CSE 473: Artificial Intelligence
Spring 2012

Adversarial Search: Expectimax
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Based on slides from
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Space of Search Strategies

- Blind Search
  - DFS, BFS, IDS
- Informed Search
  - Systematic: Uniform cost, greedy, A*, IDA*
  - Stochastic: Hill climbing w/ random walk & restarts
- Constraint Satisfaction
  - Backtracking=DFS, FC, k-consistency, exploiting structure
- **Adversary Search**
  - Mini-max
  - Alpha-beta
  - Evaluation functions
  - Expecti-max

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Overview

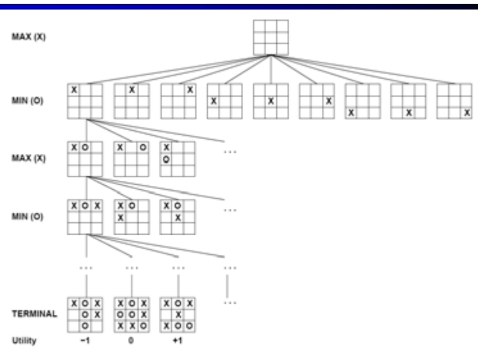
- Introduction & Agents
- Search, Heuristics & CSPs
- Adversarial Search
- Logical Knowledge Representation
- Planning & MDPs
- Reinforcement Learning
- Uncertainty & Bayesian Networks
- Machine Learning
- NLP & Special Topics

Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon, monopoly
imperfect information	stratego	bridge, poker, scrabble, nuclear war

Number of Players? 1, 2, …?

Tic-tac-toe Game Tree



Mini-Max

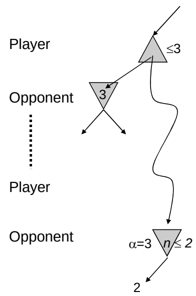
- Assumptions
  - High scoring leaf == good for you (bad for opp)
  - Opponent is super-smart, rational; never errs
  - Will play optimally against you
- Idea
  - Exhaustive search
  - Alternate: best move for you; best for opponent

Max Min

- Guarantee
  - Will find best move for you (given assumptions)

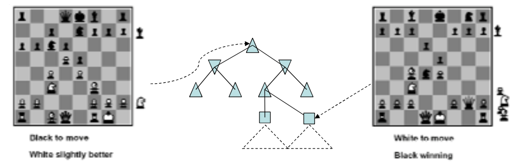
α-β Pruning General Case

- Add α,β bounds to each node
- α is the best value that MAX can get at any choice point along the current path
- If value of *n* becomes worse than α, MAX will avoid it, so can stop considering *n*'s other children
- Define β similarly for MIN



Heuristic Evaluation Function

- Function which scores non-terminals



$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

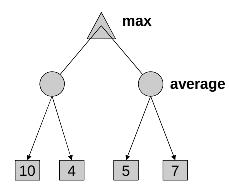
- Ideal function: returns the utility of the position
- In practice: typically weighted linear sum of features:
  - e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Modeling Opponent

- So far assumed
  Opponent = rational, infinitely smart
- What if
  Opponent = random?
  2 player w/ random opponent = 1 player stochastic
- Later…
  Sorta smart
  Infinitely smart, but actions have chance

Stochastic Single-Player

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, mine locations
- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Max nodes as before
  - Chance nodes take average (expectation) of value of children



Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  an agent should chose the action which maximizes its expected utility, given its knowledge
  - General principle for decision making
  - Often taken as the definition of rationality
  - **We'll see this idea over and over in this course!**
- Let's decompress this definition…

Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes
- Example: traffic on freeway?
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.55, P(T=heavy) = 0.20
- Some laws of probability (more later):
  - Probabilities are always ∈ [0, 1]
  - Probabilities (over all possible outcomes) sum to one
- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.20, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

What are Probabilities?

- Objectivist / frequentist answer:
  - Averages over repeated *experiments*
  - E.g. empirically estimating P(rain) from historical observation
  - E.g. pacman's estimate of what the ghost will do, given what it has done in the past
  - Assertion about how future experiments will go (in the limit)
  - Makes one think of *inherently random* events, like rolling dice
- Subjectivist / Bayesian answer:
  - Degrees of belief about unobserved variables
  - E.g. an agent's belief that it's raining, given the temperature
  - E.g. pacman's belief that the ghost will turn left, given the state
  - Often *learn* probabilities from past experiences (more later)
  - New evidence *updates beliefs* (more later)

Uncertainty Everywhere

- Not just for games of chance!
  - I'm sick: will I sneeze this minute?
  - Email contains "FREE!": is it spam?
  - Tooth hurts: have cavity?
  - 60 min enough to get to the airport?
  - Robot rotated wheel three times, how far did it advance?
  - Safe to cross street? (Look both ways!)
- Sources of uncertainty in random variables:
  - Inherently random process (dice, etc)
  - Insufficient or weak evidence
  - Ignorance of underlying processes
  - Unmodeled variables
  - The world's just noisy – it doesn't behave according to plan!

