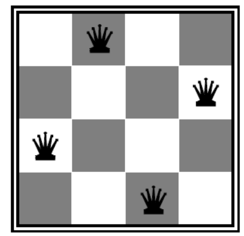
CSE 473: Artificial Intelligence

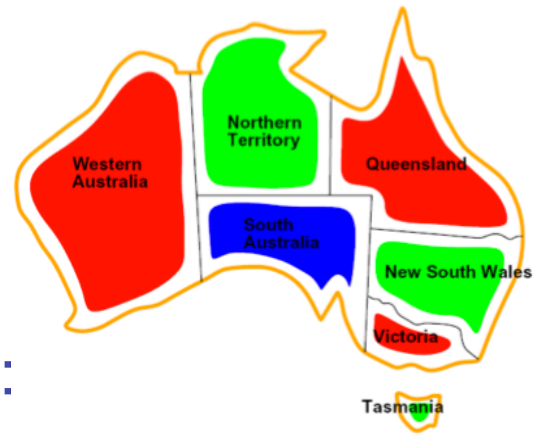
Constraint Satisfaction Examples

Cynthia Matuszek

Multiple slides adapted from Dan Klein, Stuart Russell, Andrew Moore, Paula Matuszek

Why do we care about CSPs?





- Standard search problems:
  - State is a "black box"
  - **Any function** can be goal, successor function can be anything
- Constraint satisfaction problems (CSPs):
  - Search problems that vary in the goal test.
  - State is defined by variables X_i with values from a domain D
  - Goal test is a set of constraints
- Why do we care?
  - Allows for **informed search**
  - Using structure of problems to search **wisely**

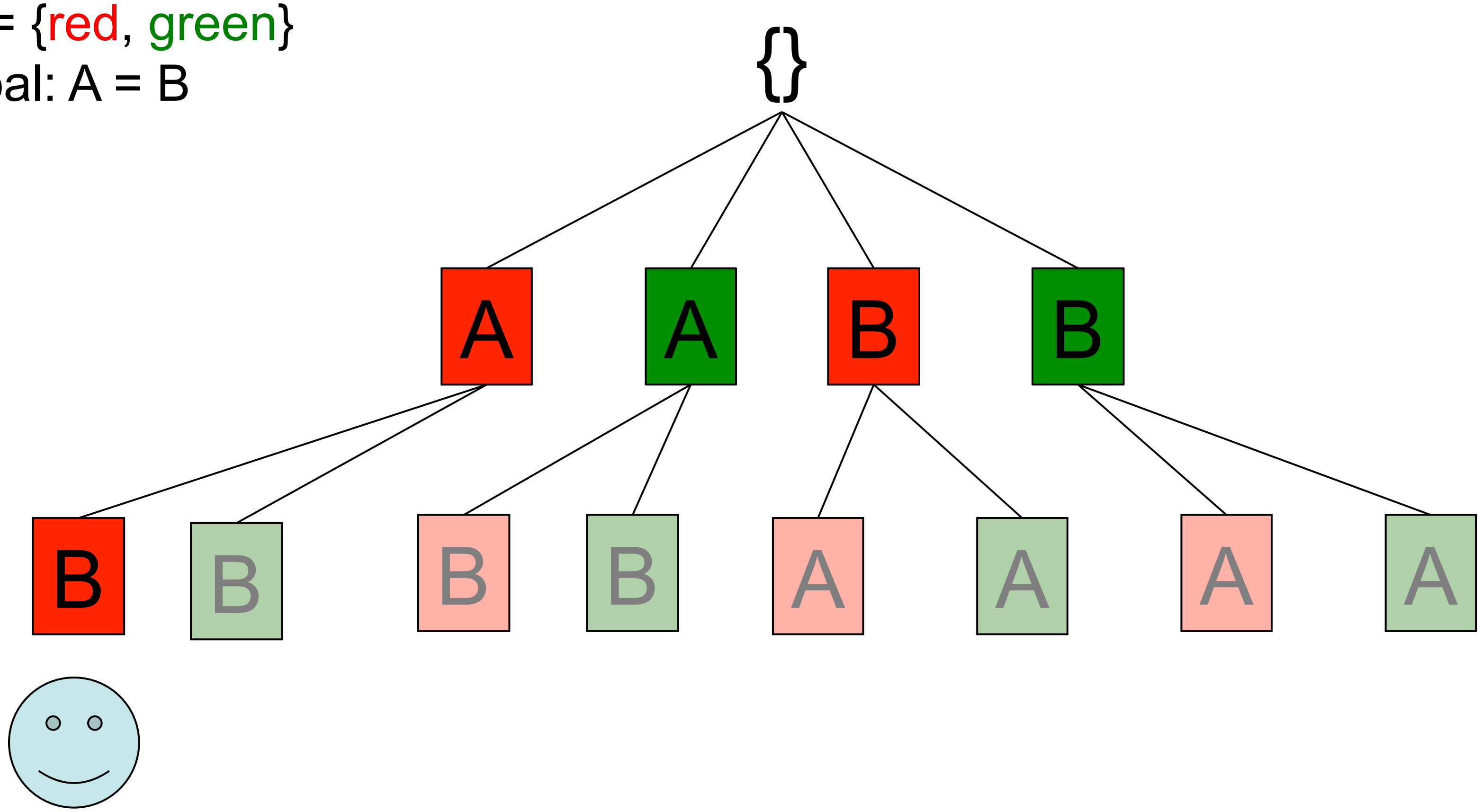
CSP heuristics & methods

Revisiting and Reviewing

- Uninformed Search for Constraint Satisfaction Problems
- Backtracking Search
- Forward Checking
- *k*-Consistency
- Ordering Heuristics
  - Minimum Remaining Values Ordering
  - Least Constraining Values
- Tree- and almost-tree CSPs

