CSE 473: Artificial Intelligence
Autumn 2014

Search: Cost & Heuristics

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With slides from
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Announcements

Piazza?

Project 1: Search
- online tomorrow
- due Monday 10/13

Search thru a Problem Space / State Space

- Input:
  - Set of states
  - Operators [and costs]
  - Start state
  - Goal state [test]
- Output:
  - Path: start ⇒ a state satisfying goal test
  - [May require shortest path]
  - [Sometimes just need state passing test]

Graduation?

- Getting a BS in CSE as a search problem?
  (don't think too hard)
- Space of States
- Operators
- Initial State
- Goal State

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Search Methods

Blind search

- Depth first search (DFS)
- Breadth first search (BFS)
- Iterative deepening depth-first search (IDS)

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Search Methods

- Depth first search (DFS)
- Breadth first search (BFS)
- Iterative deepening depth-first search (IDS)

Heuristic search

- Best first search
- Uniform cost search (UCS)
- Greedy search
- A*
- Iterative Deepening A* (IDA*)
- Beam search
- Hill climbing

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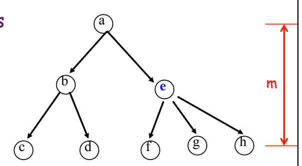
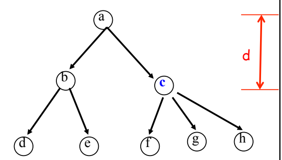
Depth First Search

- Maintain ***stack*** of nodes to visit
  - Check path to root to prune duplicates
- Evaluation
  - Complete?
    Not for infinite spaces
  - Time Complexity?
    $O(b^m)$
  - Space Complexity?
    $O(bm)$



Breadth First Search

- Maintain ***queue*** of nodes to visit
- Evaluation
  - Complete?
    Yes
  - Time Complexity?
    $O(b^d)$
  - Space Complexity?
    $O(b^d)$



Memory a Limitation?

- Suppose:
  - 4 GHz CPU
  - 16 GB main memory
  - 100 instructions / expansion
  - 10 bytes / node
- 400,000 expansions / sec
  - Memory filled in 400 sec … < 7 min

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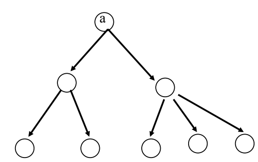
Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation
  - Complete?
  - Time Complexity?
  - Space Complexity?



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Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation
  - Complete?
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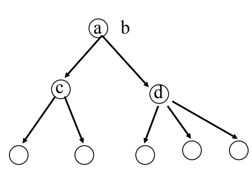

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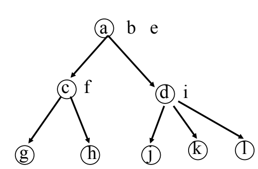
Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation
  - Complete?
  - Time Complexity?
  - Space Complexity?



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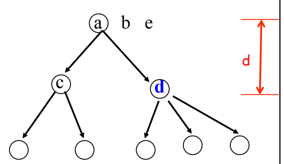
Iterative Deepening Search

- DFS with limit; incrementally grow limit
- Evaluation
  - Complete?

    Yes
  - Time Complexity?

    O(b^d)
  - Space Complexity?

    O(b^d)



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Cost of Iterative Deepening

b	ratio ID to DFS
2	3
3	2
5	1.5
10	1.2
25	1.08
100	1.02

Speed

Assuming 10M nodes/sec & sufficient memory

	BFS Nodes	BFS Time		Iter. Deep. Nodes	Iter. Deep. Time
8 Puzzle	10^5	.01 sec		10^5	.01 sec
2x2x2 Rubik's	10^6	.2 sec		10^6	.2 sec
15 Puzzle	10^{13}	6 days	1Mx	10^{17}	20k yrs
3x3x3 Rubik's	10^{19}	68k yrs	8x	10^{20}	574k yrs
24 Puzzle	10^{25}	12B yrs		10^{37}	10^{23} yrs

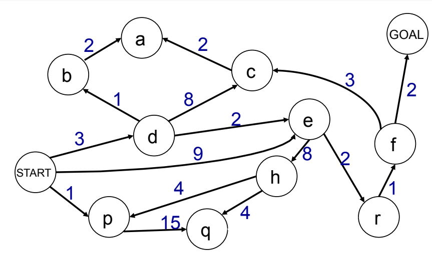
Why the difference?

