# CSE 473: Artificial Intelligence
## Spring 2017

Problem Spaces & Search

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With slides from
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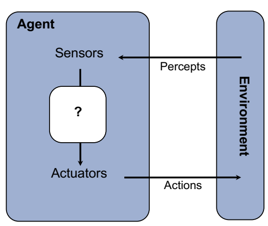
## Outline

- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

## Agent *vs.* Environment

- An **agent** is an entity that *perceives* and *acts*.
- A **rational agent** selects actions that maximize its **utility function**.
- Characteristics of the **percepts, environment,** and **action space** dictate techniques for selecting rational actions.




## Types of Agents

- Reflex

- Goal oriented

- Utility-based

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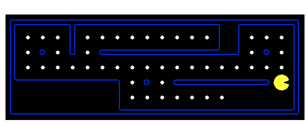
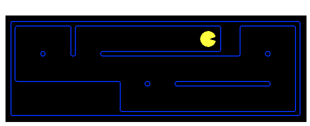
## Goal Based Agents

- Plan ahead
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Act on how the world WOULD BE

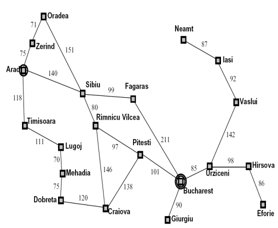




## Search thru a Problem Space (aka State Space)

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state [test]
- Output:
  - Path: start → a state satisfying goal test
    [May require shortest path]
    [Sometimes just need a state that passes test]

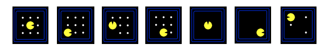
## Example: Traveling in Romania



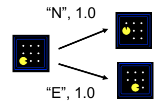

- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

## Example: Simplified Pac-Man

- Input:
  - A state space

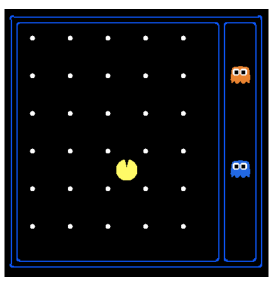


  - A successor function




  - A start state
  - A goal test
- Output:

## State Space Sizes?

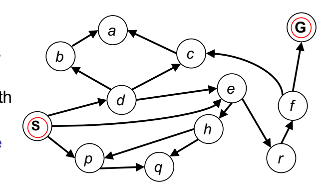


- Search Problem:
  Eat all of the food
- Pacman positions:
  10 x 12 = 120
- Pacman facing:
  up, down, left, right
- Food configurations: $2^{30}$
- Ghost1 positions: 12
- Ghost 2 positions: 11

$120 \times 4 \times 2^{30} \times 12 \times 11 = 6.8 \times 10^{13}$

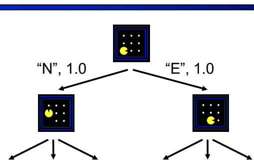
## State Space Graphs

- State space graph:
  - Each node is a state
  - The successor function is represented by arcs
  - Edges may be labeled with costs
- In a search graph, each state occurs only once!
- We can rarely build this graph in memory (so we don't)




*Ridiculously tiny search graph for a tiny search problem*

## Search Trees

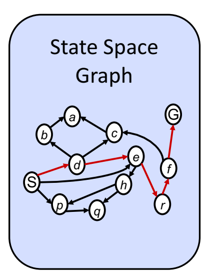



This is now / start

Possible futures

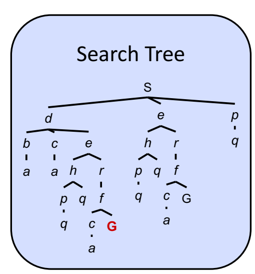
- A search tree:
  - Start state at the root node
  - Children correspond to successors
  - Nodes contain states, correspond to PLANS to those states
  - Edges are labeled with actions and costs
  - For most problems, we can never actually build the whole tree

## State Space Graphs vs. Search Trees



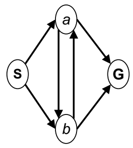

*Each NODE in in the search tree is an entire PATH in the state space graph.*

*We construct both on demand – and we construct as little as possible.*



## State Space Graphs vs. Search Trees

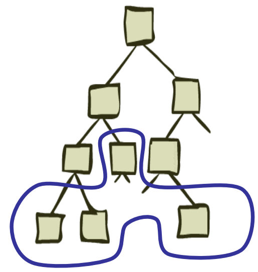
Consider this 4-state graph:



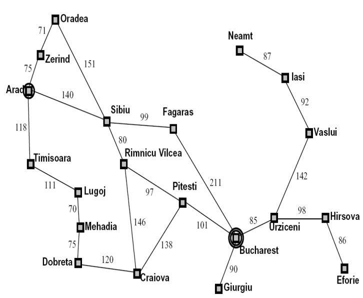

How big is its search tree (from S)?

