CSE 473: Artificial Intelligence
Autumn 2014

Problem Spaces & Search

Dieter Fox

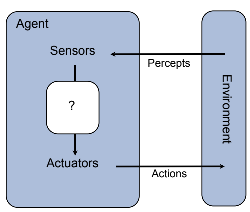
With slides from
Dan Weld, Pieter Abbeel, Dan Klein, Stuart Russell, Andrew Moore, Luke Zettlemoyer

Outline

- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

Agent *vs.* Environment

- An agent is an entity that perceives and acts.
- A rational agent selects actions that maximize its utility function.
- Characteristics of the percepts, environment, and action space dictate techniques for selecting rational actions.



Types of Agents

- Reflex

- Goal oriented

- Utility-based

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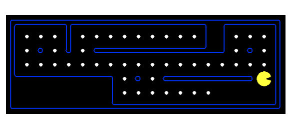
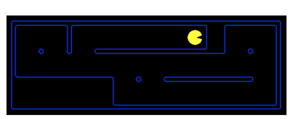
Goal Based Agents

- Plan ahead
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Act on how the world WOULD BE

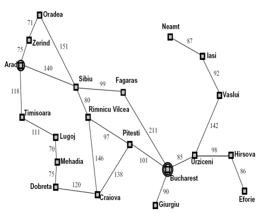




Search thru a Problem Space (aka State Space)

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state [test]
- Output:
  - Path: start → a state satisfying goal test
    [May require shortest path]
    [Sometimes just need a state that passes test]

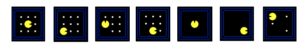
Example: Traveling in Romania



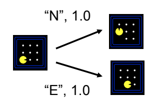
- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

Example: Simplified Pac-Man

- Input:
  - A state space



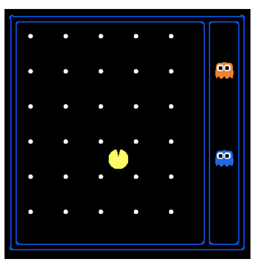
  - A successor function



  - A start state
  - A goal test
- Output:

State Space Sizes?

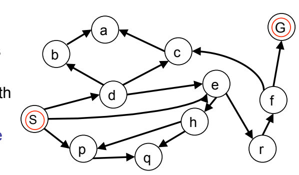
- Search Problem:
  Eat all of the food
- Pacman positions:
  10 x 12 = 120
- Pacman facing:
  up, down, left, right
- Food configurations: 2^{30}
- Ghost1 positions: 12
- Ghost 2 positions: 11



$120 \times 4 \times 2^{30} \times 12 \times 11 = 6.8 \times 10^{13}$

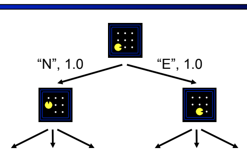
State Space Graphs

- State space graph:
  - Each node is a state
  - The successor function is represented by arcs
  - Edges may be labeled with costs
- In a search graph, each state occurs only once!
- We can rarely build this graph in memory (so we don't)



Ridiculously tiny search graph for a tiny search problem

Search Trees

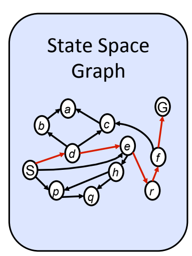


This is now / start

Possible futures

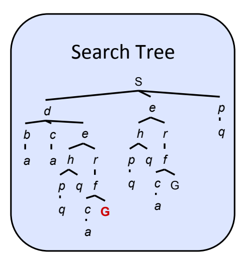
- A search tree:
  - Start state at the root node
  - Children correspond to successors
  - Nodes contain states, correspond to PLANS to those states
  - Edges are labeled with actions and costs
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees



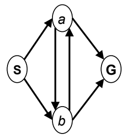
Each NODE in in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.



State Space Graphs vs. Search Trees

Consider this 4-state graph:

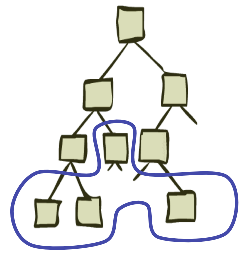


How big is its search tree (from S)?