Bread-first search & CSPs

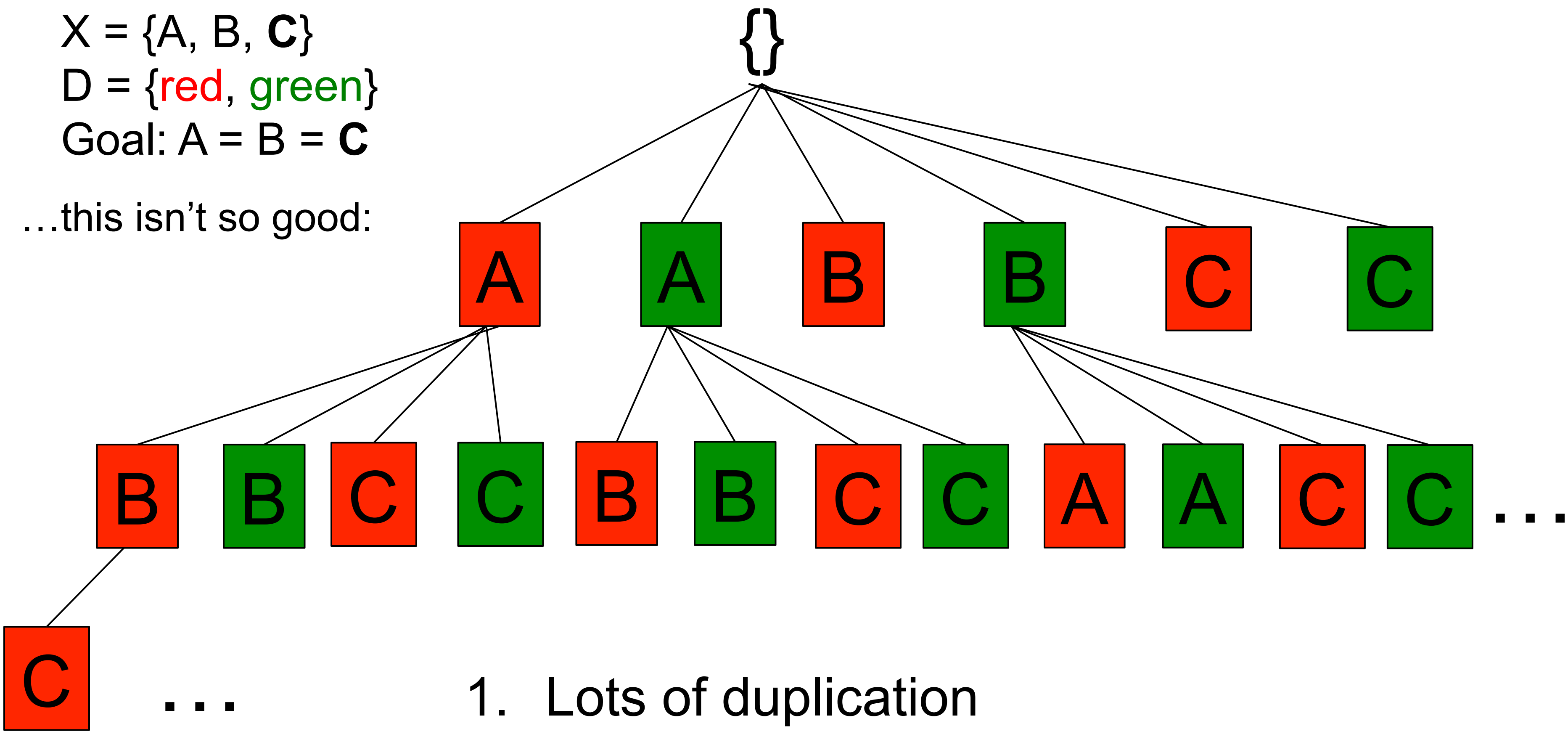
X = {A, B}
D = {red, green}
Goal: A = B



Bread-first search & CSPs

X = {A, B, **C**}
D = {red, green}
Goal: A = B = **C**

…this isn't so good:



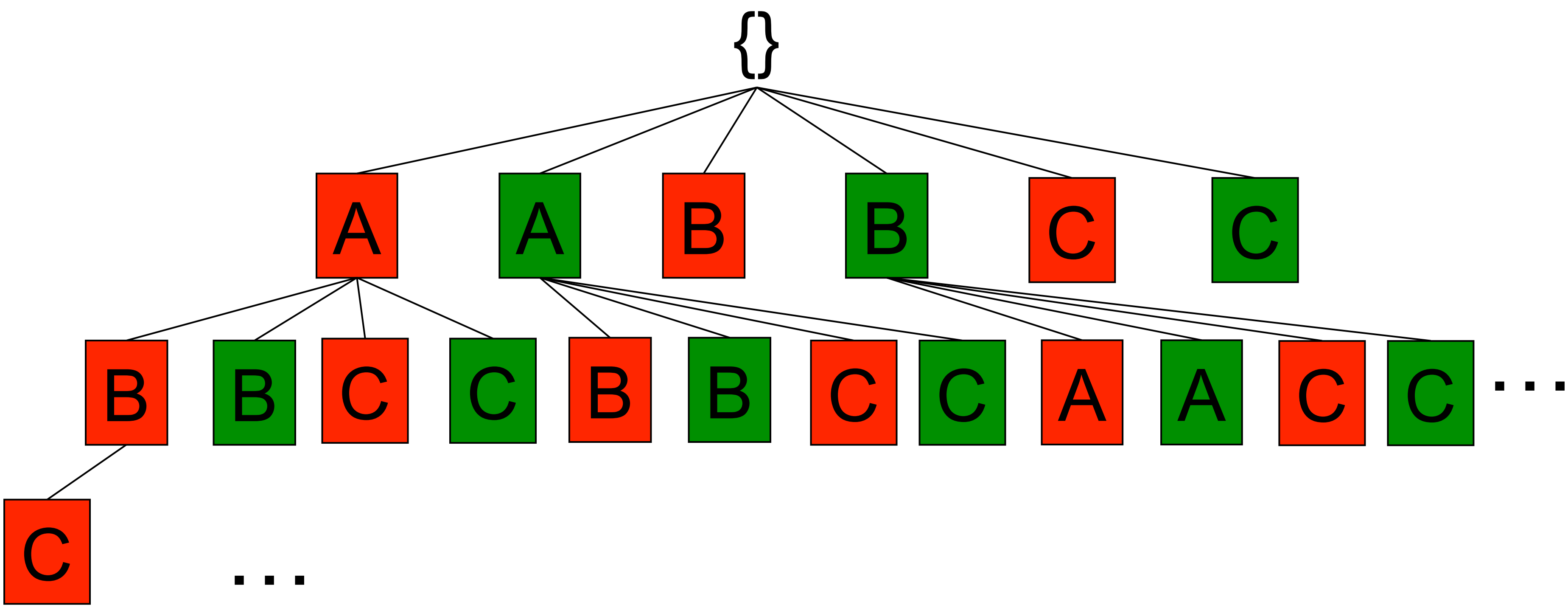
1. Lots of duplication
2. BFS always fills out the top of the search tree, when the solutions are at the bottom