Rubik has higher branch factor
15 puzzle has greater depth

\# of duplicates

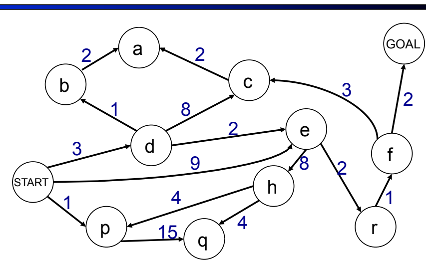
Slide adapted from Richard Korf presentation

Costs on Actions



Objective: Path with smallest overall cost

Costs on Actions



What will BFS return?

… finds the shortest path in terms of number of transitions.
It does ***not*** find the least-cost path.

Best-First Search

- Generalization of breadth-first search
- Fringe = ***Priority*** queue of nodes to be explored
- Cost function f(n) applied to each node

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Priority Queue Refresher

- A priority queue is a data structure in which you can insert and retrieve (key, value) pairs with the following operations:

pq.push(key, value)	inserts (key, value) into the queue.
pq.pop()	returns the key with the lowest value, and removes it from the queue.

- You can decrease a key's priority by pushing it again
- Unlike a regular queue, insertions aren't constant time, usually O(log n)
- We'll need priority queues for cost-sensitive search methods

Best-First Search

- Generalization of breadth-first search
- Fringe = ***Priority*** queue of nodes to be explored
- Cost function f(n) applied to each node

```
Add initial state to priority queue
While queue not empty
      Node = head(queue)
      If goal?(node) then return node
      Add children of node to queue
```

"expanding the node"

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Old Friends

- Breadth First =
  - Best First
  - with f(n) = depth(n)
- Dijkstra's Algorithm (Uniform cost) =
  - Best First
  - with f(n) = the sum of edge costs from start to n

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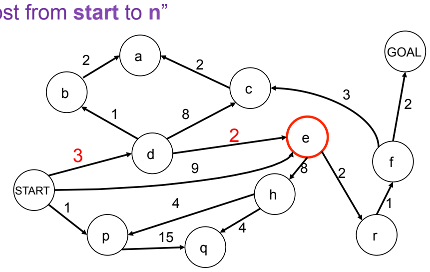
Uniform Cost Search

Best first, where

f(n) = "cost from **start** to **n**"

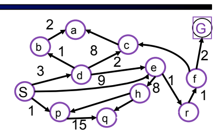


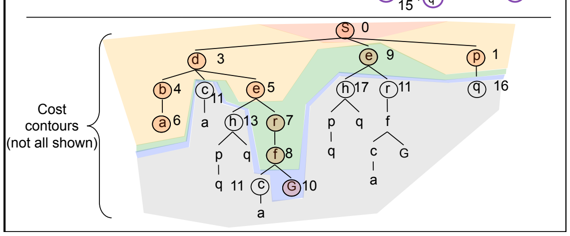
aka "Dijkstra's Algorithm"

Uniform Cost Search

Expansion order:

S, p, d, b, e, a, r, f, e, G

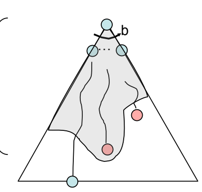




Uniform Cost Search

Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Y if finite	N	$O(b^m)$	$O(bm)$
BFS		Y	Y*	$O(b^d)$	$O(b^d)$
UCS		Y*	Y	$O(b^{C^*/\varepsilon})$	$O(b^{C^*/\varepsilon})$