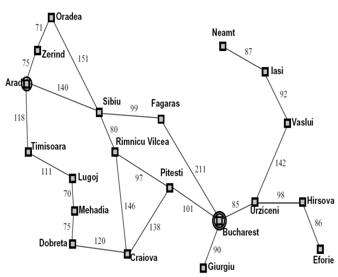
∞

Important: Lots of repeated structure in the search tree!

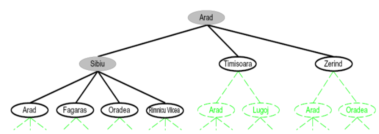
Tree Search



Search Example: Romania



Searching with a Search Tree



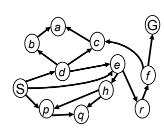
- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a fringe of partial plans under consideration
  - Try to expand as few tree nodes as possible

General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

Tree Search Example

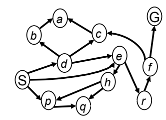


Depth-First Search

Depth-First Search

Strategy: expand a deepest node first

Implementation: Fringe is a LIFO stack



Depth-First Search

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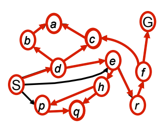
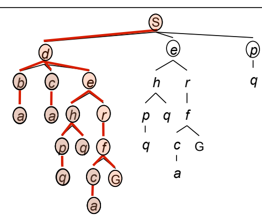
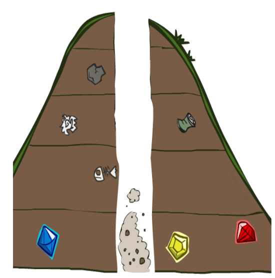


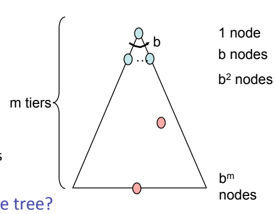


Search Algorithm Properties



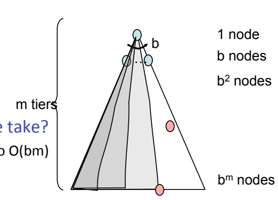
Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - b is the branching factor
  - m is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + \ldots.\ b^m = O(b^m)$

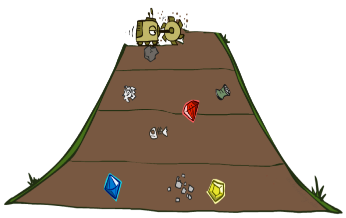


Depth-First Search (DFS) Properties

- What nodes does DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so O(bm)
- Is it complete?
  - m could be infinite, so only if we prevent cycles
- Is it optimal?
  - No, it finds the "leftmost" solution, regardless of depth or cost



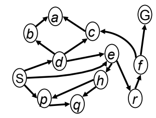
Breadth-First Search

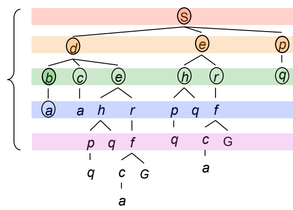


Breadth-First Search

Strategy: expand a shallowest node first

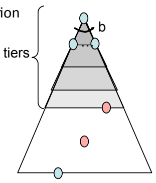
Implementation: Fringe is a FIFO queue



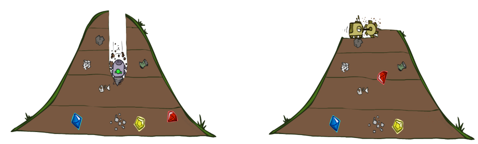


Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be d
  - Search takes time O(b^d)
- How much space does the fringe take?
  - Has roughly the last tier, so O(b^d)
- Is it complete?
  - d must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



DFS vs BFS



Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	N unless finite	N	O(b^m)	O(bm)
BFS		Y	Y	O(b^d)	O(b^d)

Memory a Limitation?

- Suppose:
  - 4 GHz CPU
  - 32 GB main memory
  - 100 instructions / expansion
  - 5 bytes / node

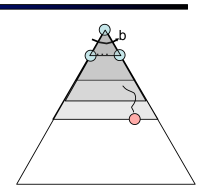
- 40 M expansions / sec
  - Memory filled in 160 sec … 3 min

Iterative Deepening

Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less.
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

….and so on.



Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y	N	O(b^m)	O(bm)
BFS		Y	Y	O(b^d)	O(b^d)
ID		Y	Y	O(b^d)	O(bd)

BFS vs. Iterative Deepening

- For b = 10, d = 5:
- BFS = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
- IDS = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456
- Overhead = (123,456 - 111,111) / 111,111 = 11%
- Memory BFS: 100,00; IDS: 50

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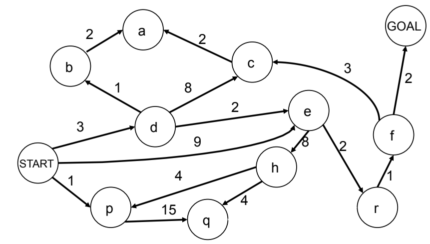
Costs on Actions

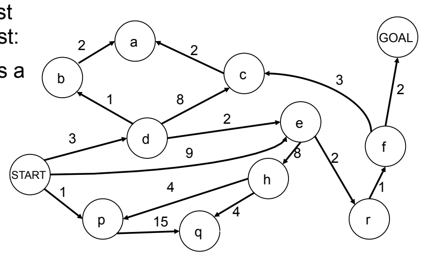


Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

Uniform Cost Search

Expand cheapest node first:

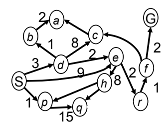
Fringe is a priority queue

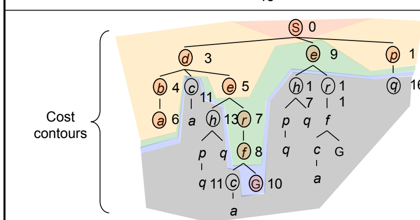


Uniform Cost Search

Strategy: expand a cheapest node first:

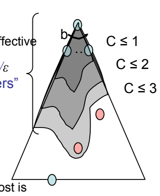
Fringe is a priority queue (priority: cumulative cost)





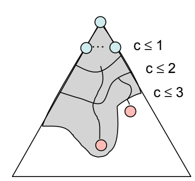
Uniform Cost Search (UCS) Properties

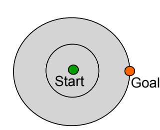
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε, then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



Uniform Cost Search

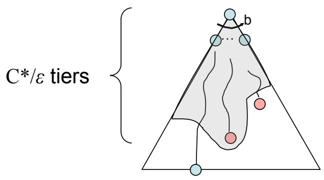
- Strategy: expand lowest path cost
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location





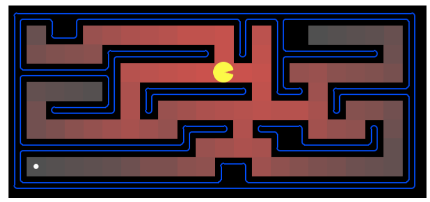
Uniform Cost Search

Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y	N	$O(b^m)$	$O(bm)$
BFS		Y	Y	$O(b^d)$	$O(b^d)$
UCS		Y*	Y	$O(b^{C*/\varepsilon})$	$O(b^{C*/\varepsilon})$



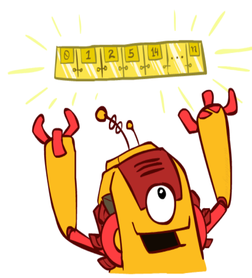
Uniform Cost: Pac-Man

- Cost of 1 for each action
- Explores all of the states, but one



The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



To Do:

- Look at the course website:
  - http://www.cs.washington.edu/cse473/15sp
- Do the readings (Ch 3)
- Do PS0 if new to Python
- Start PS1, when it is posted