Can We Do Better?

- It’s actually hard to understand why uninformed search does so badly. Why?
- Because you would never implement these problems that way.
  - Better successor functions, internal checks, …
- Hence, “uninformed”

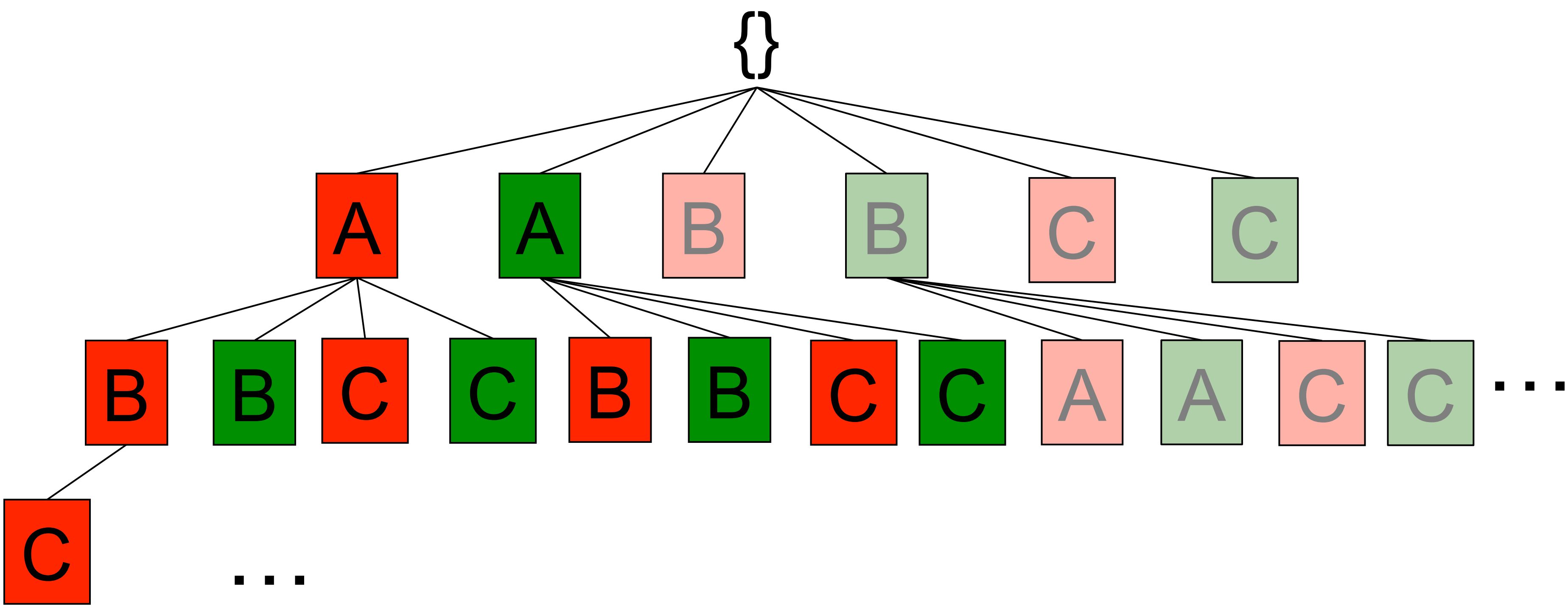
Improvement 1: Commutativity

- Idea 1: Only consider a single variable at each point
  - Variable assignments are commutative, so fix ordering
  - I.e., [A = red then B = green] same as [B = green then A = red]
  - Only need to consider assignments to a single variable at each step



