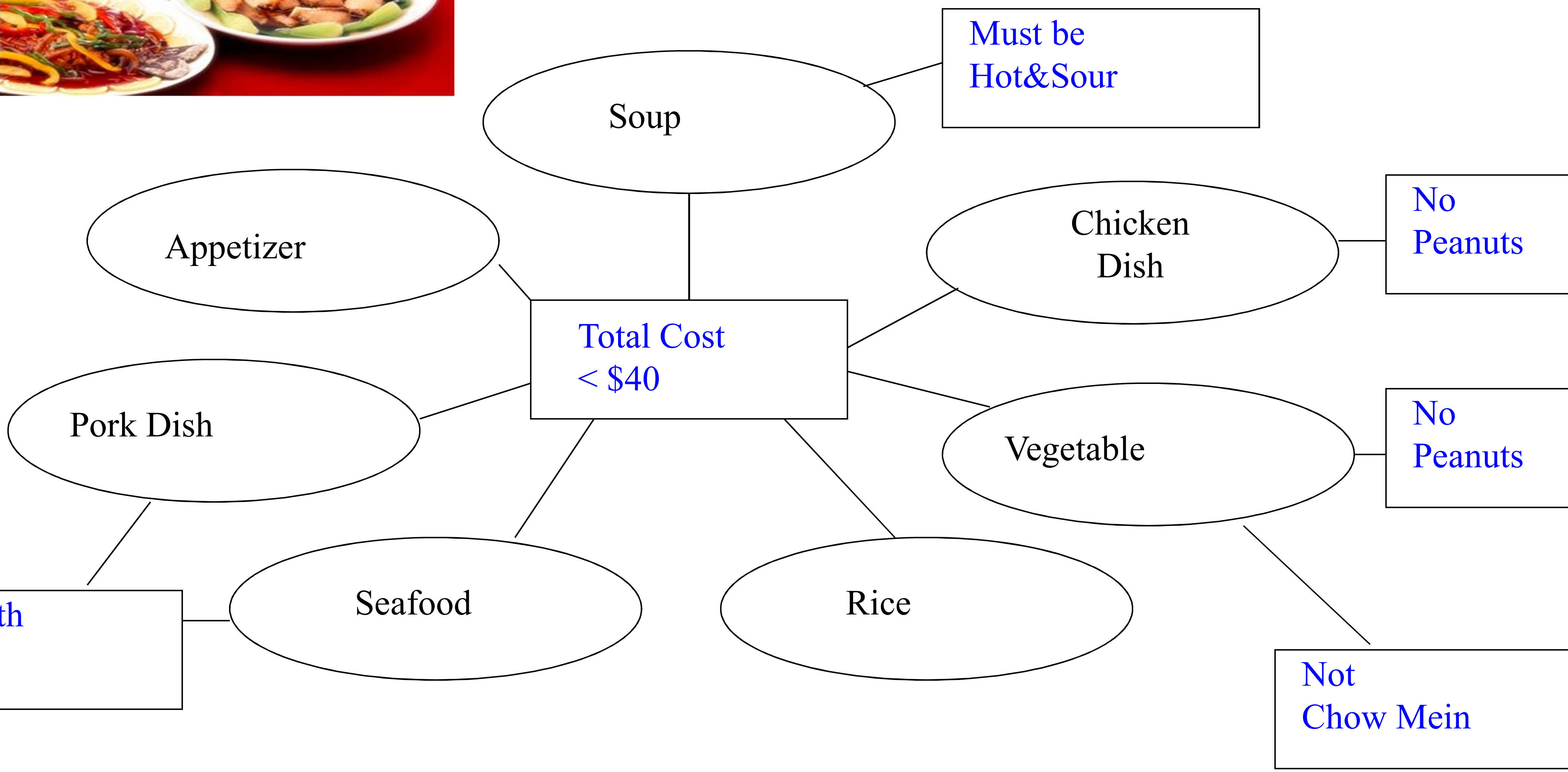
$O + O = R + 10 \cdot X_1$

$\cdots$

```
    T W O
+   T W O
---------
  F O U R
```



Chinese Constraint Network

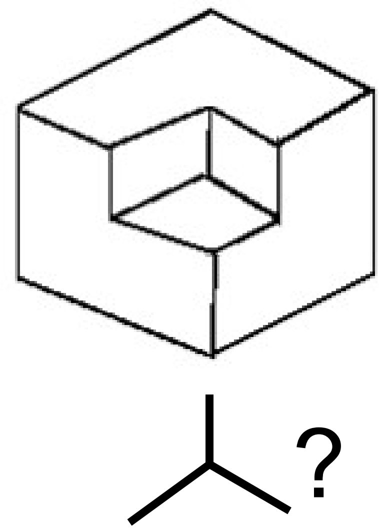


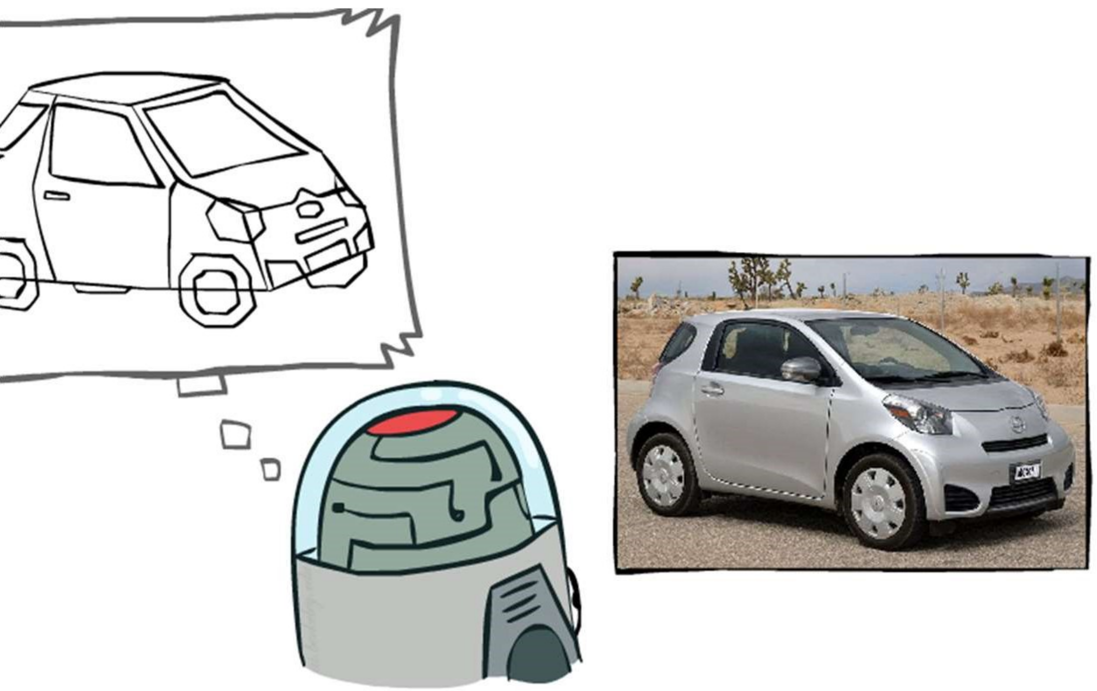
Real-World CSPs

- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Gate assignment in airports
- Space Shuttle Repair
- Transportation scheduling
- Factory scheduling
- … lots more!

Example: The Waltz Algorithm

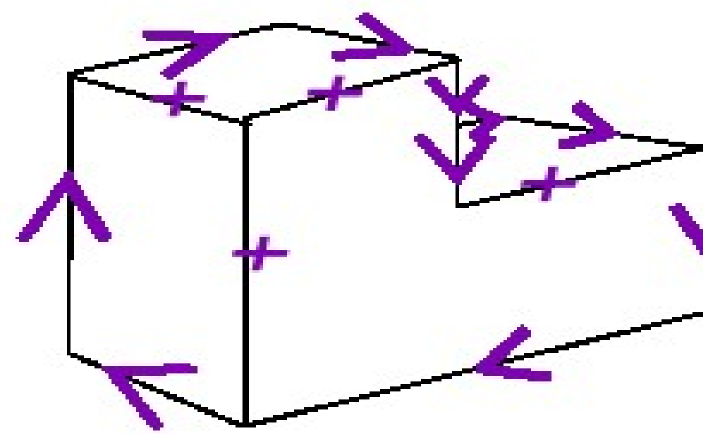
- he Waltz algorithm is for interpreting ne drawings of solid polyhedra as 3D bjects
- n early example of an AI computation osed as a CSP





