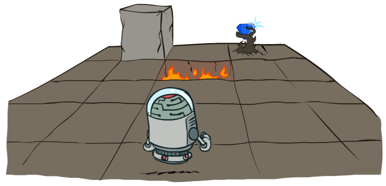
CS 473: Artificial Intelligence

Markov Decision Processes



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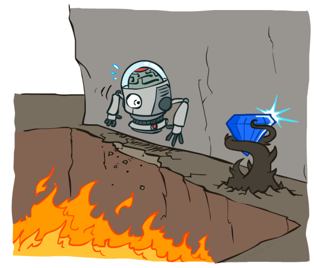
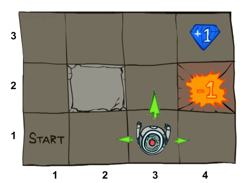
Non-Deterministic Search



Example: Grid World

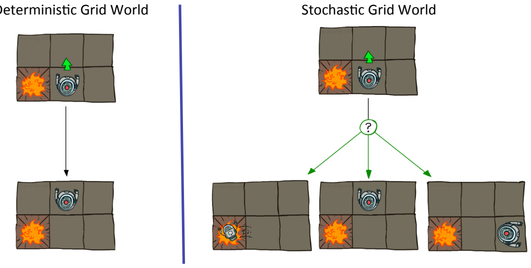
- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards



Grid World Actions

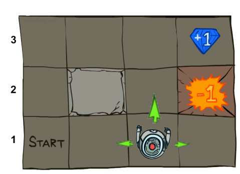
Deterministic Grid World

Stochastic Grid World



Markov Decision Processes

- An MDP is defined by:
  - A set of states s in S
  - A set of actions a in A
  - A transition function T(s, a, s')
    - Probability that a from s leads to s', i.e., P(s'| s, a)
    - Also called the model or the dynamics



$T(s_{11}, E, \ldots$
$\ldots$
$T(s_{31}, N, s_{11}) = 0$
$\ldots$
$T(s_{31}, N, s_{32}) = 0.8$
$T(s_{31}, N, s_{21}) = 0.1$
$T(s_{31}, N, s_{41}) = 0.1$
$\ldots$

T is a Big Table!
11 X 4 x 11 = 484 entries

For now, we give this as input to the agent

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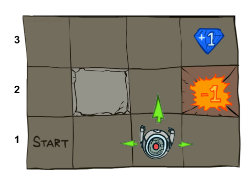
  - A reward function R(s, a, s')



$\ldots$
$R(s_{32}, N, s_{33}) = -0.01$ ← ***Cost of breathing***
$\ldots$
$R(s_{32}, N, s_{42}) = -1.01$
$\ldots$
$R(s_{33}, E, s_{43}) = 0.99$

R is also a Big Table!

For now, we also give this to the agent

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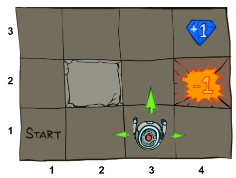
    - Sometimes just R(s) or R(s')

…

$R(s_{33}) = -0.01$

$R(s_{42}) = -1.01$

$R(s_{43}) = 0.99$



Markov Decision Processes

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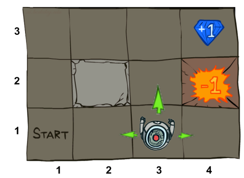
  - A reward function R(s, a, s')
    - Sometimes just R(s) or R(s')
  - A start state
  - Maybe a terminal state
- MDPs are non-deterministic search problems
  - One way to solve them is with expectimax search
  - We'll have a new tool soon



[Demo – gridworld manual intro (L8D1)]

What is Markov about MDPs?

- "Markov" generally means that given the present state, the future and the past are independent
- For Markov decision processes, "Markov" means action outcomes depend only on the current state

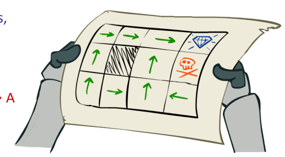
$$P(S_{t+1} = s'|S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$
$$=$$
$$P(S_{t+1} = s'|S_t = s_t, A_t = a_t)$$

- This is just like search, where the successor function could only depend on the current state (not the history)

Andrey Markov
(1856-1922)

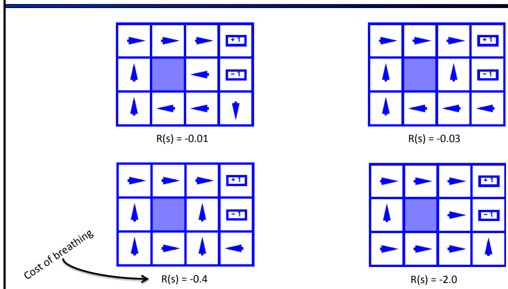
Policies

- In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal
- For MDPs, we want an optimal policy π*: S → A
  - A policy π gives an action for each state
  - An optimal policy is one that maximizes expected utility if followed
  - An explicit policy defines a reflex agent
- Expectimax didn't compute entire policies
  - It computed the action for a single state only