Important: Lots of repeated structure in the search tree!

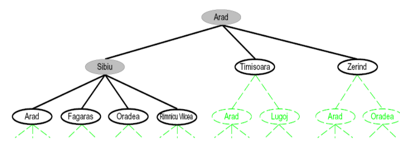
## Tree Search



## Search Example: Romania




## Searching with a Search Tree



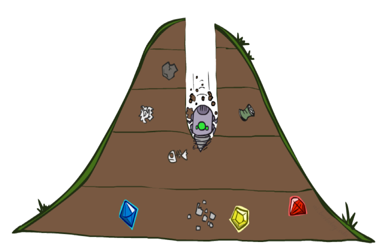

- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

## General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

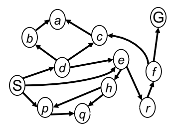
## Depth-First Search



## Depth-First Search

*Strategy: expand a deepest node first*

*Implementation: Fringe is a LIFO stack*



## Depth-First Search

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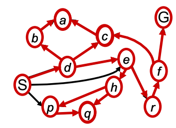
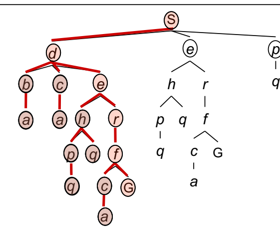
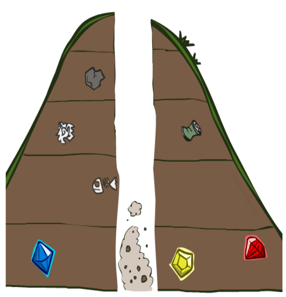


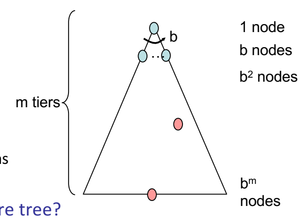


## Search Algorithm Properties



## Search Algorithm Properties

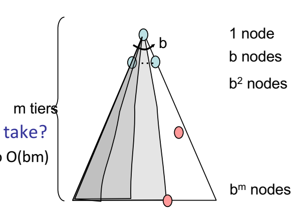
- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths



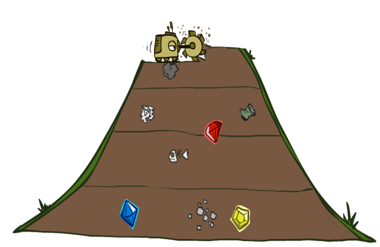

- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots. b^m = O(b^m)$

## Depth-First Search (DFS) Properties

- What nodes does DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost



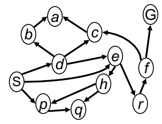

## Breadth-First Search

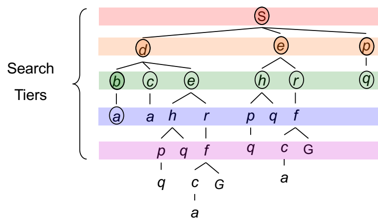


## Breadth-First Search

*Strategy: expand a shallowest node first*

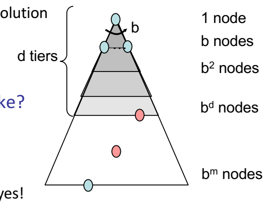
*Implementation: Fringe is a FIFO queue*



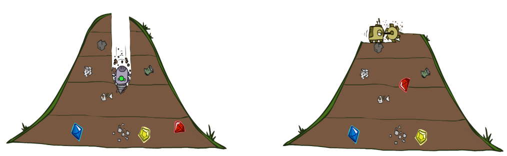




## Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be d
  - Search takes time O($b^d$)
- How much space does the fringe take?
  - Has roughly the last tier, so O($b^d$)
- Is it complete?
  - d must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)




## DFS vs BFS



| Algorithm | | Complete | Optimal | Time | Space |
|---|---|---|---|---|---|
| DFS | w/ Path Checking | N unless finite | N | $O(b^m)$ | $O(bm)$ |
| BFS | | Y | Y | $O(b^d)$ | $O(b^d)$ |

## Memory a Limitation?

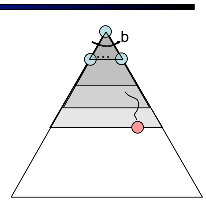
- Suppose:
  - 4 GHz CPU
  - 32 GB main memory
  - 100 instructions / expansion
  - 5 bytes / node
- 40 M expansions / sec
  - Memory filled in 160 sec … 3 min

## Iterative Deepening

Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less.
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

….and so on.