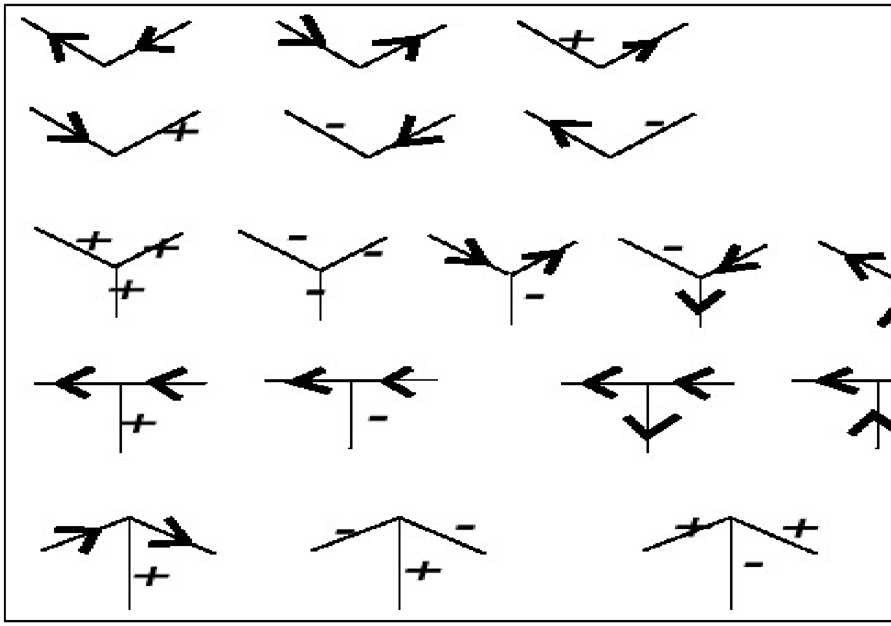
Waltz on Simple Scenes

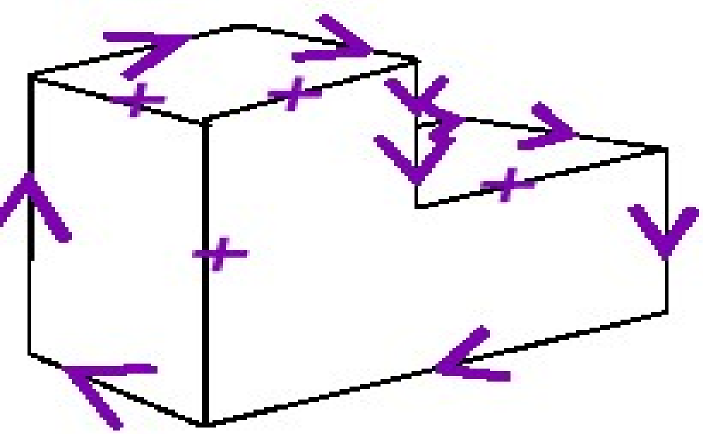
- Assume all objects:
  - Have no shadows or cracks
  - Three-faced vertices
  - “General position”: no junctions change with small movements of the eye.
- Then each line on image is one of the following:
  - Boundary line (edge of an object) (>) with right hand of arrow denoting “solid” and left hand denoting “space”
  - Interior convex edge (+)
  - Interior concave edge (-)



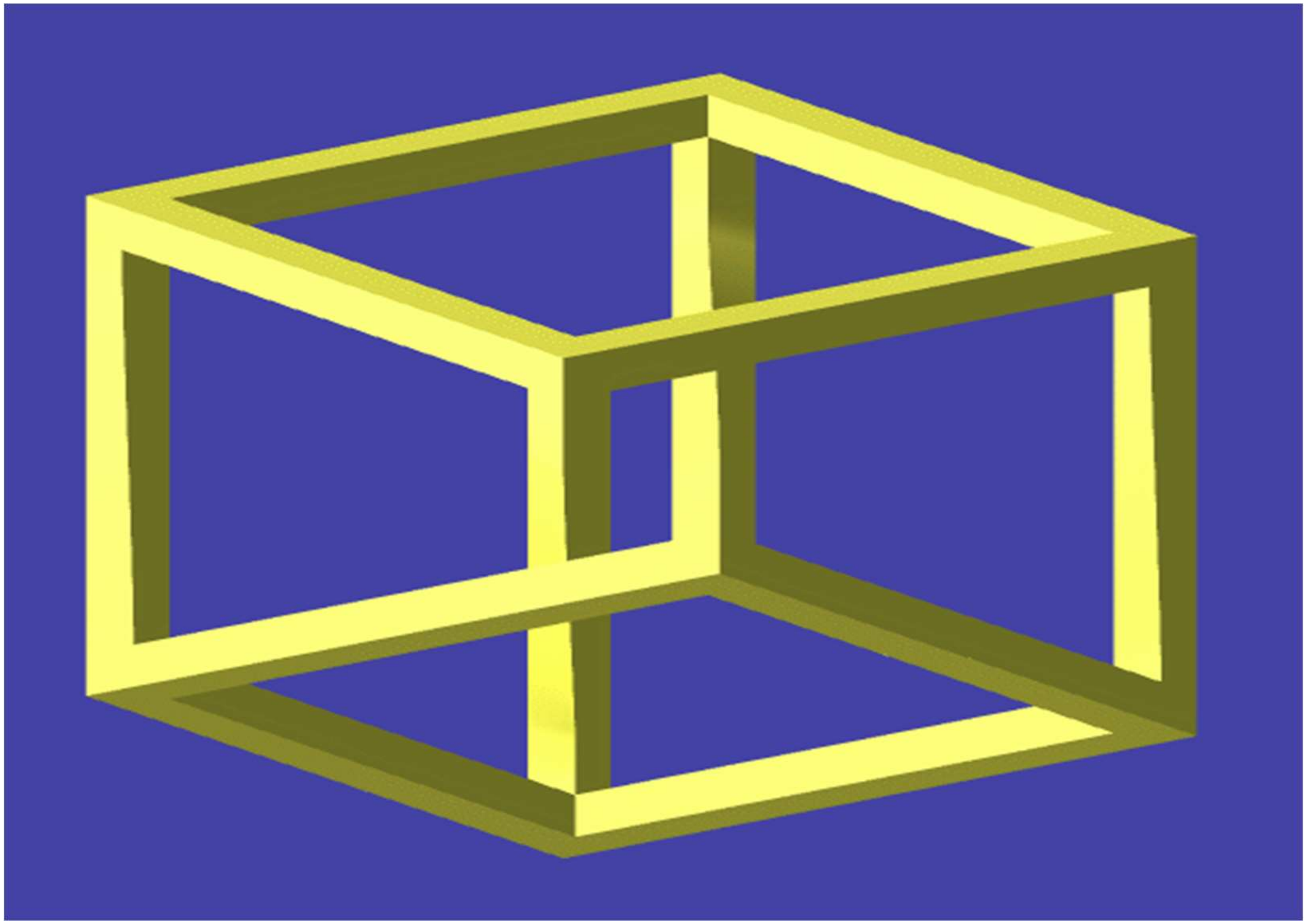
Legal Junctions

- Only certain junctions are physically possible
- How can we formulate a CSP to label an image?
- **Variables:** edges
- **Domains:** >, <, +, -
- **Constraints:** legal junction types

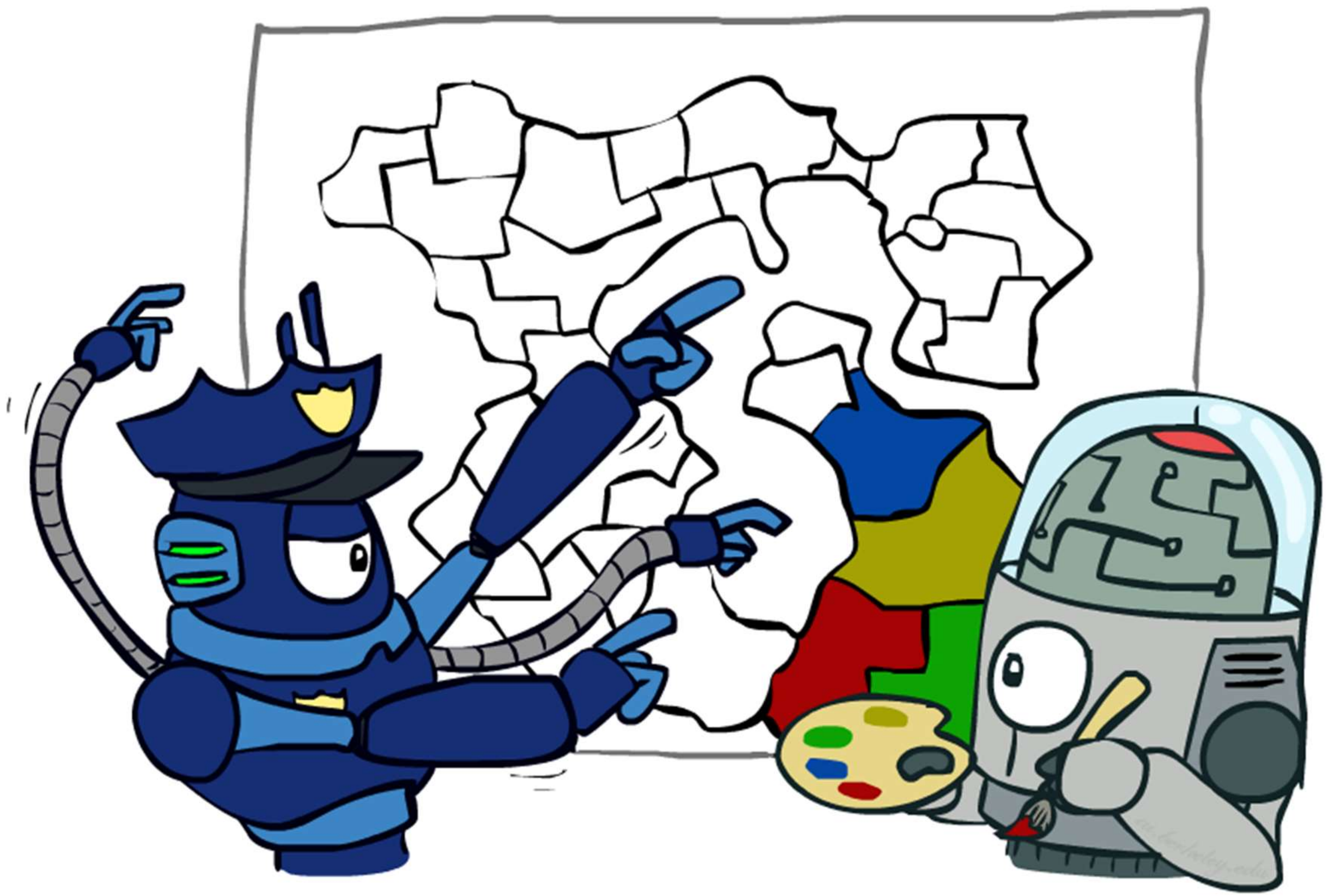




Slight Problem: Local *vs* Global Consistency

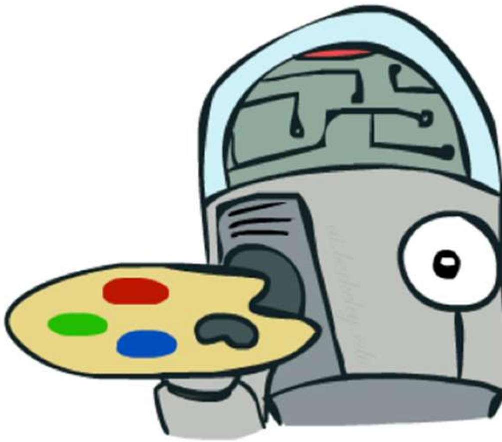


Varieties of CSPs



Varieties of CSP Variables

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by linear program methods (see CSE 521 for a bit of LP theory)