Optimal policy when R(s, a, s') = -0.03 for all non-terminals s

Optimal Policies



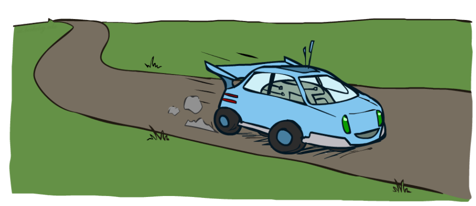
R(s) = -0.01

R(s) = -0.03

R(s) = -0.4

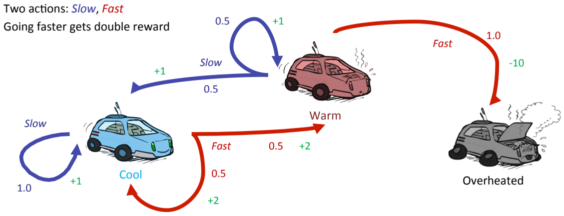
R(s) = -2.0

Example: Racing

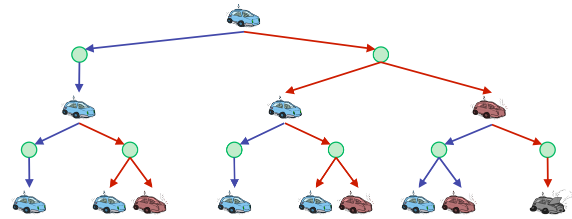


Example: Racing

- A robot car wants to travel far, quickly
- Three states: Cool, Warm, Overheated
- Two actions: *Slow*, *Fast*
- Going faster gets double reward

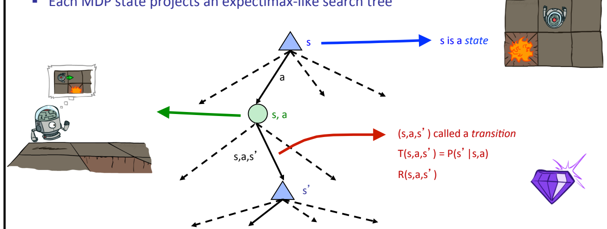


Racing Search Tree

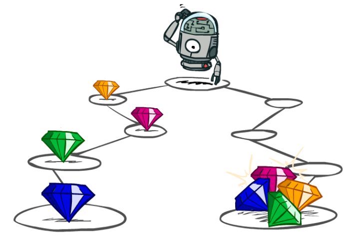


MDP Search Trees

- Each MDP state projects an expectimax-like search tree

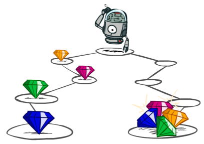


Utilities of Sequences



Utilities of Sequences

- What preferences should an agent have over reward sequences?
- More or less? [1, 2, 2] or [2, 3, 4]
- Now or later? [0, 0, 1] or [1, 0, 0]



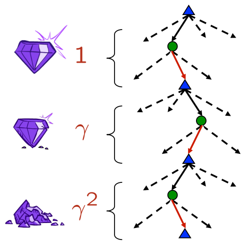
Discounting

- It's reasonable to maximize the sum of rewards
- It's also reasonable to prefer rewards now to rewards later
- One solution: values of rewards decay exponentially

1	γ	γ^2
Worth Now	Worth Next Step	Worth In Two Steps

Discounting

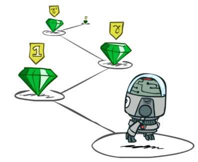
- How to discount?
  - Each time we descend a level, we multiply in the discount once
- Why discount?
  - Sooner rewards probably do have higher utility than later rewards
  - Also helps our algorithms converge
- Example: discount of 0.5
  - U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3
  - U([1,2,3]) < U([3,2,1])



Stationary Preferences

- Theorem: if we assume stationary preferences:

$$[a_1, a_2, \ldots] \succ [b_1, b_2, \ldots]$$
$$\Updownarrow$$
$$[r, a_1, a_2, \ldots] \succ [r, b_1, b_2, \ldots]$$



- Then: there are only two ways to define utilities
  - Additive utility: $U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \cdots$
  - Discounted utility: $U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 \cdots$

Quiz: Discounting

- Given:

10				1
a	b	c	d	e

  - Actions: East, West, and Exit (only available in exit states a, e)
  - Transitions: deterministic

- Quiz 1: For γ = 1, what is the optimal policy?

10				1

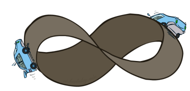
- Quiz 2: For γ = 0.1, what is the optimal policy?

10				1

- Quiz 3: For which γ are West and East equally good when in state d?

Infinite Utilities?!

- Problem: What if the game lasts forever? Do we get infinite rewards?
- Solutions:
  - Finite horizon: (similar to depth-limited search)
    - Terminate episodes after a fixed T steps (e.g. life)
    - Gives nonstationary policies (γ depends on time left)



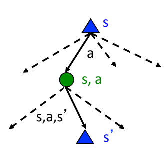
  - Discounting: use 0 < γ < 1

$$U([r_0, \ldots r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq R_{\max}/(1-\gamma)$$

    - Smaller γ means smaller "horizon" – shorter term focus
  - Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

Recap: Defining MDPs

- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards