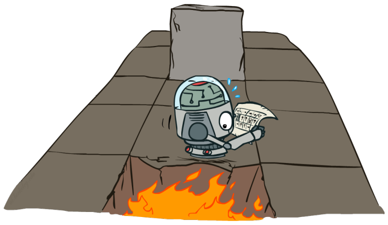
CS 188: Artificial Intelligence

Markov Decision Processes II



Instructors: Dan Klein and Pieter Abbeel --- University of California, Berkeley

Stochastic Planning: MDPs

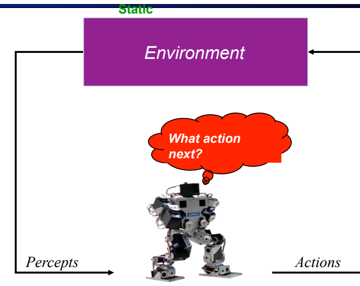
Static

Fully Observable

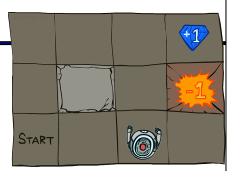
Perfect

Stochastic

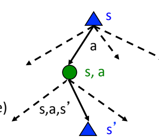
Instantaneous



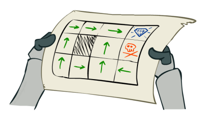
Recap: MDPs



- Markov decision processes:
  - States S
  - Actions A
  - Transitions P(s'|s,a) (or T(s,a,s'))
  - Rewards R(s,a,s') (and discount γ)
  - Start state s_0
- Quantities:
  - Policy = map of states to actions
  - Utility = sum of discounted rewards
  - Values = expected future utility from a state (max node)
  - Q-Values = expected future utility from a q-state (chance node)



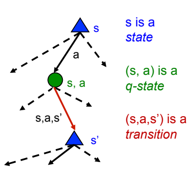
Solving MDPs



- Value Iteration
- ~~Real-Time Dynamic Programming~~
- Policy Iteration
- Reinforcement Learning

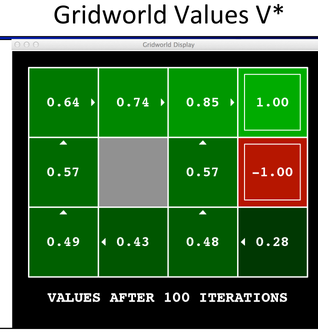
Optimal Quantities

- The value (utility) of a state s:
  $V^*(s)$ = expected utility starting in s and acting optimally
- The value (utility) of a q-state (s,a):
  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally
- The optimal policy:
  $\pi^*(s)$ = optimal action from state s

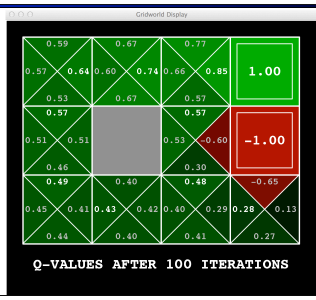


[Demo: gridworld values (L9D1)]

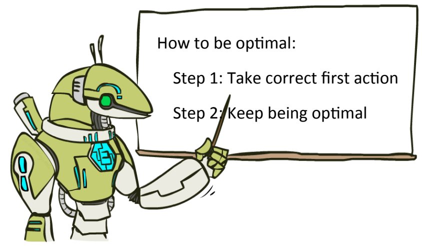
Gridworld Values V*



Gridworld: Q*



The Bellman Equations



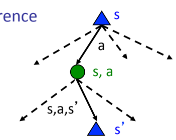
The Bellman Equations

- Definition of "optimal utility" via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

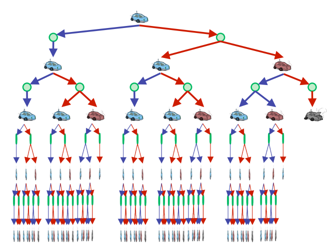
$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$



- These are the Bellman equations, and they characterize optimal values in a way we'll use over and over

