# CSE 473: Artificial Intelligence Reinforcement Learning 



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## Reinforcement Learning



## Reinforcement Learning

State: s
Reward: r


- Basic idea:
- Receive feedback in the form of rewards
- Agent's utility is defined by the reward function
- Must (learn to) act so as to maximize expected rewards
- All learning is based on observed samples of outcomes!


## Example 2 - More Animal Learning



## Example: Animal Learning

- RL studied experimentally for more than 60 years in psychology
- Rewards: food, pain, hunger, drugs, etc.
- Mechanisms and sophistication debated
- Example: foraging
- Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
- Bees have a direct neural connection from nectar intake measurement to motor planning area


## Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to $\mathrm{V}(\mathrm{s})$ using a neural network
- Combined with depth 3 search, one of the top 3 players in the world
- You could imagine training Pacman this way...
- ... but it's tricky! (It's also PS 3)




## Example: Learning to Walk



Finished


## Video of Demo Crawler Bot

| More demos at: $\underline{\text { http://inst.eecs.berkeley.edu/~ee128/fa11/videos.html }}$ |
| :--- | :--- |

## "Few driving tasks are as intimidating as parallel parking....

## Parallel Parking

"Few driving tasks are as intimidating as parallel parking....
https://www.youtube.com/watch?v=pB iFY2ild


## Other Applications

- Go playing
- Robotic control
- helicopter maneuvering, autonomous vehicles
- Mars rover - path planning, oversubscription planning
- elevator planning
- Game playing - backgammon, tetris, checkers
- Neuroscience
- Computational Finance, Sequential Auctions
- Assisting elderly in simple tasks
- Spoken dialog management
- Communication Networks - switching, routing, flow control
- War planning, evacuation planning


## Reinforcement Learning

- Still assume a Markov decision process (MDP):
- A set of states $s \in S$
- A set of actions (per state) A
- A model T(s,a,s')
- A reward function $R\left(s, a, s^{\prime}\right)$ \& discount $\gamma$
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R
- I.e. we don't know which states are good or what the actions do
- Must actually try actions and states out to learn


## Offline (MDPs) vs. Online (RL)



Offline Solution (Planning)


Monte Carlo
Planning


Online Learning (RL)

## Three Key Ideas for RL

- Model-based vs model-free learning
- What function is being learned?
- Approximating the Value Function
- Smaller $\rightarrow$ easier to learn \& better generalization
- Exploration-exploitation tradeoff


## Exploration-Exploitation tradeoff

- You have visited part of the state space and found a reward of 100
- is this the best you can hope for???
- Exploitation: should I stick with what I know and find a good policy w.r.t. this knowledge?
- at risk of missing out on a better reward somewhere
- Exploration: should I look for states w/ more reward?
- at risk of wasting time \& getting some negative reward



## Model-Based Learning



## Model-Based Learning

- Model-Based Idea:
- Learn an approximate model based on experiences
- Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model

- Count outcomes s' for each s, a
- Normalize to give an estimate of $\widehat{T}\left(s, a, s^{\prime}\right)$
- Discover each $\hat{R}\left(s, a, s^{\prime}\right)$ when we experience ( $s, \mathrm{a}, \mathrm{s}^{\prime}$ )
- Step 2: Solve the learned MDP

- For example, use value iteration, as before


## Example: Model-Based Learning

| Random $\pi$ |  |  | Observed Episodes (Training) |  | Learned Model$\widehat{T}\left(s, a, s^{\prime}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Episode 1 | Episode 2 |  |
| B D | A | D | B, east, $C,-1$ <br> $C$, east, $D,-1$ <br> $D$, exit, $x,+10$ | B, east, $C,-1$ <br> $C$, east, $D,-1$ <br> $D$, exit, $x,+10$ | $T(B$, east, $C)=1.00$ $T(C$, east, $D)=0.75$ $T(C$, east, $A)=0.25$ $\ldots$ |
|  |  |  |  | Episode 4E, north, $C,-1$  <br> C, east, A, -1 <br> A, exit, x, -10 | $\widehat{R}\left(s, a, s^{\prime}\right)$ $\begin{aligned} & \text { R(B, east, C) }=-1 \\ & \mathrm{R}(\mathrm{C}, \text { east, } \mathrm{D})=-1 \\ & \mathrm{R}(\mathrm{D}, \text { exit, } \mathrm{x})=+10\end{aligned}$ $\ldots$ |

## Convergence

- If policy explores "enough" - doesn't starve any state
- Then T \& R converge
- So, VI, PI, Lao* etc. will find optimal policy
- Using Bellman Equations
- When can agent start exploiting??
- (We'll answer this question later)


## Two main reinforcement learning approaches

- Model-based approaches:

```
Learn T + R
    |S\mp@subsup{|}{}{2}|A|+|S|A| parameters (40,400)
```

- Model-free approach: Learn Q

$$
|S||A| \text { parameters }
$$

(400)


## Model-Free Learning



## Reminder: Q-Value Iteration

- Forall s, a
- Initialize $\mathrm{Q}_{0}(\mathrm{~s}, \mathrm{a})=0$
no time steps left means an expected reward of zero
- K = 0
- Repeat
do Bellman backups
For every (s,a) pair:

$$
\begin{aligned}
& \quad Q_{k+1}(s, a) \leftarrow \sum_{s^{\prime}} T\left(s, a, s^{\prime}\right)\left[R\left(s, a, s^{\prime}\right)+\gamma \max _{a^{\prime}} Q_{k}\left(s^{\prime}, a^{\prime}\right)\right] \\
& \mathrm{K}+=1
\end{aligned}
$$