Varieties of CSP Constraints

Varieties of Constraints

- Unary constraints involve a single variable (equivalent reducing domains), e.g.:

$$SA \neq green$$

- Binary constraints involve pairs of variables, e.g.:

$$SA \neq WA$$

- Higher-order constraints involve 3 or more variables:
  e.g., cryptarithmetic column constraints

Preferences (soft constraints):

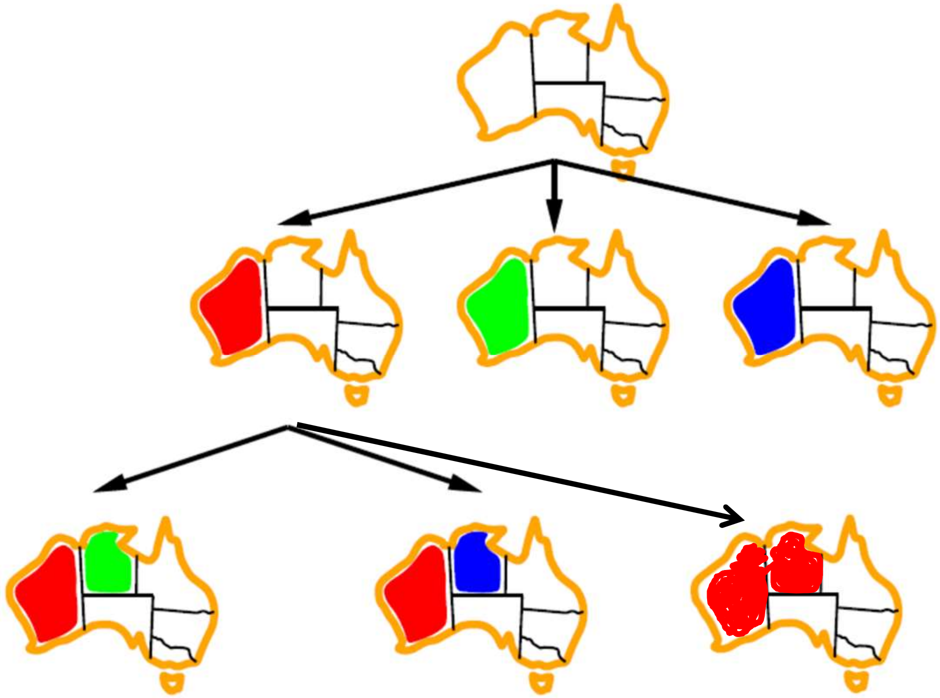
- E.g., red is better than green
- Often representable by a cost for each variable assignment
- Gives constrained optimization problems
- (We'll ignore these until we get to Bayes' nets)