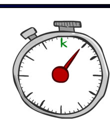
Racing Search Tree

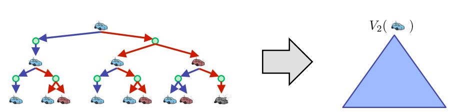
- We're doing way too much work with expectimax!
- Problem: States are repeated
  - Idea: Only compute needed quantities once
- Problem: Tree goes on forever
  - Idea: Do a depth-limited computation, but with increasing depths until change is small
  - Note: deep parts of the tree eventually don't matter if $\gamma < 1$



Time-Limited Values

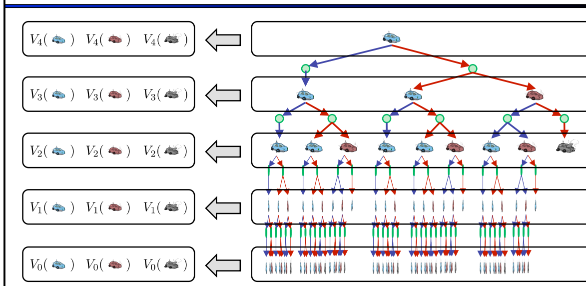
- Key idea: time-limited values
- Define $V_k(s)$ to be the optimal value of s if the game ends in k more time steps
  - Equivalently, it's what a depth-k expectimax would give from s



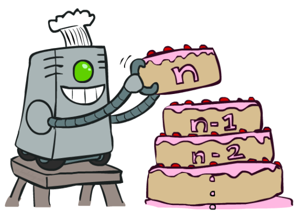


[Demo – time-limited values (L8D6)]

Time-Limited Values: Avoiding Redundant Computation

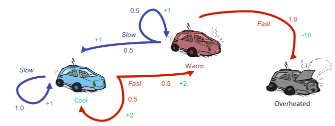


Value Iteration



Example: Value Iteration

V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V_k(s')\right]$$

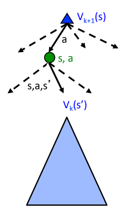
Assume no discount (gamma=1) to keep math simple!

Value Iteration

Called a "Bellman Backup"

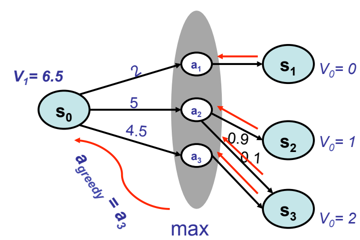
- **Start with $V_0(s) = 0$:** *no time steps left means an expected reward sum of zero*
- **Given vector of $V_k(s)$ values, do one ply of expectimax from each state:**

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V_k(s')\right]$$



- **Repeat until convergence**
  (trust me, it does)

Example: Bellman Backup

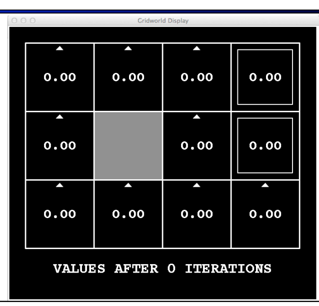


$Q_1(s,a_1) = 2 + \gamma\, 0$
~ 2

$Q_1(s,a_2) = 5 + \gamma\, 0.9\sim 1$
$+ \gamma\, 0.1\sim 2$
~ 6.1

$Q_1(s,a_3) = 4.5 + \gamma\, 2$
~ 6.5

k=0



Noise = 0.2
Discount = 0.9
Living reward = 0

k=1

If agent is in 4,3, it only has one legal action: get jewel. It gets a reward and the game is over.
If agent is in the pit, it has only one legal action, die. It gets a penalty and the game is over.

Agent does NOT get a reward for moving INTO 4,3.