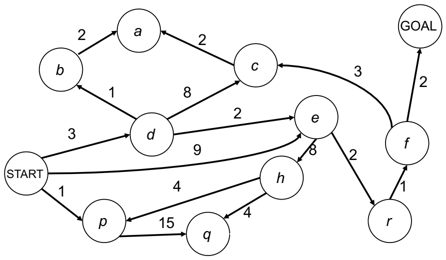

| Algorithm | | Complete | Optimal | Time | Space |
|---|---|---|---|---|---|
| DFS | w/ Path Checking | Y | N | $O(b^m)$ | $O(bm)$ |
| BFS | | Y | Y | $O(b^d)$ | $O(b^d)$ |
| ID | | Y | Y | $O(b^d)$ | $O(bd)$ |

## BFS vs. Iterative Deepening

- For b = 10, d = 5:
- BFS = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
- IDS = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456
- Overhead = (123,456 - 111,111) / 111,111 = 11%
- Memory BFS: 100,000; IDS: 50

30

## Costs on Actions

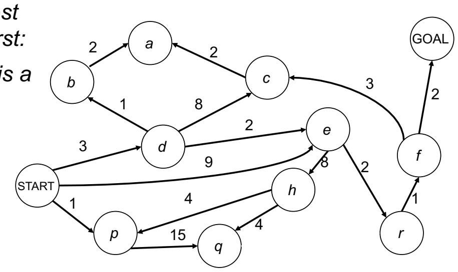



Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

## Uniform Cost Search

*Expand cheapest node first:*

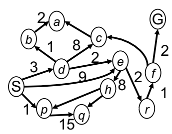
*Fringe is a priority queue*

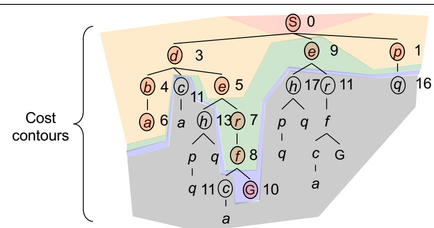



## Uniform Cost Search

*Strategy: expand a cheapest node first:*

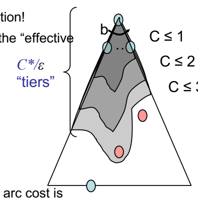
*Fringe is a priority queue (priority: cumulative cost)*





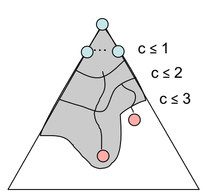

## Uniform Cost Search (UCS) Properties

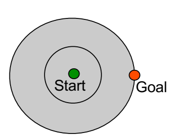
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs $C^*$ and arcs cost at least $\varepsilon$ , then the "effective depth" is roughly $C^*/\varepsilon$
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes!




## Uniform Cost Search

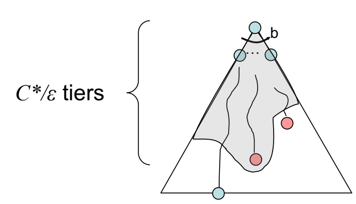
- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location



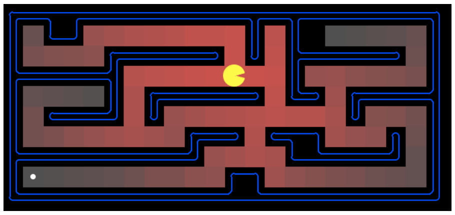

## Uniform Cost Search

| Algorithm | | Complete | Optimal | Time | Space |
|---|---|---|---|---|---|
| DFS | w/ Path Checking | Y | N | $O(b^m)$ | $O(bm)$ |
| BFS | | Y | Y | $O(b^d)$ | $O(b^d)$ |
| UCS | | Y* | Y | $O(b^{C^*/\varepsilon})$ | $O(b^{C^*/\varepsilon})$ |



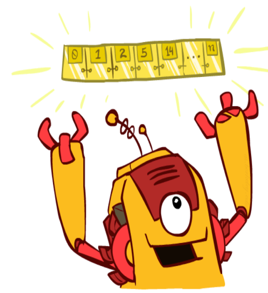
## Uniform Cost: Pac-Man

- Cost of 1 for each action
- Explores all of the states, but one



## The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



## To Do:

- Look at the course website:
  - http://www.cs.washington.edu/cse473/17sp
- Do the readings (Ch 3)
- Do PS0 if new to Python
- Start PS1, when it is posted