- Until convergence



## Puzzle: Q-Learning

- Forall s, a
- Initialize $\mathbf{Q}_{\mathbf{0}}(\mathbf{s}, \mathbf{a})=\mathbf{0}$ no time steps left means an expected reward of zero
- K = 0
- Repeat do Bellman backups

For every ( $\mathrm{s}, \mathrm{a}$ ) pair:

- Until convergence

Q: How can we compute without $\mathrm{R}, \mathrm{T}$ ?!?
A: Compute averages using sampled outcomes

## Simple Example: Expected Age

Goal: Compute expected age of CSE students

| Known P(A) |  |  |
| :---: | :---: | :---: |
| $E[A]=\sum_{a} P(a) \cdot a \quad=0.35 \times 20+\ldots$ |  |  |
| Note: never know |  |  |

Without $P(A)$, instead collect samples $\left[a_{1}, a_{2}, \ldots a_{N}\right]$

| Unknown P(A): "Model Based" |  |
| :---: | :---: |
|  |  |
| s this | $\hat{P}(a)=\frac{\operatorname{num}(a)}{N}$ |
| y you right <br> . | $E[A] \approx \sum_{a} \hat{P}(a) \cdot a$ |



## Anytime Model-Free Expected Age

Goal: Compute expected age of CSE students

Let $A=0$
Loop for $\mathrm{i}=1$ to $\infty$
$\mathrm{a}_{\mathrm{i}} \leftarrow$ ask "what is your age?"
$A \leftarrow(1-\alpha)^{*} A+\alpha^{*} a_{i}$
Without $P(A)$, instead collect samples $\left[a_{1}, a_{2}, \ldots a_{N}\right]$

Let $A=0$
Loop for $\mathrm{i}=1$ to $\infty$
$a_{i} \leftarrow$ ask "what is your age?"
$A \leftarrow(i-1) / i^{*} A+(1 / i)^{*} a_{i}$

Unknown P(A): "Model Free"

$$
E[A] \approx \frac{1}{N} \sum_{i} a_{i}
$$

## Exponential Moving Average

- Exponential moving average
- The running interpolation update: $\quad \bar{x}_{n}=(1-\alpha) \cdot \bar{x}_{n-1}+\alpha \cdot x_{n}$
- Makes recent samples more important:

$$
\bar{x}_{n}=\frac{x_{n}+(1-\alpha) \cdot x_{n-1}+(1-\alpha)^{2} \cdot x_{n-2}+\ldots}{1+(1-\alpha)+(1-\alpha)^{2}+\ldots}
$$

- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages
- E.g., $\alpha=1 / \mathrm{i}$


## Sampling Q-Values

- Big idea: learn from every experience!
- Follow exploration policy a $\leftarrow \pi(s)$
- Update $Q(s, a)$ each time we experience a transition ( $s, a, s^{\prime}, r$ )
- Likely outcomes s' will contribute updates more often
- Update towards running average:


Get a sample of $Q(s, a): \quad$ sample $=R\left(s, a, s^{\prime}\right)+\gamma$ Max $_{a^{\prime}} Q\left(s^{\prime}, a^{\prime}\right)$
Update to $\mathrm{Q}(\mathrm{s}, \mathrm{a}): \quad \mathrm{Q}(\mathrm{s}, \mathrm{a}) \leftarrow(1-\alpha) \mathrm{Q}(\mathrm{s}, \mathrm{a})+(\boldsymbol{\alpha})$ sample

## Q Learning

- Forall s, a
- Initialize Q(s, a) = 0
- Repeat Forever

Where are you? s.
Choose some action a
Execute it in real world: (s, a, r, s')
Do update:

$$
Q(s, a) \leftarrow(1-\alpha) Q(s, a)+(\alpha)\left[r+\gamma \max _{a^{\prime}} Q\left(s^{\prime}, a^{\prime}\right)\right]
$$

## Example

Observed Transition:

## B, east, C, -2



$$
\begin{array}{ccccc}
Q(s, a) \leftarrow(1-\alpha) Q(s, a)+(\alpha)\left[r+\gamma \max _{a^{\prime}} Q\left(s^{\prime}, a^{\prime}\right)\right] \\
-1 & 1 / 2 & 0 & 1 / 2 & -2
\end{array}
$$

## Example

Assume: $\gamma=1, \alpha=1 / 2$

Observed Transition:
$B$, east, $C,-2$
C, east, $D,-2$

$Q(s, a) \leftarrow(1-\alpha) Q(s, a)+(\alpha)\left[r+\gamma \max _{a^{\prime}} Q\left(s^{\prime}, a^{\prime}\right)\right]$
3
$1 / 2$
0
$\begin{array}{ll}1 / 2 & -2\end{array}$
8

## Example

Observed Transition: $B$, east, $C,-2$ C, east, $D,-2$

$Q(s, a) \leftarrow(1-\alpha) Q(s, a)+(\alpha)\left[r+\gamma \max _{a^{\prime}} Q\left(s^{\prime}, a^{\prime}\right)\right]$