C^*/ε tiers

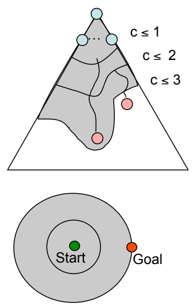


C^* = Optimal cost

ε = Minimum cost of an action

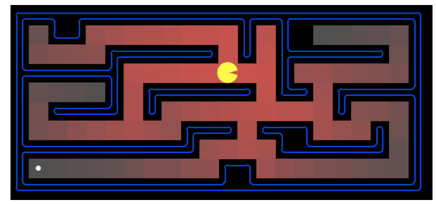
Uniform Cost Issues

- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location



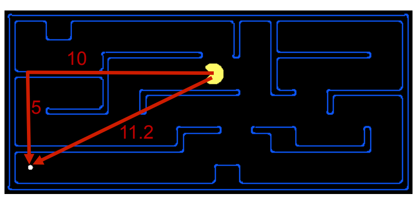
Uniform Cost: Pac-Man

- Cost of 1 for each action
- Explores all of the states, but one



What is a *Heuristic*?

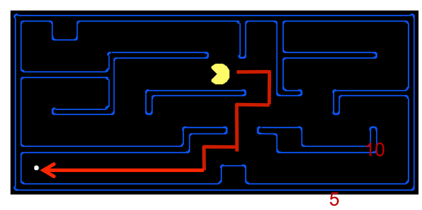
- An **estimate** of how close a state is to a goal
- Designed for a particular search problem



- Examples: Manhattan distance: 10+5 = 15
  Euclidean distance: 11.2

What is a *Heuristic*?

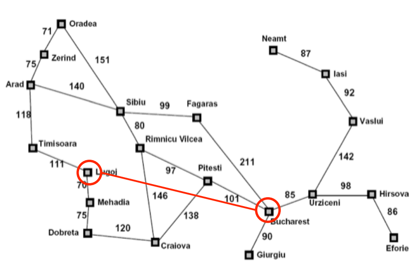
- An **estimate** of how close a state is to a goal
- Designed for a particular search problem



- Actual distance to goal: 2+4+2+1+8=

Greedy Search

Best first with f(n) = heuristic estimate of distance to goal

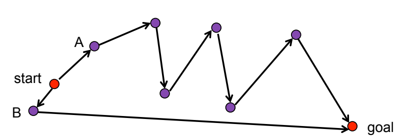


Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search

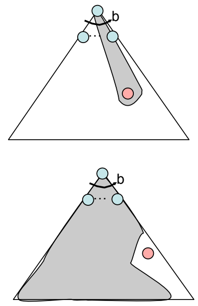
Expand the node that seems closest…



What can go wrong?

Greedy Search

- A common case:
  - Best-first takes you straight to a (suboptimal) goal
- Worst-case: like a badly-guided DFS in the worst case
  - Can explore everything
  - Can get stuck in loops if no cycle checking
- Like DFS in completeness (if finite # states w/ cycle checking)



A* Search

Hart, Nilsson & Rafael 1968

Best first search with f(n) = g(n) + h(n)

- g(n) = sum of costs from start to n
- h(n) = estimate of lowest cost path n → goal

  h(goal) = 0

If h(n) is admissible and monotonic

then A* is optimal

Underestimates cost of reaching goal from node

f values increase from node to descendants (triangle inequality)

A* Search

Hart, Nilsson & Rafael 1968

Best first search with f(n) = g(n) + h(n)

- g(n) = sum of costs from start to n
- h(n) = estimate of lowest cost path n → goal

  h(goal) = 0

Can view as cross-breed:

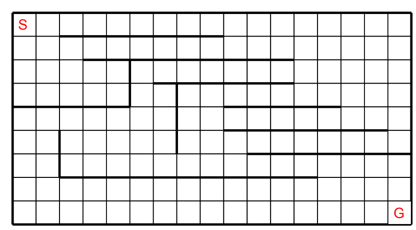
g(n) ~ uniform cost search

h(n) ~ greedy search

Best of both worlds…

Is Manhattan distance **admissible**?

- Underestimate?