Noise = 0.2
Discount = 0.9
Living reward = 0

k=2

Noise = 0.2
Discount = 0.9
Living reward = 0

k=3

Noise = 0.2
Discount = 0.9
Living reward = 0

k=4

Noise = 0.2
Discount = 0.9
Living reward = 0

k=5

Noise = 0.2
Discount = 0.9
Living reward = 0

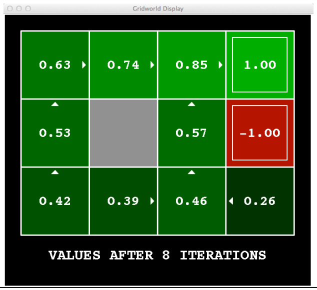
k=6

Noise = 0.2
Discount = 0.9
Living reward = 0

k=7

Noise = 0.2
Discount = 0.9
Living reward = 0

k=8



Noise = 0.2
Discount = 0.9
Living reward = 0

k=9

Noise = 0.2
Discount = 0.9
Living reward = 0

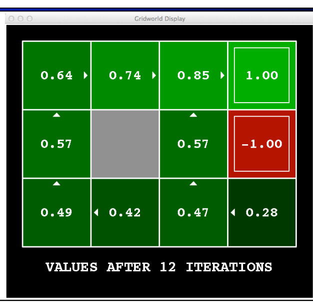
k=10

Noise = 0.2
Discount = 0.9
Living reward = 0

k=11

Noise = 0.2
Discount = 0.9
Living reward = 0

k=12



Noise = 0.2
Discount = 0.9
Living reward = 0

k=100

Noise = 0.2
Discount = 0.9
Living reward = 0

Value Iteration

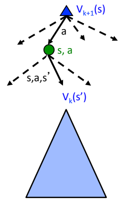
- **Start with $V_0(s) = 0$:**
- **Given vector of $V_k(s)$ values, do one ply of expectimax from each state:**

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- **Repeat until convergence**

- **Complexity of each iteration: $O(S^2A)$**
- **Number of iterations: poly(|S|, |A|, 1/(1-g))**

- **Theorem: will converge to unique optimal values**



Value Iteration

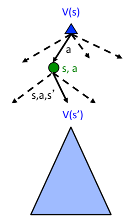
- Bellman equations characterize the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

- Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- Value iteration is just a fixed point solution method
  - … though the V_k vectors are also interpretable as time-limited values

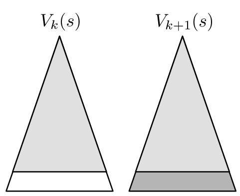


Convergence*

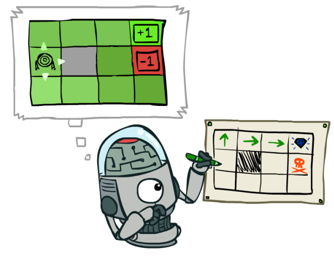
- How do we know the V_k vectors will converge?

- Case 1: If the tree has maximum depth M, then V_M holds the actual untruncated values

- Case 2: If the discount is less than 1
  - Sketch: For any state V_k and V_{k+1} can be viewed as depth k+1 expectimax results in nearly identical search trees
  - The max difference happens if big reward at k+1 level
  - That last layer is at best all R_{MAX}
  - But everything is discounted by γ^k that far out
  - So V_k and V_{k+1} are at most γ^k max|R| different
  - So as k increases, the values converge



Policy Extraction



Computing Actions from Values

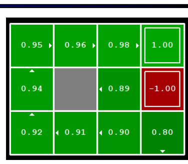
- Let's imagine we have the optimal values V*(s)

- How should we act?
  - It's not obvious!

- We need to do a mini-expectimax (one step)

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

- This is called policy extraction, since it gets the policy implied by the values



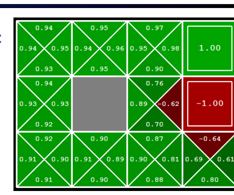
Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:

- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

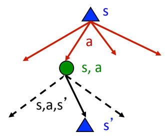
- Important lesson: actions are easier to select from q-values than values!