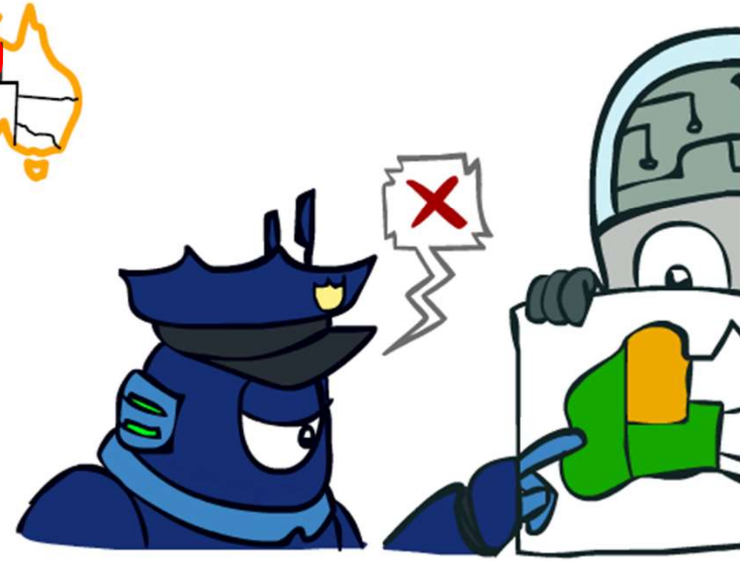
Solving CSPs

CSP as Search

- States
- Operators
- Initial State
- Goal State

Standard Depth First Search





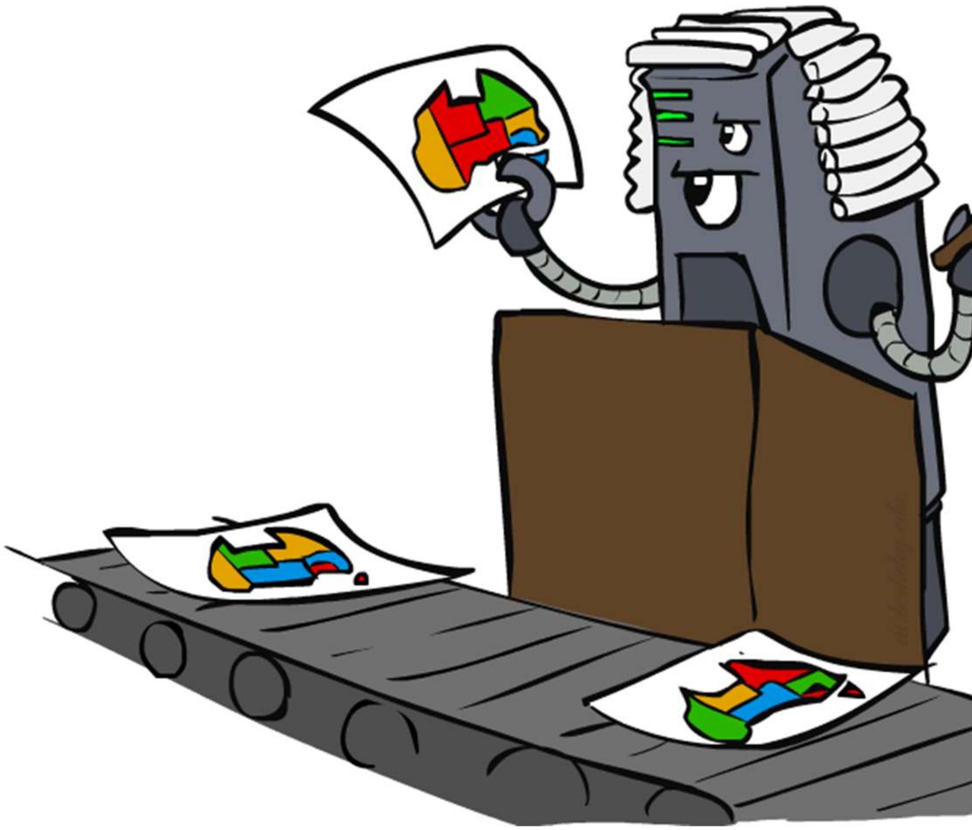
Standard Search Formulation

Standard search formulation of CSPs

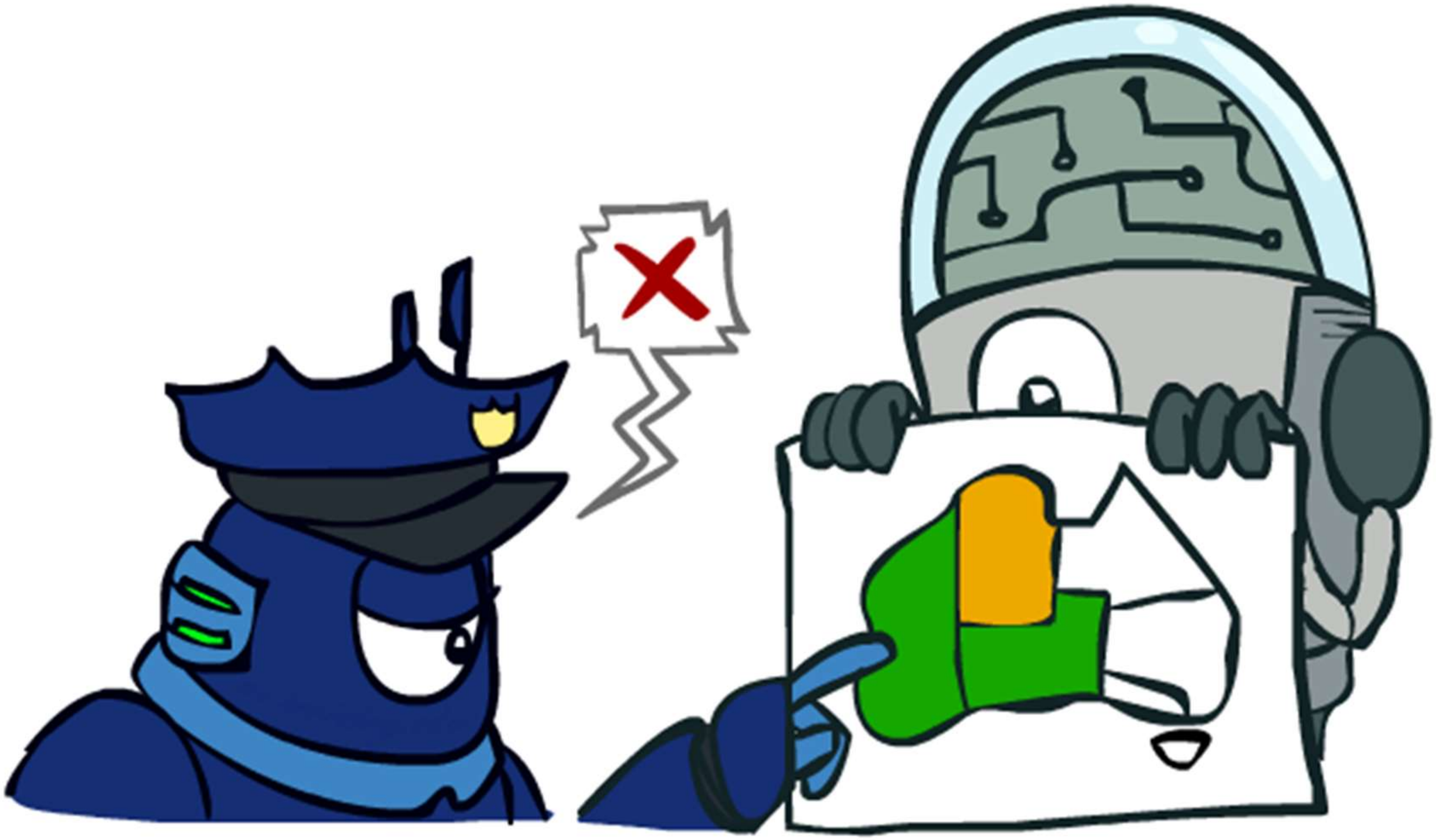
States defined by the values assigned so far (partial assignments)