Reminder: Expectations

- We can define function f(X) of a random variable X
- The expected value of a function is its average value, weighted by the probability distribution over inputs
- Example: How long to get to the airport?
  - Length of driving time as a function of traffic:
    L(none) = 20, L(light) = 30, L(heavy) = 60
  - What is my expected driving time?
    - Notation: $E_{P(T)}[\, L(T)\,]$
    - Remember, P(T) = {none: 0.25, light: 0.5, heavy: 0.25}
    - E[L(T)] = L(none) * P(none) + L(light) * P(light) + L(heavy) * P(heavy)
    - E[L(T)] = (20 * 0.25) + (30 * 0.5) + (60 * 0.25) = 35

Expectations II

- Real valued functions of random variables:

$$f : X \rightarrow R$$

- Expectation of a function of a random variable

$$E_{P(X)}[f(X)] = \sum_x f(x)P(x)$$

- Example: Expected value of a fair die roll

X	P	*f*
1	1/6	1
2	1/6	2
3	1/6	3
4	1/6	4
5	1/6	5
6	1/6	6

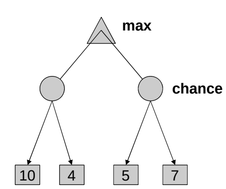
$$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$
$$= 3.5$$

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any set of preferences between outcomes can be summarized as a utility function (provided the preferences meet certain conditions)
- In general, we hard-wire utilities and let actions emerge (why don't we let agents decide their own utilities?)
- More on utilities soon…

Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, next card is unknown
  - In minesweeper, mine locations
  - In pacman, the ghosts act randomly
- Can do **expectimax search**
  - Chance nodes, like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Max nodes as in minimax search
  - Chance nodes take average (expectation) of value of children

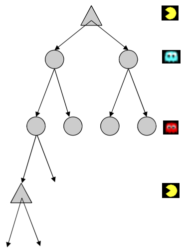


Later, we'll learn how to formalize the underlying problem as a

Markov Decision Process

Expectimax Search

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model can be a simple uniform distribution (roll a die)
  - Model can be sophisticated and require a great deal of computation
  - We have a node for every outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume for any state we magically have a distribution to assign probabilities to opponent actions / environment outcomes

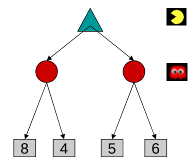


Expectimax Pseudocode

```
def value(s)
    if s is a max node return maxValue(s)
    if s is an exp node return expValue(s)
    if s is a terminal node return evaluation(s)

def maxValue(s)
    values = [value(s′) for s′ in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s′) for s′ in successors(s)]
    weights = [probability(s, s′) for s′ in successors(s)]
    return expectation(values, weights)
```



Which Algorithm?

Minimax: no point in trying

QuickTime™ and a GIF decompressor are needed to see this picture.

3 ply look ahead, ghosts move randomly

Which Algorithm?

Expectimax: wins some of the time

QuickTime™ and a GIF decompressor are needed to see this picture.

3 ply look ahead, ghosts move randomly

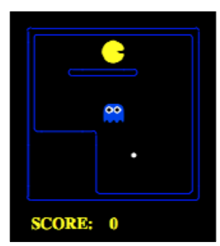
Expectimax for Pacman

- Notice that we′ve stopped thinking that the ghosts are trying to minimize pacman′s score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act
- Quiz: Can we see minimax as a special case of expectimax?
- Quiz: what would pacman′s computation look like if we assumed that the ghosts were doing 1-ply minimax and taking the result 80% of the time, otherwise moving randomly?

Expectimax for Pacman

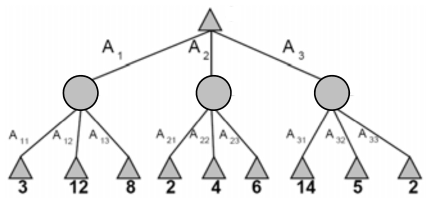
Results from playing 5 games

	Minimizing Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 493	Won 5/5 Avg. Score: 483
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503



Pacman does depth 4 search with an eval function that avoids trouble
Minimizing ghost does depth 2 search with an eval function that seeks Pacman

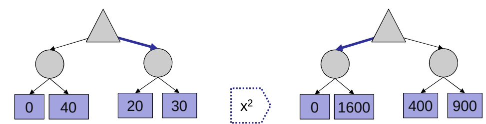
Expectimax Pruning?



- Not easy
  - exact: need bounds on possible values
  - approximate: sample high-probability branches

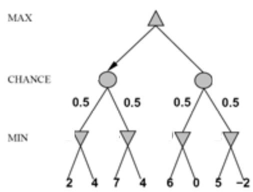
Expectimax Evaluation

- Evaluation functions quickly return an estimate for a node's true value (which value, expectimax or minimax?)
- For minimax, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For expectimax, we need *magnitudes* to be meaningful



Stochastic Two-Player

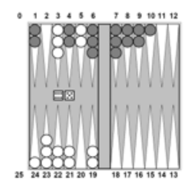
- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
  return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
  return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
  return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon H 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 = 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning → world-champion level play

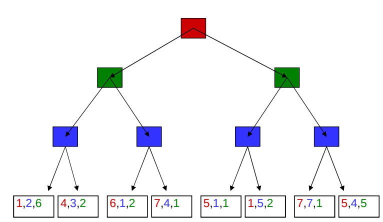


Multi-player Non-Zero-Sum Games

	B = silent (cooperates)	B = confesses (defects)
A = silent (cooperates)	A: 1 month B: 1 month	A: 10 years B: 0
A = confesses (defects)	A: 0 B: 3 years	A: 3 years B: 3 years

Multi-player Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children
  - Can give rise to cooperation and competition dynamically…



Types of Games

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imperfect information	stratego	bridge, poker, scrabble, nuclear war

Number of Players? 1, 2, …?