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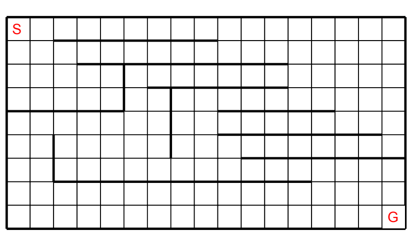
Is Manhattan distance **monotonic**?

- f values increase from node to children
- (triangle inequality)



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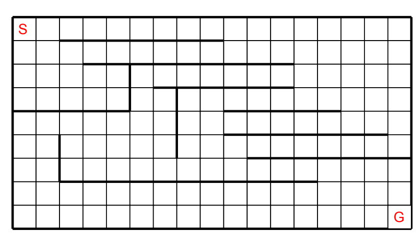
Euclidean Distance

- Admissible?
- Monotonic?



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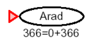
A* Example



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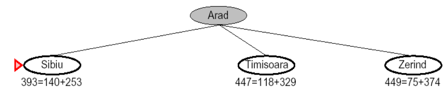
A* Example



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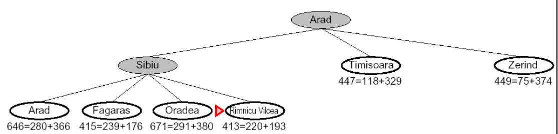
A* Example



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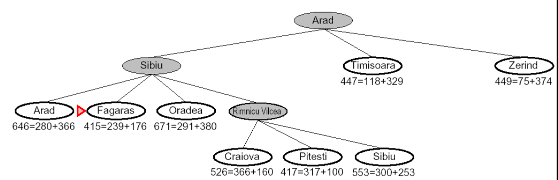
A* Example



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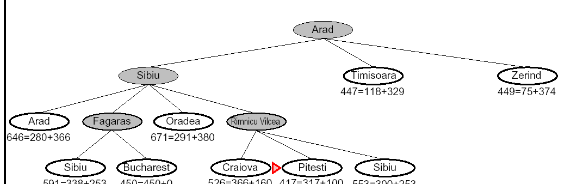
A* Example



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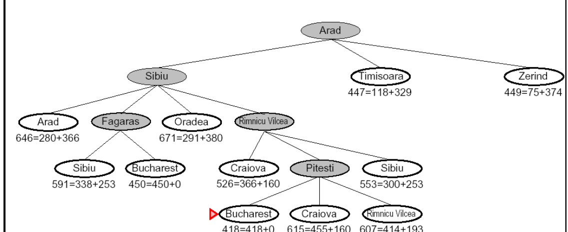
A* Example



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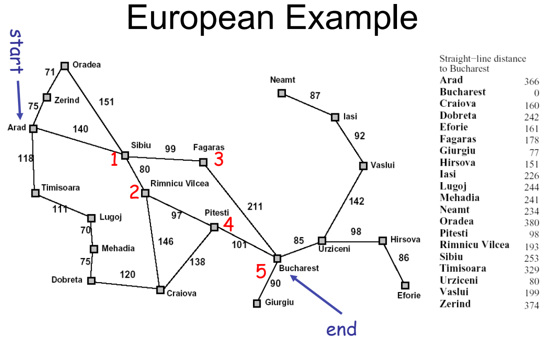
European Example



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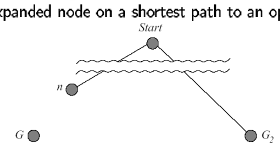
Optimality of A*

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

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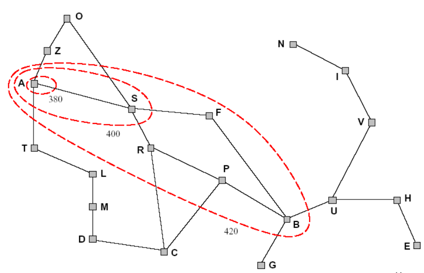
Optimality Continued

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



A* Summary

- Pros

  Produces optimal cost solution!

  Does so quite quickly (focused)

- Cons

  Maintains priority queue…

  Which can get exponentially big ☹

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