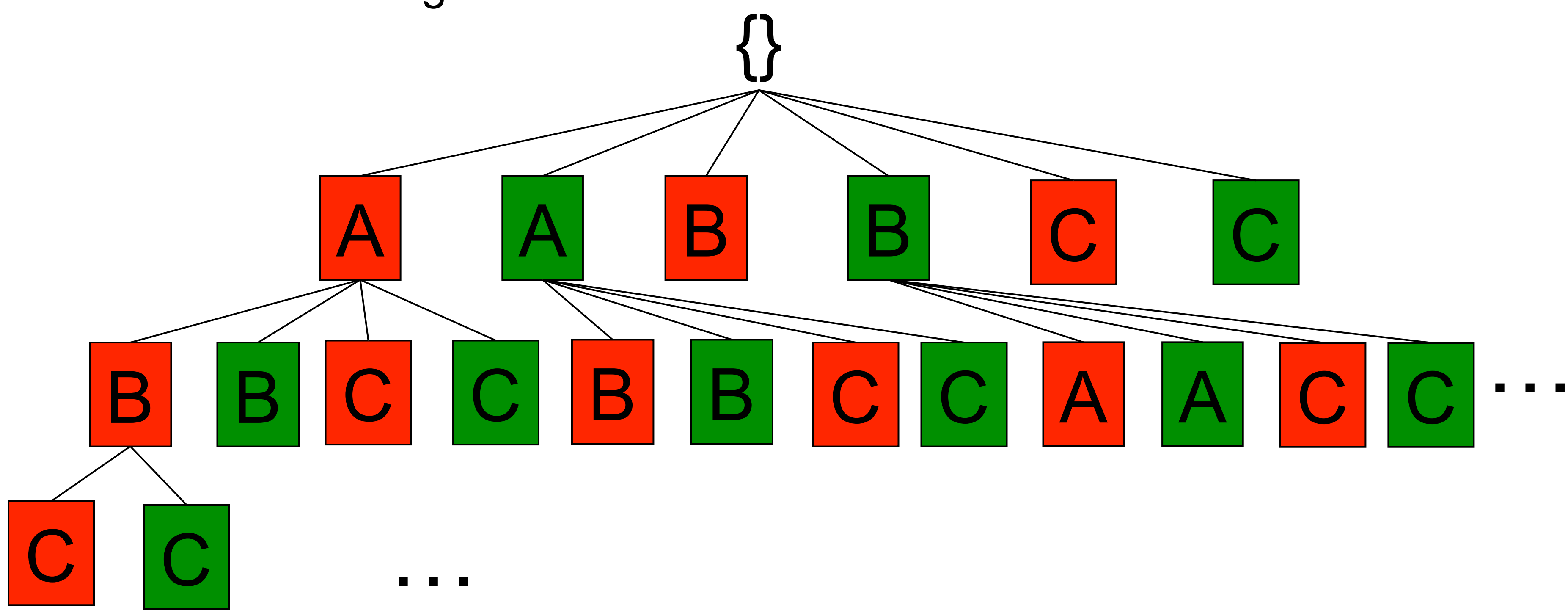
Improvement 1: Commutativity

- Idea 1: Only consider a single **variable** at each point
  - Variable assignments are commutative, so fix ordering
  - I.e., [A = red then B = green] same as [B = green then A = red]
  - Only need to consider assignments to a single variable at each step



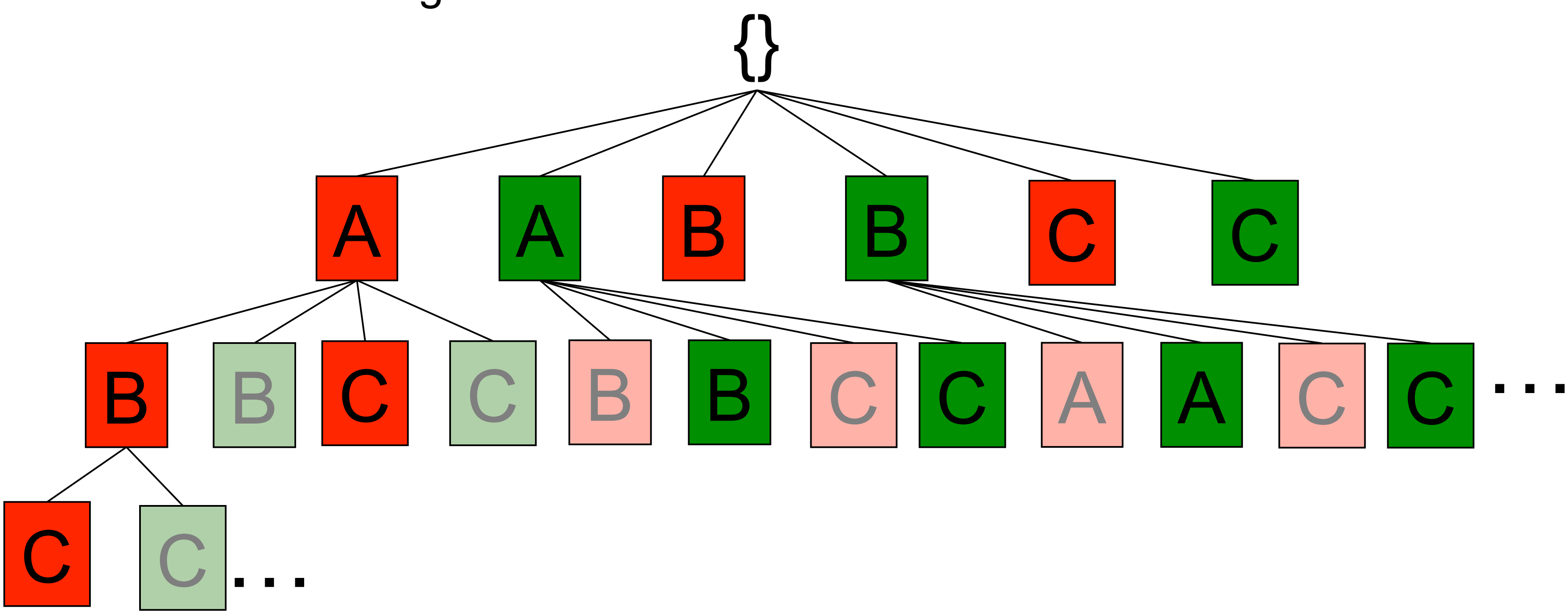
Improvement 2: Legal Assignments

- Idea 2: Only allow legal assignments at each point
  - Only assign values which don't ***eventually*** doom the search
  - Might have to do some extra computation
  - "Incremental goal test"



Improvement 2: Legal Assignments

- Idea 2: Only allow legal assignments at each point
  - Only assign values which do not conflict with existing assignments
  - Might have to do some extra computation
  - “Incremental goal test”