Problems with Value Iteration

- Value iteration repeats the Bellman updates:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$



- Problem 1: It's slow – O(S^2A) per iteration
- Problem 2: The "max" at each state rarely changes
- Problem 3: The policy often converges long before the values

[Demo: value iteration (L9D2)]

VI → Asynchronous VI

- Is it essential to back up ***all*** states in each iteration?
  - No!
- States may be backed up
  - many times or not at all
  - in any order
- As long as no state gets starved…
  - convergence properties still hold!!

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Prioritization of Bellman Backups

- Are all backups equally important?
- Can we avoid some backups?
- Can we schedule the backups more appropriately?

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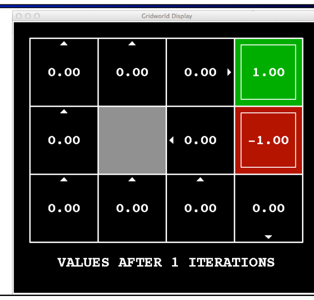
k=1



Noise = 0.2
Discount = 0.9
Living reward = 0

k=2

Noise = 0.2
Discount = 0.9
Living reward = 0

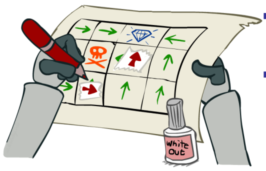
k=3

Noise = 0.2
Discount = 0.9
Living reward = 0

Asynch VI: Prioritized Sweeping

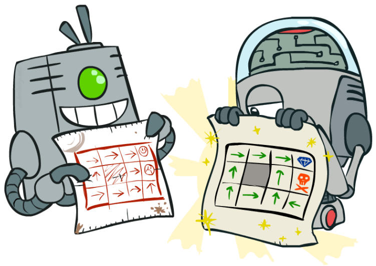
- Why backup a state if values of successors same?
- Prefer backing a state
  - whose successors had most change
- Priority Queue of (state, expected change in value)
- Backup in the order of priority
- After backing a state update priority queue
  - for all predecessors

Solving MDPs

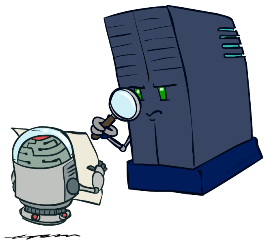


- Value Iteration
- Policy Iteration
- Reinforcement Learning

Policy Methods

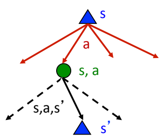


Policy Evaluation

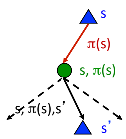


Fixed Policies

Do the optimal action



Do what π says to do

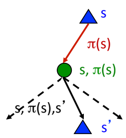


- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state
  - … though the tree's value would depend on which policy we fixed

Utilities for a Fixed Policy

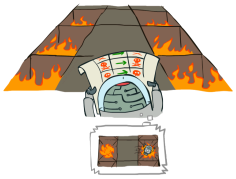
- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy
- Define the utility of a state s, under a fixed policy π:
  $V^\pi(s)$ = expected total discounted rewards starting in s and following π
- Recursive relation (one-step look-ahead / Bellman equation):

$$V^\pi(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

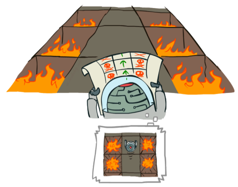


Example: Policy Evaluation

Always Go Right



Always Go Forward



Example: Policy Evaluation

Always Go Right

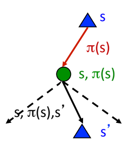
Always Go Forward

Policy Evaluation

- How do we calculate the V's for a fixed policy π?
- Idea 1: Turn recursive Bellman equations into updates (like value iteration)

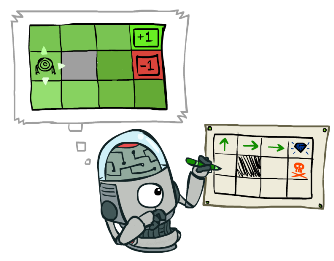
$$V_0^{\pi}(s) = 0$$

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$



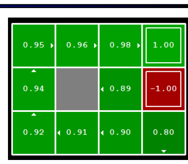
- Efficiency: $O(S^2)$ per iteration
- Idea 2: Without the maxes, the Bellman equations are just a linear system
  - Solve with Matlab (or your favorite linear system solver)

