

CSE 473  
**Automated Planning**  
 Dan Weld  
 (With slides by UW AI faculty & Dana Nau)

**I have a plan - a plan that cannot possibly fail.**  
 - Inspector Clousseau



## Logistics

- HW1 due in one week (Fri 5/4)
  - Parts due in between:
    - Monday    draft answer to problem 1
    - Wed        give feedback on another person's answer

## Overview

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- Introduction & Agents
- Search, Heuristics & CSPs
- Adversarial Search
- **Logical Knowledge Representation**
- **Planning** & MDPs
- Reinforcement Learning
- Uncertainty & Bayesian Networks
- Machine Learning
- NLP & Special Topics

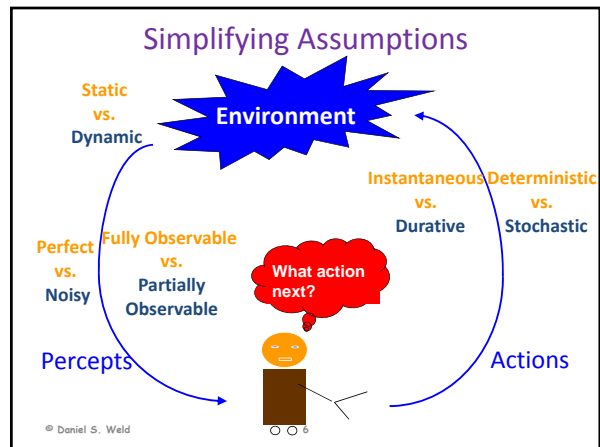
## Today's Topics

- Logic for specifying planning domains
- Planning graph for computing heuristics
- Compiling planning to SAT

## Planning

- **Given**
  - a logical description of the **initial situation**,
  - a logical description of the **goal conditions**, and
  - a logical description of a set of **possible actions**,
- **Find**
  - a **sequence of actions** (a **plan of actions**) that brings us from the initial situation to a situation in which the goal conditions hold.

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## Classical Operators

**unstack(x,y)**  
 Precond: on(x,y), clear(x), handempty  
 Effects:  $\neg$ on(x,y),  $\neg$ clear(x),  $\neg$ handempty, holding(x), clear(y)

**stack(x,y)**  
 Precond: holding(x), clear(y)  
 Effects:  $\neg$ holding(x),  $\neg$ clear(y), on(x,y), clear(x), handempty

**pickup(x)**  
 Precond: ontable(x), clear(x), handempty  
 Effects:  $\neg$ ontable(x),  $\neg$ clear(x),  $\neg$ handempty, holding(x)

**putdown(x)**  
 Precond: holding(x)  
 Effects:  $\neg$ holding(x), ontable(x), clear(x), handempty

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## Driving

```

(defschema (drive)
  :parameters (?v ?s ?d)
  :precondition (and (vehicle ?v) (at ?v ?s)
                    (location ?s) (location ?d)
                    (road-connected ?s ?d))
  :effect (and (at ?v ?d) (not (at ?v ?s))
              (forall (object ?o)
                (when (in ?o ?v)
                  (and (at ?o ?v)) (not (at ?o ?s))))))
  
```

Static facts

Universally quantified conditional schemata for driving.

## Planning vs. Problem-Solving ?

Basic difference: **Explicit, logic-based representation**

- States/Situations:** descriptions of the world by logical formulae  
 → agent can explicitly reason about the world.
- Goal conditions** as logical formulae vs. goal test (black box)  
 → agent can reflect on its goals.
- Operators/Actions:** Transformations on logical formulae  
 → agent can reason about the effects of actions by inspecting the definition of its operators.

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## Forward World-Space Search

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## Heuristics for State-Space Search

- Count number of false goal propositions in current state**  
 Admissible?  
 NO
- Subgoal independence assumption:**
  - Cost of solving conjunction is sum of cost of solving each subgoal independently
  - Optimistic: ignores negative interactions
  - Pessimistic: ignores redundancy

– Admissible? No  
 – Can you make this admissible?