Idea 1 + Idea 2 = Backtracking

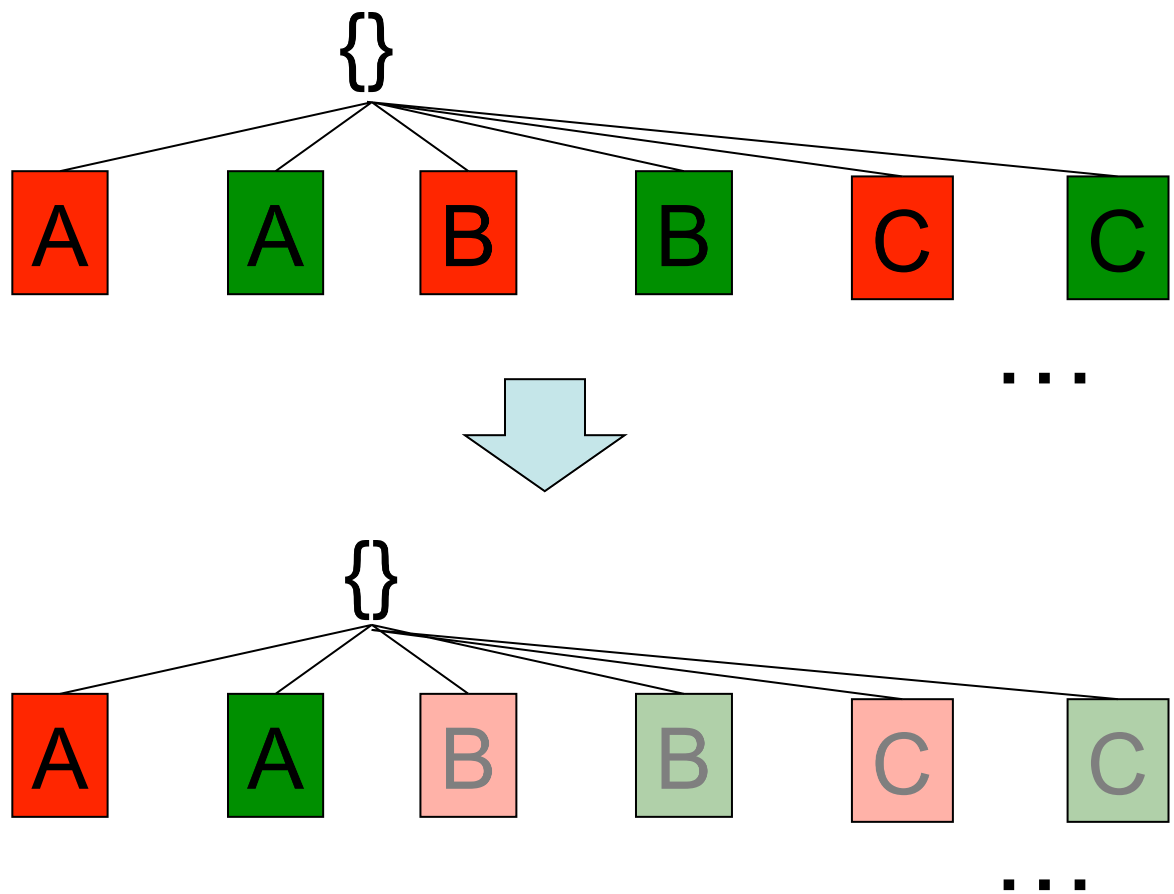
- Depth-first search for CSPs with these fixes is ***backtracking search***
  - Backtrack when there's no legal assignment for the next variable
- Backtracking search is the basic uninformed algorithm for CSPs