- Initial state: the empty assignment, {}
- Successor function: assign a value to an unassigned variable
- **Goal test: the current assignment is complete and satisfies all constraints**

We'll start with the straightforward, naïve approach, then improve it

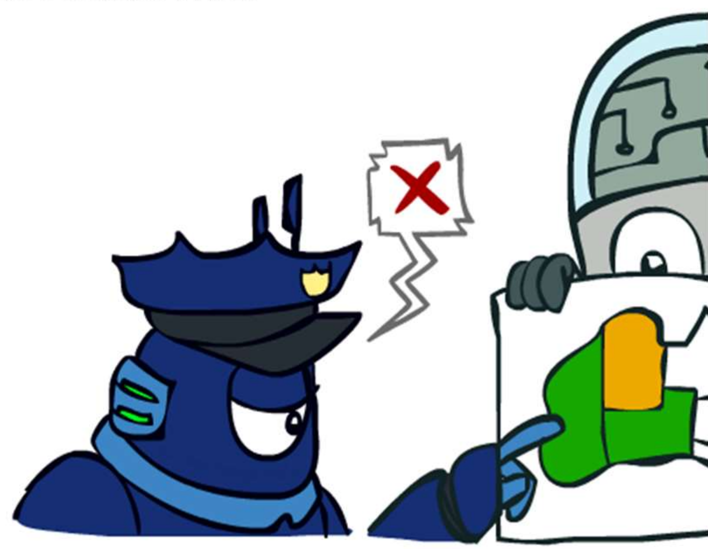


Backtracking Search

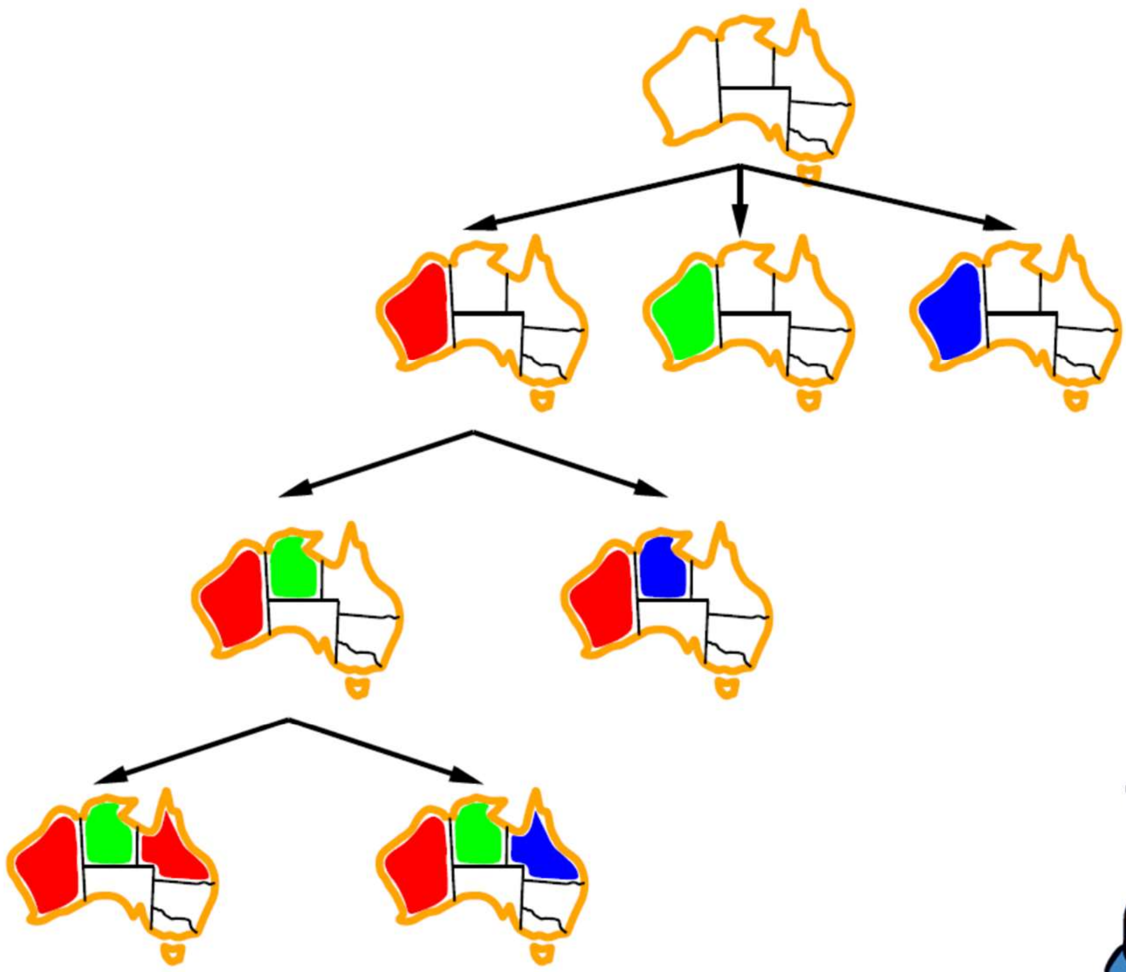


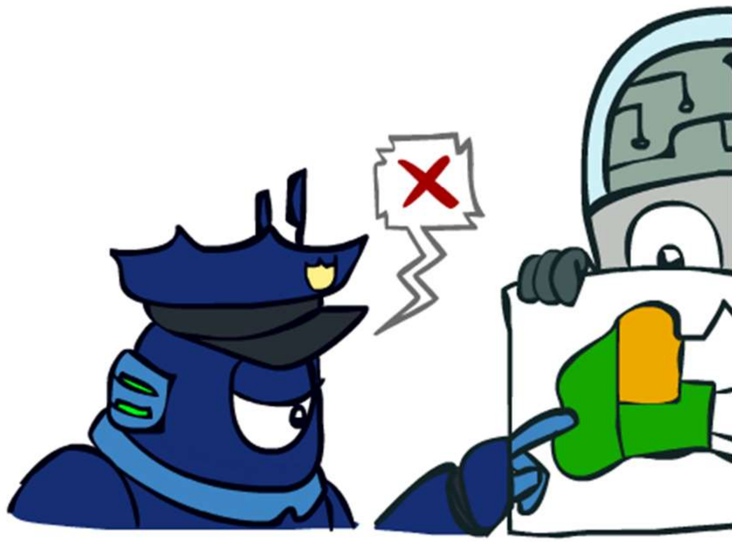
Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the cons
  - "Incremental goal test"
- Depth-first search with these two improvements is called *backtracking search*
- Can solve n-queens for n $\approx$ 25



Backtracking Example





Backtracking Search

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment given CONSTRAINTS[csp] then