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## Heuristic Generation II

**unstack(x,y)**  
 Precond: ~~on(x,y), clear(x), handempty~~  
 Effects:  $\neg$ on(x,y),  $\neg$ clear(x),  $\neg$ handempty, holding(x), clear(y)

**stack(x,y)**  
 Precond: ~~holding(x), clear(y)~~  
 Effects:  $\neg$ holding(x),  $\neg$ clear(y), on(x,y), clear(x), handempty

**pickup(x)**  
 Precond: ~~ontable(x), clear(x), handempty~~  
 Effects:  $\neg$ ontable(x),  $\neg$ clear(x),  $\neg$ handempty, holding(x)

**putdown(x)**  
 Precond: ~~holding(x)~~  
 Effects:  $\neg$ holding(x), ontable(x), clear(x), handempty

Delete **preconditions**

Solve relaxed planning problem

Admissible?

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### Heuristic Generation III

<p>unstack(x,y)                  Precond: on(x,y), clear(x), handempty                  Effects: <del>on(x,y), clear(x), handempty,</del>                  holding(x), clear(y)</p>	<p>Delete <b>negative effects</b></p> <p>Solve relaxed planning problem</p> <p>Admissable?</p>
<p>stack(x,y)                  Precond: holding(x), clear(y)                  Effects: <del>holding(x), clear(y),</del>                  on(x,y), clear(x), handempty</p>	
<p>pickup(x)                  Precond: ontable(x), clear(x), handempty                  Effects: <del>ontable(x), clear(x), handempty,</del>                  holding(x)</p>	
<p>putdown(x)                  Precond: holding(x)                  Effects: <del>holding(x),</del> ontable(x),                  clear(x), handempty</p>	

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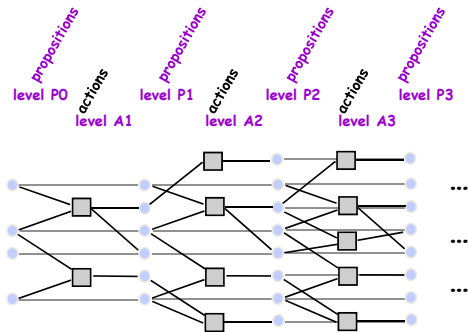
### Planning Graph: Basic idea

- Construct a planning graph: encodes constraints on possible plans
- Use this planning graph to compute an informative heuristic (Forward A\*)
- Planning graph can be built for each problem in polynomial time

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15

### The Planning Graph



Note: a few noops missing for clarity

16

### PG Example

Init(Have(Cake))  
 Goal(Have(Cake)  $\wedge$  Eaten(Cake))  
 Action(Eat(Cake),  
 PRECOND: Have(Cake)  
 EFFECT:  $\neg$ Have(Cake)  $\wedge$  Eaten(Cake))  
 Action(Bake(Cake),  
 PRECOND:  $\neg$  Have(Cake)  
 EFFECT: Have(Cake))

### PG Example

$S_0$                        $A_0$                        $S_1$

Have(Cake)

$\neg$ Eaten(Cake)

Create level 0 from initial problem state.

### Graph Expansion

**Proposition level 0**  
 initial conditions

**Action level i**  
 no-op for each proposition at level i-1  
 action for each operator instance whose preconditions exist at level i-1

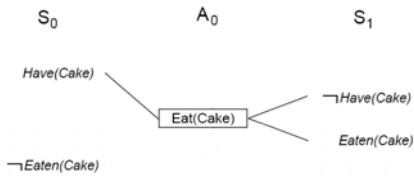
**Proposition level i**  
 effects of each no-op and action at level i

**No-op-action(P),**  
 PRECOND: P  
 EFFECT: P  
 Have a no-op action for each ground fact

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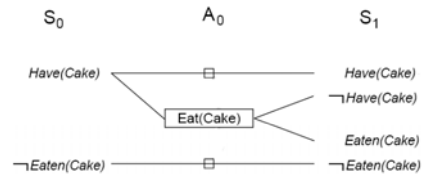
21

## PG Example



Add all applicable actions.  
Add all effects to the next state.