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)
function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
       if value is consistent with assignment given CONSTRAINTS[csp] then
           add {var = value} to assignment
           result ← RECURSIVE-BACKTRACKING(assignment, csp)
           if result ≠ failure then return result
           remove {var = value} from assignment
   return failure
```

Backtracking

- Idea 1

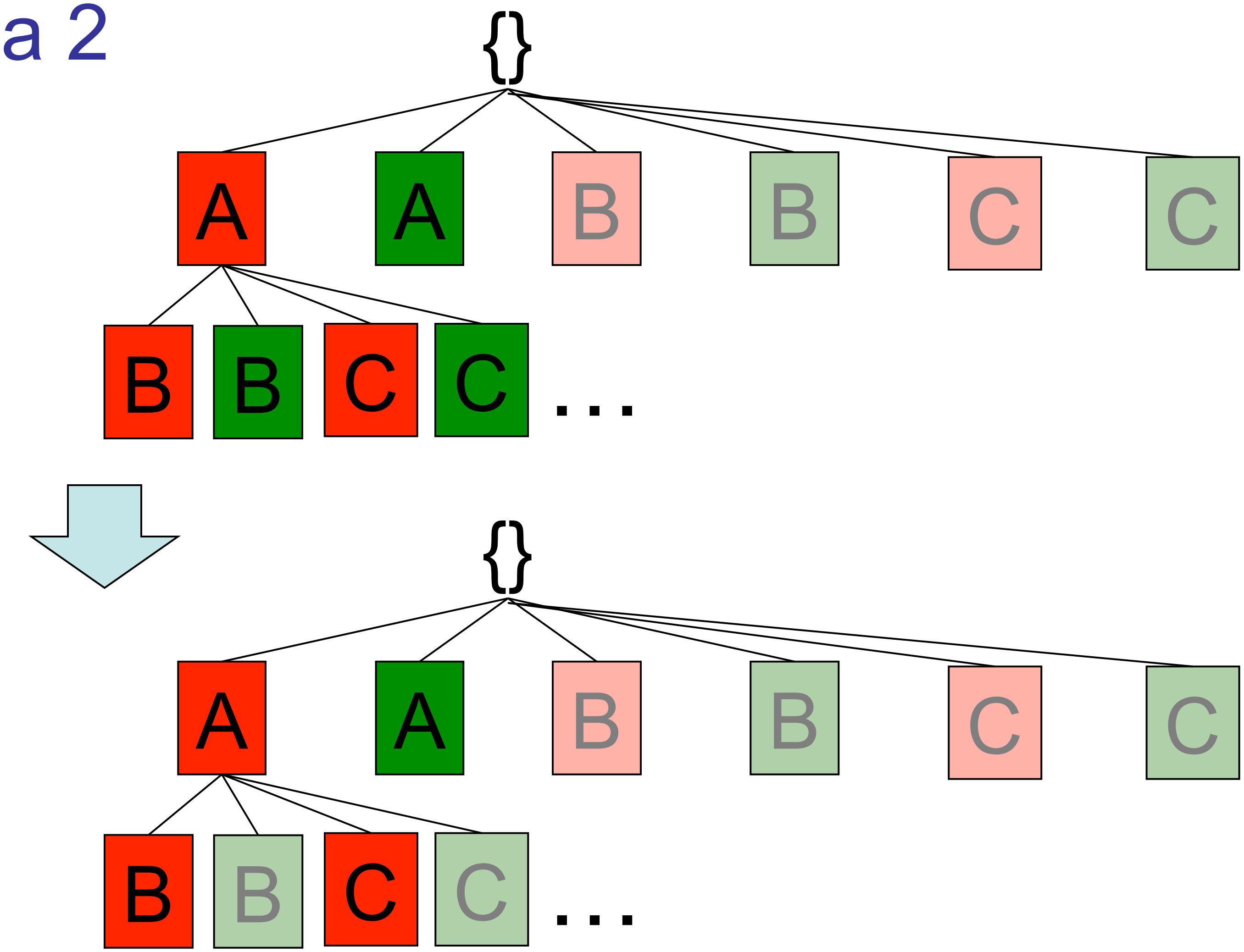
X = {A, B, C}
D = {red, green}
Goal: A = B = C



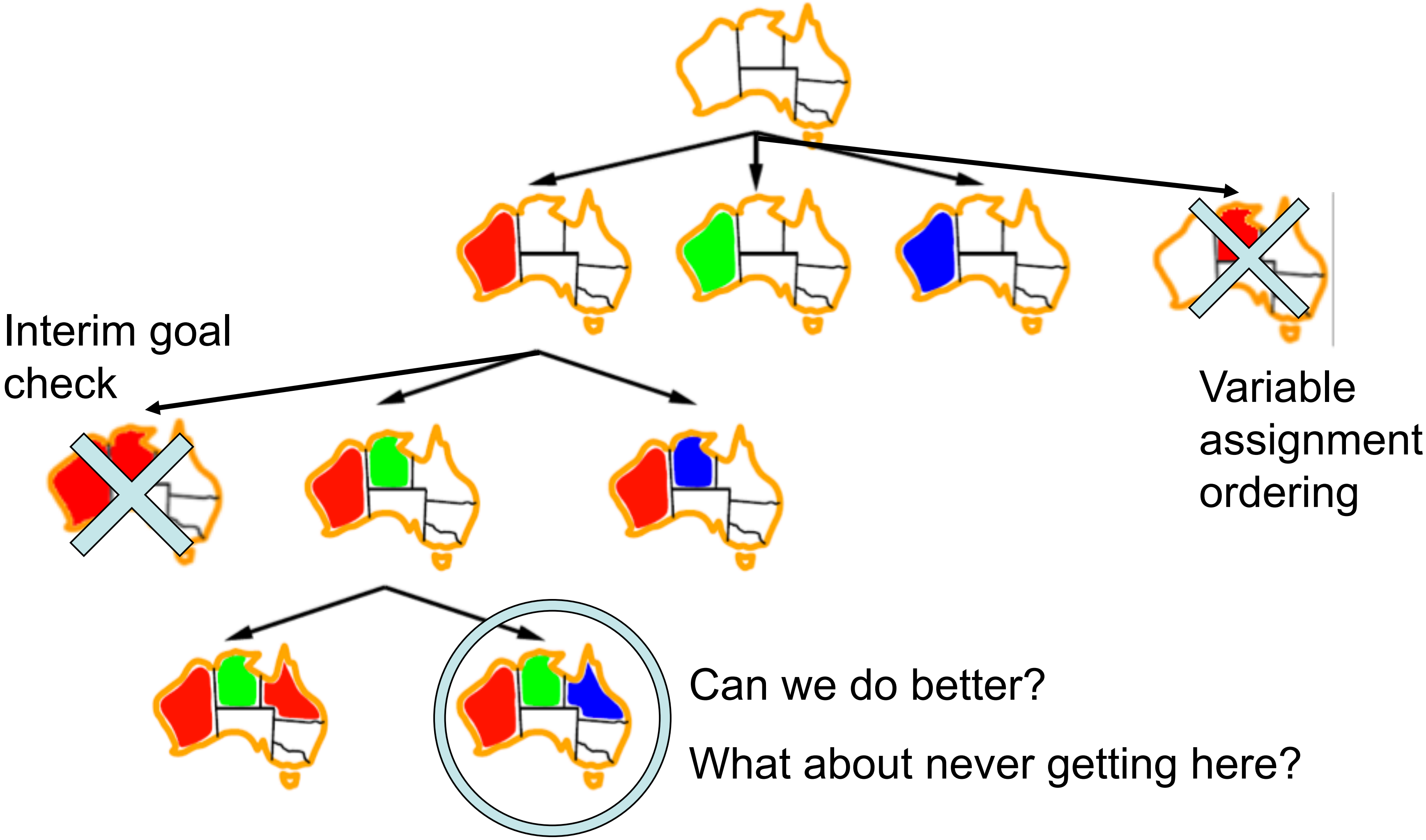
Backtracking

- Plus Idea 2

X = {A, B, C}
D = {red, green}
Goal: A = B = C



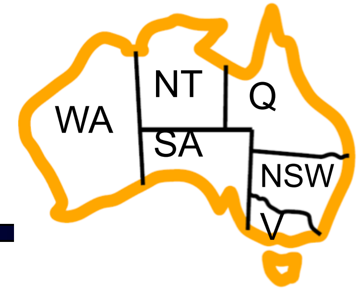
