CSE 473: Artificial Intelligence

Constraint Satisfaction Problems II

Factored (aka Structured) Search

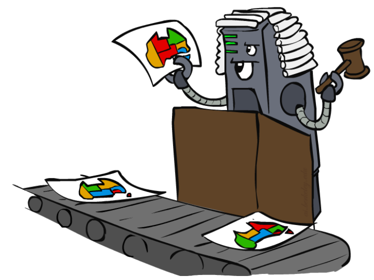
[With many slides by Dan Klein and Pieter Abbeel (UC Berkeley) available at http://ai.berkeley.edu.]

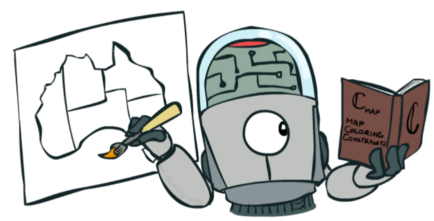
What is Search For?

- **Planning:** sequences of actions
  - The ***path to the goal*** is the important thing
  - Paths have various costs, depths
  - Assume little about problem structure
- **Identification:** assignments to variables
  - The ***goal itself*** is important, ***not the path***
  - All paths at the same depth (for some formulations)

Constraint Satisfaction Problems

- Standard search problems:
  - State is a “black box”: arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Making use of CSP formulation allows for optimized algorithms
  - Typical example of trading generality for utility (in this case, speed)





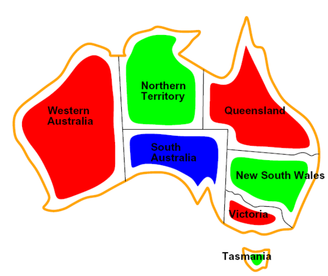
CSP Example: Map Coloring

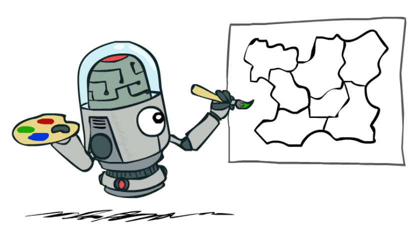
- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $D = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors

  Implicit: $\text{WA} \neq \text{NT}$

  Explicit: $(\text{WA}, \text{NT}) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$
- Solutions are assignments satisfying all constraints, e.g.:

  {WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}





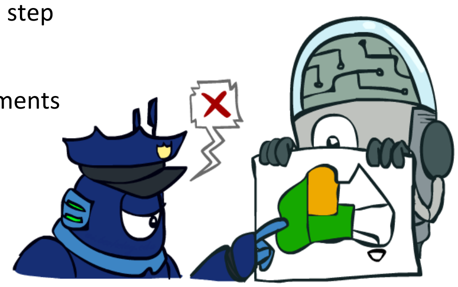
Systematic Search to Solve CSP

- States – partial assignments to variables
- Operators – assign another variable
- Initial State – no variables assigned
- Goal State – all vars assigned & constraints satisfied

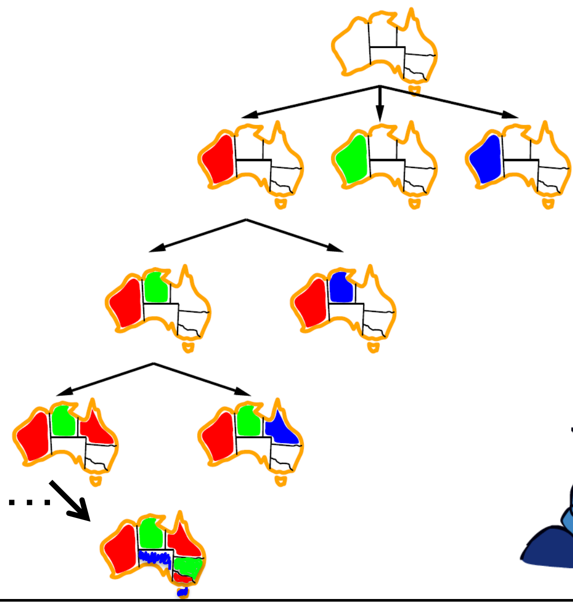
We'll improve this basic method to exploit structure

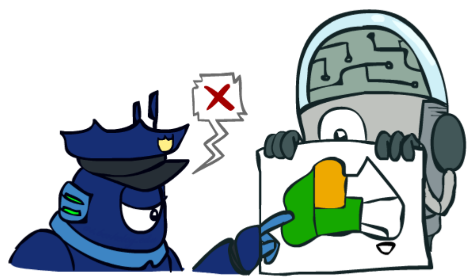
Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs
- Start with Depth First Search
  - "backtracking search" **IS a Kind of** depth first search with these 2 details:
- Idea 1: One variable at a time
  - Variable assignments are commutative, so **fix ordering**
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step
- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constraints
  - "Incremental goal test"
- Can solve n-queens for $n \approx 25$



Backtracking Example





Backtracking Search

function BACKTRACKING-SEARCH(*csp*) **returns** solution/failure
  return RECURSIVE-BACKTRACKING({ }, *csp*)

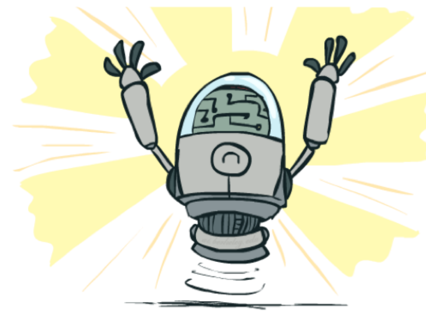
function RECURSIVE-BACKTRACKING(*assignment, csp*) **returns** soln/failure
  if *assignment* is complete **then return** *assignment*
  var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*], *assignment, csp*)
  for each *value* **in** ORDER-DOMAIN-VALUES(*var, assignment, csp*) **do**
    if *value* is consistent with *assignment* given CONSTRAINTS[*csp*] **then**
      add $\{var = value\}$ to *assignment*
      result ← RECURSIVE-BACKTRACKING(*assignment, csp*)
      if $result \neq failure$ **then return** *result*
      remove $\{var = value\}$ from *assignment*
  return *failure*

- What are the choice points?