## PG Example

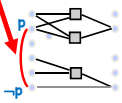


Add *persistence actions* (aka no-ops) to map all literals in state  $S_i$  to state  $S_{i+1}$ .

## Mutual Exclusion

Two propositions are mutex if

- one is the negation of the other

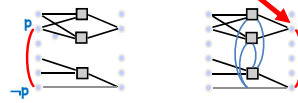


24

## Mutual Exclusion

Two proposition are mutex if

- one is the negation of the other
- all ways of achieving them are mutex



25

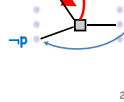
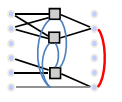
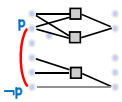
## Mutual Exclusion

Two actions are mutex if

- they have mutex preconditions
- one clobbers the other's preconditions or effects

Two proposition are mutex if

- one is the negation of the other
- all ways of achieving them are mutex

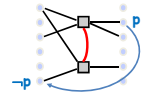


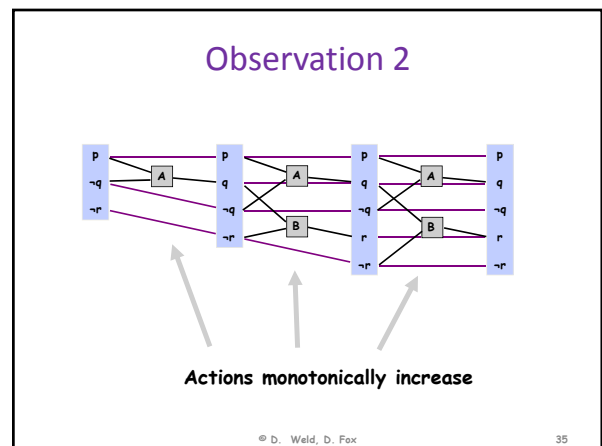
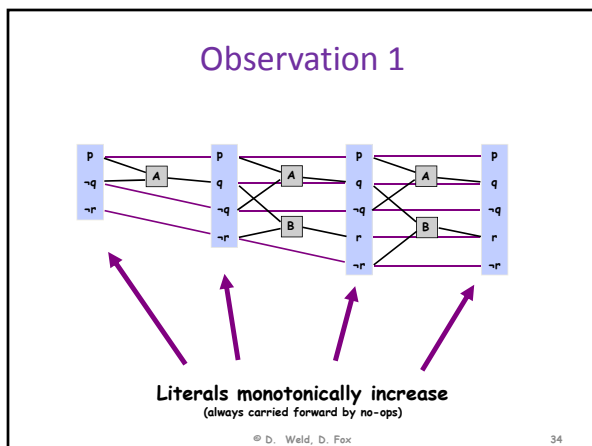
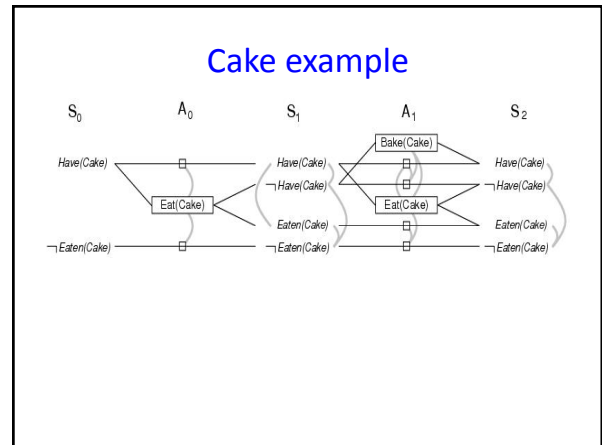
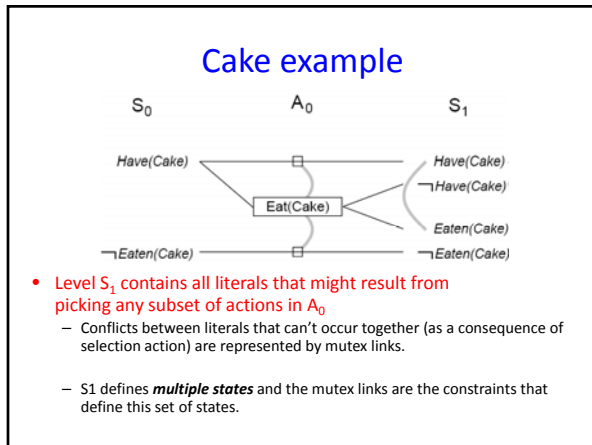
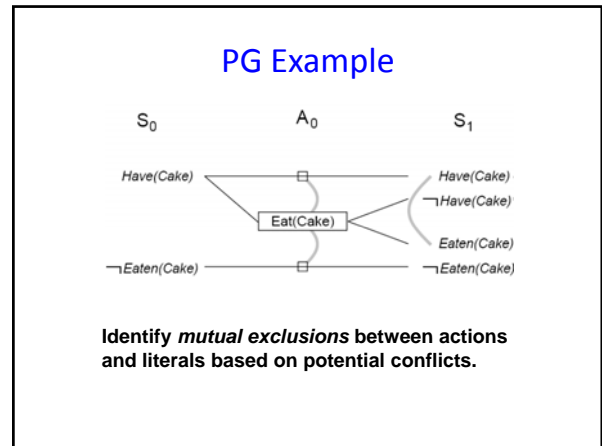
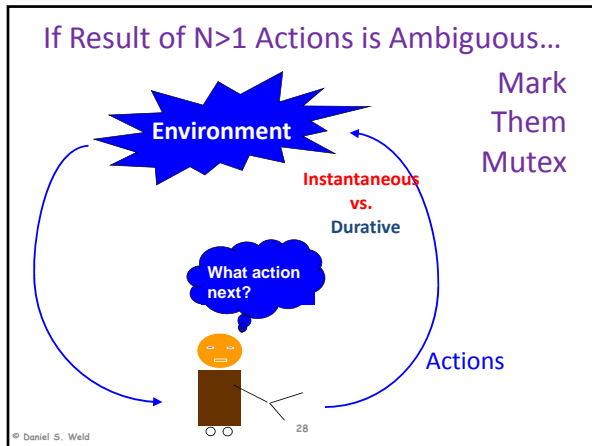
26