Backtracking Example



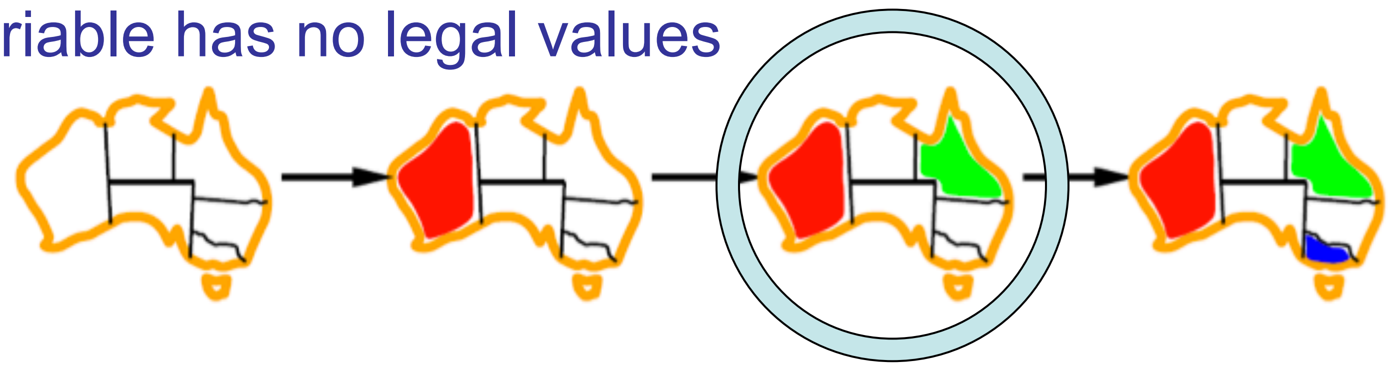
Forward Checking

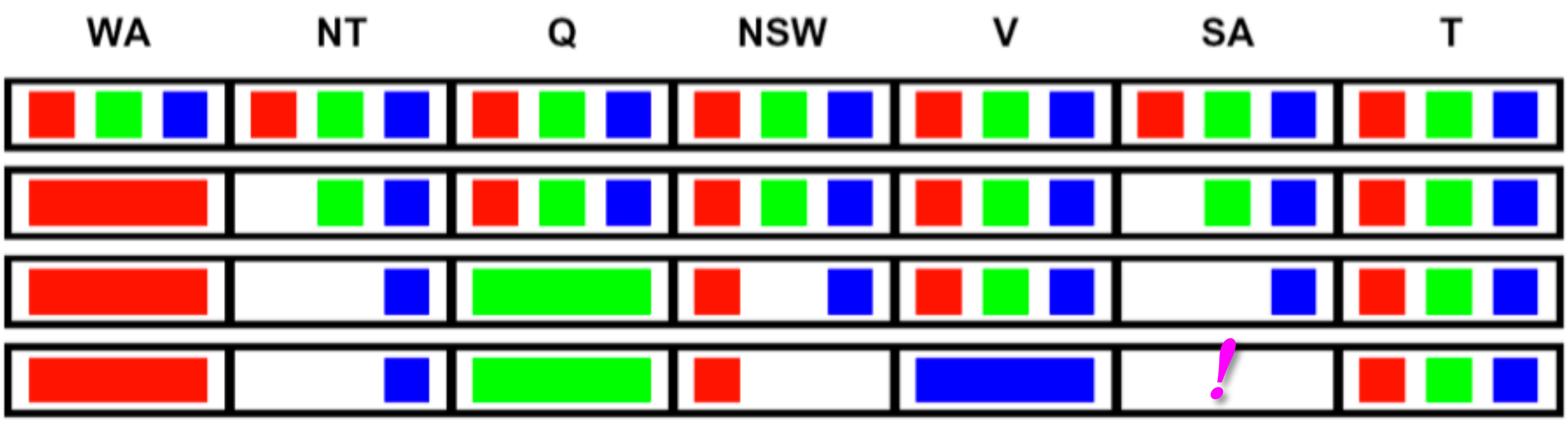
- Idea: Keep track of remaining legal values for unassigned variables (using immediate constraints)
- Terminate/prune when any ***as-yet-unassigned*** variable has no legal values

Forward Checking



- Idea: Keep track of remaining legal values for unassigned variables (using immediate constraints)
- Terminate/prune when any ***as-yet-unassigned*** variable has no legal values

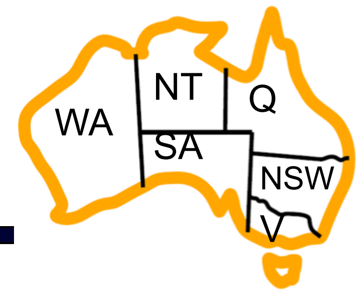




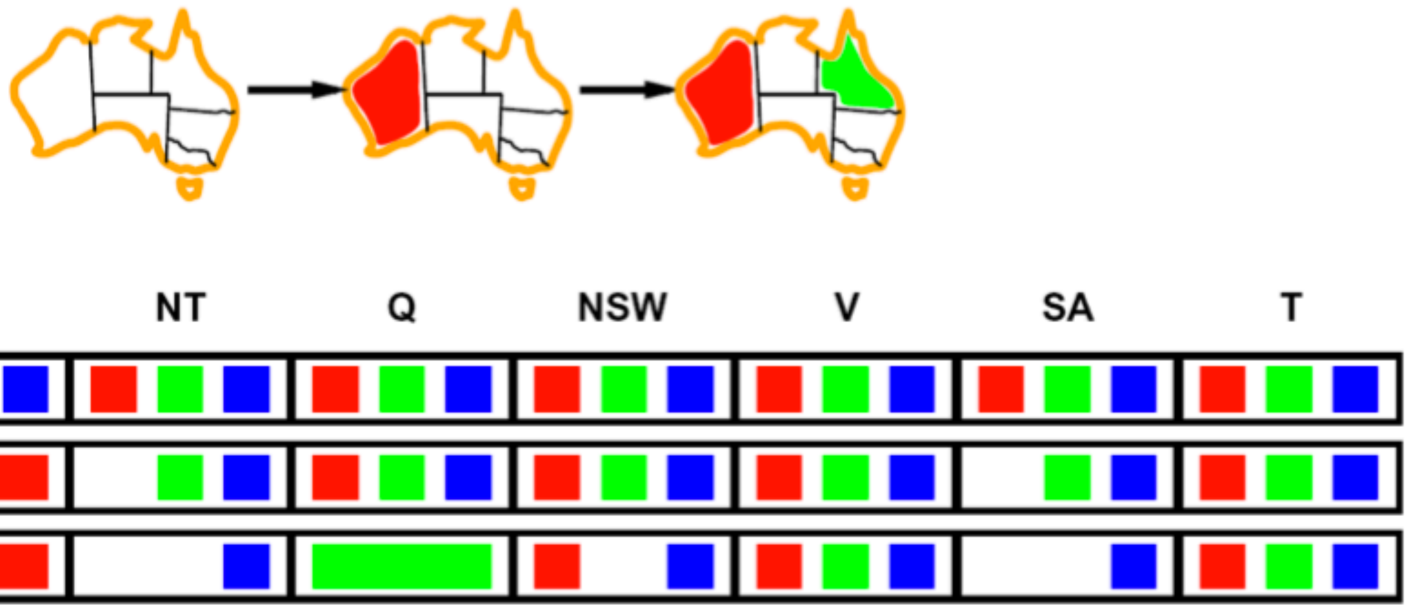
Improving Forward Checking

- Why does forward checking allow this?