[Demo: coloring -- backtracking]

Improving Backtracking

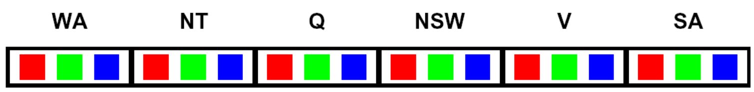
- General-purpose ideas give huge gains in speed
- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
- Filtering: Can we detect inevitable failure early?
- Structure: Can we exploit the problem structure?



Filtering

Filtering: Forward Checking

- Filtering: Keep track of domains for unassigned variables and cross off bad options
- Forward checking: Cross off values that violate a constraint when added to the existing assignment

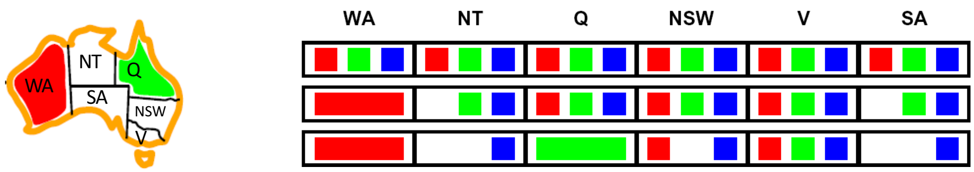


[Demo: coloring -- forward checking]

Video of Demo Coloring – Backtracking with Forward Checking

Filtering: Constraint Propagation

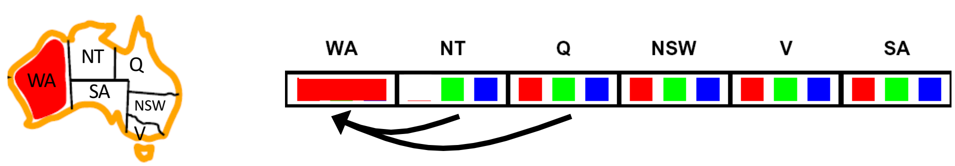
- Forward checking only propagates information from assigned to unassigned
- It doesn't catch when ***two unassigned variables*** have no consistent assignment:

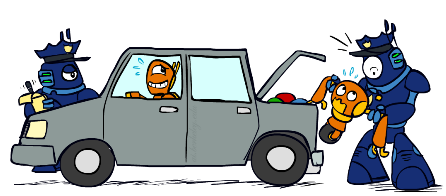


- NT and SA cannot both be blue!
- Why didn't we detect this yet?
- *Constraint propagation:* reason from constraint to constraint

Consistency of a Single Arc

- An arc X $\rightarrow$ Y is consistent iff for *every* x in the tail there is *some* y in the head which could be assigned without violating a constraint



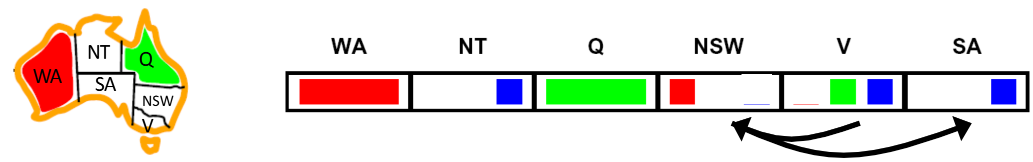


Delete from the tail!

- Forward checking: Enforcing consistency ***of arcs pointing to each new assignment***

Arc Consistency of an **Entire CSP**

- A simple form of propagation makes sure all arcs are consistent:



- Important: If X loses a value, neighbors of X need to be rechecked!
- Arc consistency detects failure ***earlier*** than forward checking
- Can be run as a preprocessor ***or*** after each assignment
- What's the ***downside*** of enforcing arc consistency?

Remember: Delete from the tail!

AC-3 algorithm for Arc Consistency

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X1, X2, ..., Xn}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (Xi, Xj) <- REMOVE-FIRST(queue)
      if REMOVE-INCONSISTENT-VALUES(Xi, Xj) then
         for each Xk in NEIGHBORS[Xi] do
            add (Xk, Xi) to queue
```

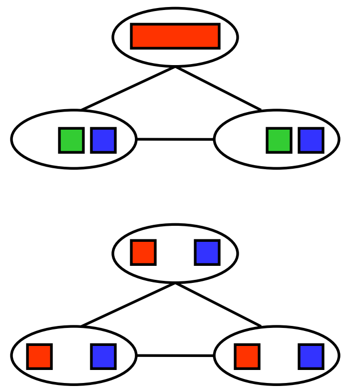
```
function REMOVE-INCONSISTENT-VALUES( Xi, Xj) returns true iff succeeds
   removed <- false
   for each x in DOMAIN[Xi] do
      if no value y in DOMAIN[Xj] allows (x,y) to satisfy the constraint Xi <-> Xj
         then delete x from DOMAIN[Xi]; removed <- true
   return removed
```

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting ***all*** possible future problems is NP-hard – why?

[Demo: CSP applet (made available by aispace.org) -- n-queens]

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)
- Arc consistency still runs inside a backtracking search!



What went wrong here?

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]