## Mutual Exclusion

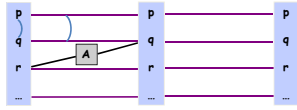
Light-fuse(*match*, *bomb*)  
Precond: lit(*match*), holding(*bomb*)  
Effects: will-explode(*bomb*)

Extinguish(*match*)  
Precond: lit(*match*)  
Effects: ¬lit(*match*)





### Observation 3

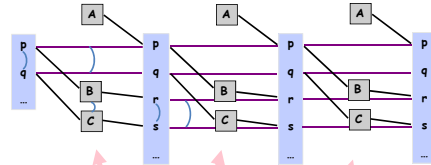


mutex relationships between literals monotonically decrease

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36

### Observation 4



Mutex relationships between actions monotonically decrease

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37

### Observation 5

#### Planning Graph 'levels off'.

- After some time  $k$ , all levels are identical
  - Because it's a finite space & monotonicity

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38

### Properties of Planning Graph

- If goal is absent from last level?
  - Then goal cannot be achieved!
- If there exists a plan to achieve goal?
  - Then goal is present in the last level &
  - No mutexes between conjuncts
- If goal is present in last level (w/ no mutexes) ?
  - There still may not exist any viable plan

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39

### Heuristics based on Planning Graph

- Construct planning graph starting from  $s$
- $h(s)$  = level at which goal appears non-mutex
  - Admissible?
  - YES

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### Planning Graph is Optimistic

Suppose you want to prepare a surprise dinner for your sleeping sweetheart

$s_0 = \{\text{garbage, cleanHands, quiet}\}$

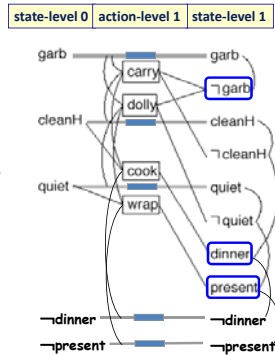
$g = \{\text{dinner, present, } \neg\text{garbage}\}$