Policy Extraction



Computing Actions from Values

- Let's imagine we have the optimal values V*(s)



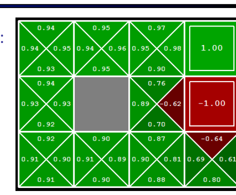
- How should we act?
  - It's not obvious!
- We need to do a mini-expectimax (one step)

$$\pi^*(s) = \arg\max_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^*(s')]$$

- This is called policy extraction, since it gets the policy implied by the values

Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:

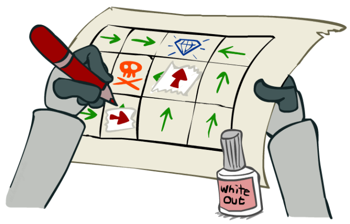


- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s, a)$$

- Important lesson: actions are easier to select from q-values than values!

Policy Iteration



Policy Iteration

- Alternative approach for optimal values:
  - Step 1: Policy evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - Step 2: Policy improvement: update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges
- This is policy iteration
  - It's still optimal!
  - Can converge (much) faster under some conditions

Policy Iteration

- Evaluation: For fixed current policy π, find values with policy evaluation:
  - Iterate until values converge:

$$V^{\pi_i}_{k+1}(s) \leftarrow \sum_{s'} T(s,\pi_i(s),s')\left[R(s,\pi_i(s),s') + \gamma V^{\pi_i}_k(s')\right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^{\pi_i}(s')\right]$$

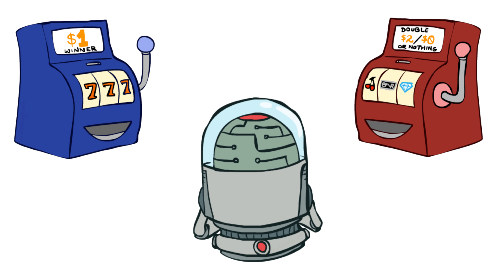
Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
  - Every iteration updates both the values and (implicitly) the policy
  - We don't track the policy, but taking the max over actions implicitly recomputes it
- In policy iteration:
  - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
  - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
  - The new policy will be better (or we're done)
- Both are dynamic programs for solving MDPs

Summary: MDP Algorithms

- So you want to….
  - Compute optimal values: use value iteration or policy iteration
  - Compute values for a particular policy: use policy evaluation
  - Turn your values into a policy: use policy extraction (one-step lookahead)
- These all look the same!
  - They basically are – they are all variations of Bellman updates
  - They all use one-step lookahead expectimax fragments
  - They differ only in whether we plug in a fixed policy or max over actions

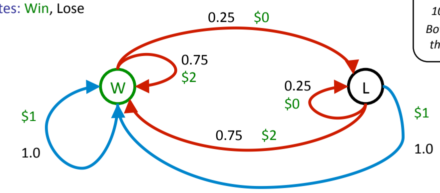
Double Bandits



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose

No discount
100 time steps
Both states have the same value

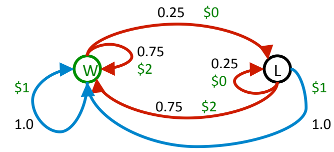


Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount
100 time steps
Both states have the same value

	Value
Play Red	150
Play Blue	100

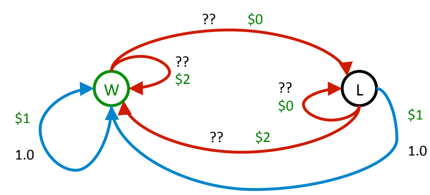


Let's Play!

$2 $2 $0 $2 $2
$2 $2 $0 $0 $0

Online Planning

- Rules changed! Red's win chance is different.



Let's Play!

$0 $0 $0 $2 $0
$2 $0 $0 $0 $0

What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

Next Time: Reinforcement Learning!