         add {var = value} to assignment
         result ← RECURSIVE-BACKTRACKING(assignment, csp)
         if result ≠ failure then return result
         remove {var = value} from assignment
   return failure
```

- What are the choice points?

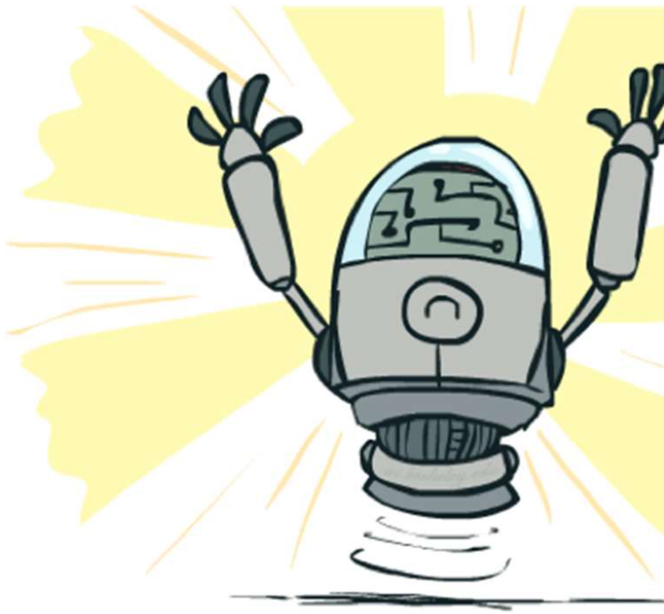
[Demo: coloring -- back

Backtracking Search

- Kind of depth first search
- Is it ***complete***?

Improving Backtracking

General-purpose ideas give huge gains in speed



Ordering:

- Which variable should be assigned next?
- In what order should its values be tried?

Filtering: Can we detect inevitable failure early?

Structure: Can we exploit the problem structure?

Next: Constraint Satisfaction Problems - Part