Action	Preconditions	Effects
cook()	cleanHands	dinner
wrap()	quiet	present
carry()	none	$\neg$ garbage, $\neg$ cleanHands
dolly()	none	$\neg$ garbage, $\neg$ quiet

Also have persistence actions: one for each literal

## Example (continued)

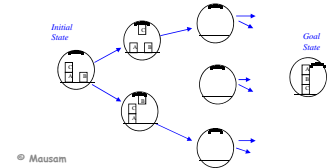
- Recall the goal is
  - $\{\neg \text{garbage}, \text{dinner}, \text{present}\}$
- Note that in state-level 1,
  - All of them are there
  - None are mutex with each other
- Thus, there's a chance that a plan exists
  - But no actual plan *does*...
  - Pairwise, the goals are consistent
  - But no consistent way to achieve all three
- Planning graph  $\sim k$  consistency
  - But with no fixed limit on  $k$



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## Fast-Forward (FF)

- Fastest classical planner  $\sim 2009$
- State space local search
  - Guided by relaxed planning graph
  - Full best-first search to escape plateaus
  - A few other bells and whistles...



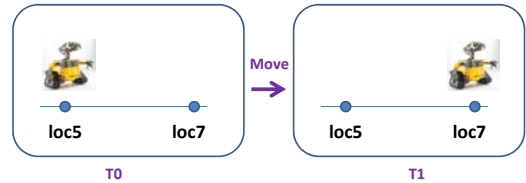
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## Today's Topics

- Logic for specifying planning domains
- Planning graph for computing heuristics
- Compiling planning to SAT

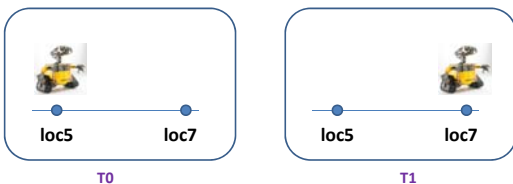
## Fluent

- Ground literal whose truth value may change over time
- Eg,  $\text{at}(\text{robbie}, \text{location5})$
- Not  $\text{robot}(\text{robbie})$



## Encoding Fluents in Logic

- $\text{at}(\text{robbie}, \text{location7}, \text{time1})$



## Encoding Initial Conditions & Goals

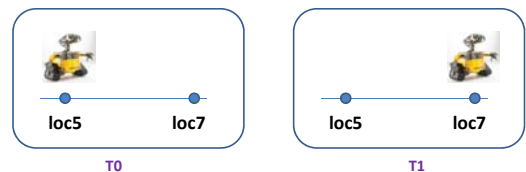
Suppose we only cared about 1-step plans

State desired end conditions:

$$\Phi = \text{at}(\text{robbie}, \text{loc5}, \text{time0}) \wedge \text{at}(\text{robbie}, \text{loc7}, \text{time1})$$

Is  $\Phi$  satisfiable?

Is  $P \wedge Q$  satisfiable?

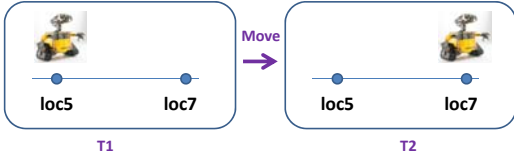


## Encoding Action Effects in Logic

`move(r, l1, l2)`  
 Precond: `robot(r), at(r,l1), ...`  
 Effects: `at(r,l2)`

$\forall r, l1, l2, t \quad \text{move}(r,l1,l2,t) \Rightarrow \text{at}(r, l2, t+1)$

$\forall r, l1, l2, t \quad \text{move}(r,l1,l2,t) \Rightarrow \text{at}(r, l, t) \rightarrow \neg \text{move}(r,l1,l2,t) \vee \text{at}(r, l, t)$



## Compiling to Propositional Logic

`move(r, l1, l2)`  
 Precond: `robot(r), at(r,l1), ...`  
 Effects: `at(r,l2)`

$\forall r, l1, l2, t \quad \text{move}(r,l1,l2,t) \Rightarrow \text{at}(r, l2, t+1)$

Infinite worlds: impossible

But suppose only 2 robots (robbie, sue), 2 locations, 1 action time

$\text{move}(\text{robbie}, \text{loc5}, \text{loc7}, 1) \Rightarrow \text{at}(\text{robbie}, \text{loc7}, 2) \wedge$   
 $\text{move}(\text{robbie}, \text{loc7}, \text{loc5}, 1) \Rightarrow \text{at}(\text{robbie}, \text{loc5}, 2) \wedge$   
 $\text{move}(\text{sue}, \text{loc5}, \text{loc7}, 1) \Rightarrow \text{at}(\text{sue}, \text{loc7}, 2) \wedge$   
 $\text{move}(\text{sue}, \text{loc7}, \text{loc5}, 1) \Rightarrow \text{at}(\text{sue}, \text{loc5}, 2)$

## Overall Approach

- A *bounded planning problem* is a pair  $(P, n)$ :
  - $P$  is a planning problem;  $n$  is a positive integer
  - Any solution for  $P$  of length  $n$  is a solution for  $(P, n)$
- Planning algorithm:
- Do iterative deepening like we did with Graphplan:
  - for  $n = 0, 1, 2, \dots$ ,
    - encode  $(P, n)$  as a satisfiability problem  $\Phi$
    - if  $\Phi$  is satisfiable, then
      - From the set of truth values that satisfies  $\Phi$ , a solution plan can be constructed, so return it and exit

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## Encoding Planning Problems

- Encode  $(P, n)$  as a formula  $\Phi$  such that
  - $\pi = \langle a_0, a_1, \dots, a_{n-1} \rangle$  is a solution for  $(P, n)$  if and only if  $\Phi$  can be satisfied in a way that makes the fluents  $a_0, \dots, a_{n-1}$  true
- Let
  - $A = \{\text{all actions in the planning domain}\}$
  - $S = \{\text{all states in the planning domain}\}$
  - $L = \{\text{all literals in the language}\}$
- $\Phi$  is the conjunct of many other formulas ...

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## Formulas in $\Phi$

- Formula describing the initial state:
 
$$\bigwedge \{l_0 \mid l \in s_0\} \wedge \bigwedge \{\neg l_0 \mid l \in L - s_0\}$$
- Formula describing the goal:
 
$$\bigwedge \{l_n \mid l \in g\} \wedge \bigwedge \{\neg l_n \mid l \in g^c\}$$
- For every action  $a$  in  $A$ , formulas describing what changes  $a$  would make if it were the  $i$ 'th step of the plan:
  - $a_i \Rightarrow \bigwedge \{p_i \mid p \in \text{Precond}(a)\} \wedge \bigwedge \{e_{i+1} \mid e \in \text{Effects}(a)\}$
- Complete exclusion axiom:
  - For all actions  $a$  and  $b$ , formulas saying they can't occur at the same time
    - $\neg a_i \vee \neg b_i$
  - this guarantees there can be only one action at a time
- Is this enough?

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## Frame Axioms

- *Frame axioms*:
  - Formulas describing what *doesn't* change between steps  $i$  and  $i+1$
- Several ways to write these
- One way: *explanatory frame axioms*
  - One axiom for every literal  $l$
  - Says that if  $l$  changes between  $s_i$  and  $s_{i+1}$ , then the action at step  $i$  must be responsible:
 
$$\neg l_i \wedge l_{i+1} \Rightarrow \bigvee_{a \in A} \{a_i \mid l \in \text{effects}^+(a)\}$$

$$\wedge (l_i \wedge \neg l_{i+1} \Rightarrow \bigvee_{a \in A} \{a_i \mid l \in \text{effects}^-(a)\})$$

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## Example

- **Planning domain:**
  - one robot r1
  - two adjacent locations l1, l2
  - one operator (move the robot)

- **Encode  $(P,n)$  where  $n = 1$**

- Initial state:  $\{at(r1,l1)\}$   
Encoding:  $at(r1,l1,0) \wedge \neg at(r1,l2,0)$
- Goal:  $\{at(r1,l2)\}$   
Encoding:  $at(r1,l2,1) \wedge \neg at(r1,l1,1)$
- Operator: see next slide

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## Example (continued)

- **Operator:**  $move(r,l,l')$   
precond:  $at(r,l)$   
effects:  $at(r,l'), \neg at(r,l)$

### Encoding:

- $move(r1,l1,l2,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l1,1)$
  - $move(r1,l2,l1,0) \Rightarrow at(r1,l2,0) \wedge at(r1,l1,1) \wedge \neg at(r1,l2,1)$
  - $move(r1,l1,l1,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l1,1) \wedge \neg at(r1,l1,1)$
  - $move(r1,l2,l2,0) \Rightarrow at(r1,l2,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l2,1)$
- } **contradictions (easy to detect)**
- $move(l1,r1,l2,0) \Rightarrow \dots$
  - $move(l2,l1,r1,0) \Rightarrow \dots$
  - $move(l1,l2,r1,0) \Rightarrow \dots$
  - $move(l2,l1,r1,0) \Rightarrow \dots$
- } **nonsensical**

- **How to avoid generating the last four actions?**
  - Assign data types to the constant symbols like we did for state-variable representation

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## Example (continued)

- **Locations:** l1, l2
- **Robots:** r1
- **Operator:**  $move(r : robot, l : location, l' : location)$   
precond:  $at(r,l)$   
effects:  $at(r,l'), \neg at(r,l)$

### Encoding:

- $move(r1,l1,l2,0) \Rightarrow at(r1,l1,0) \wedge at(r1,l2,1) \wedge \neg at(r1,l1,1)$
- $move(r1,l2,l1,0) \Rightarrow at(r1,l2,0) \wedge at(r1,l1,1) \wedge \neg at(r1,l2,1)$

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## Example (continued)

- **Complete-exclusion axiom:**  
 $\neg move(r1,l1,l2,0) \vee \neg move(r1,l2,l1,0)$
- **Explanatory frame axioms:**  
 $\neg at(r1,l1,0) \wedge at(r1,l1,1) \Rightarrow move(r1,l2,l1,0)$   
 $\neg at(r1,l2,0) \wedge at(r1,l2,1) \Rightarrow move(r1,l1,l2,0)$   
 $at(r1,l1,0) \wedge \neg at(r1,l1,1) \Rightarrow move(r1,l1,l2,0)$   
 $at(r1,l2,0) \wedge \neg at(r1,l2,1) \Rightarrow move(r1,l2,l1,0)$

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## Extracting a Plan

- **Suppose we find an assignment of truth values that satisfies  $\Phi$ .**
  - This means  $P$  has a solution of length  $n$
- **For  $i=1,\dots,n$ , there will be exactly one action  $a$  such that  $a_i = true$** 
  - This is the  $i$ 'th action of the plan.
- **Example (from the previous slides):**
  - $\Phi$  can be satisfied with  $move(r1,l1,l2,0) = true$
  - Thus  $\langle move(r1,l1,l2,0) \rangle$  is a solution for  $(P,0)$ 
    - It's the only solution - no other way to satisfy  $\Phi$

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## Planning

- **How to find an assignment of truth values that satisfies  $\Phi$ ?**
  - Use a satisfiability algorithm
- **Example: the Davis-Putnam algorithm**
  - First need to put  $\Phi$  into conjunctive normal form  
e.g.,  $\Phi = D \wedge (\neg D \vee A \vee \neg B) \wedge (\neg D \vee \neg A \vee \neg B) \wedge (\neg D \vee \neg A \vee B) \wedge A$
  - Write  $\Phi$  as a set of *clauses* (disjuncts of literals)  
 $\Phi = \{ \{D\}, \{\neg D, A, \neg B\}, \{\neg D, \neg A, \neg B\}, \{\neg D, \neg A, B\}, \{A\} \}$
  - Two special cases:
    - If  $\Phi = \emptyset$  then  $\Phi$  is always *true*
    - If  $\Phi = \{ \dots, \emptyset, \dots \}$  then  $\Phi$  is always *false* (hence unsatisfiable)

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