- Forward checking *propagates* information from assigned to adjacent unassigned variables, but doesn't detect more distant failures

Constraint Propagation

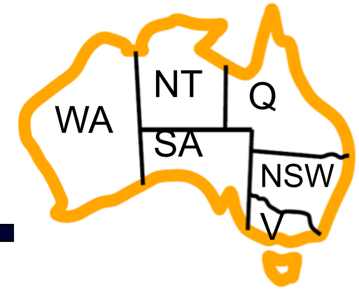


- Why does forward checking allow this?

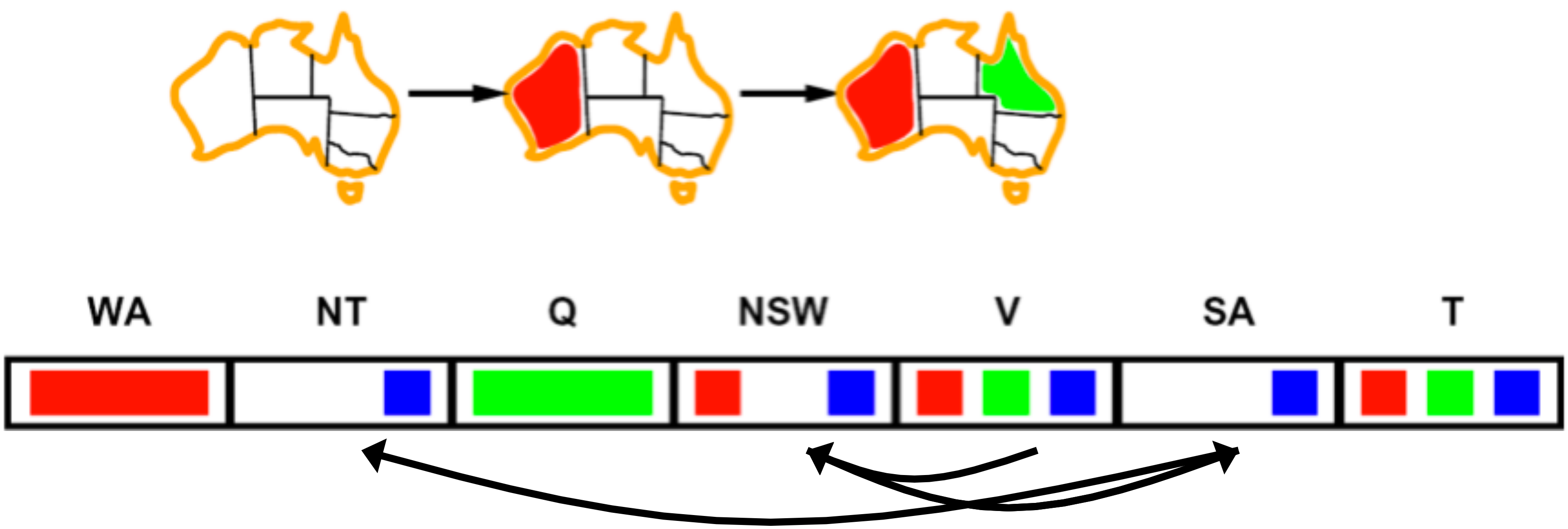


- Neither SA nor NT have *no* possible assignments.
- How do we fix it?

Arc Consistency



- Simplest form of propagation makes each arc consistent
- Every pair of variables that affect each other share an arc
  - X → Y is consistent iff for every value x there is some allowed
- If X loses a value, neighbors of X need to be rechecked!

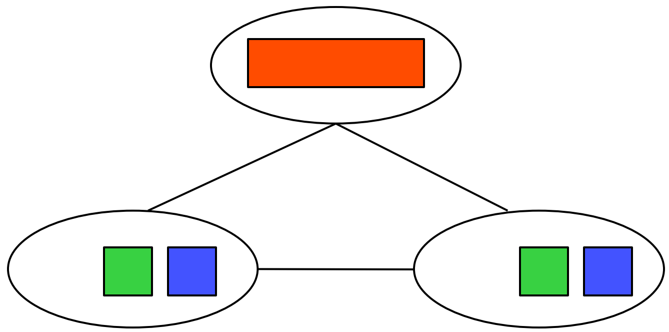


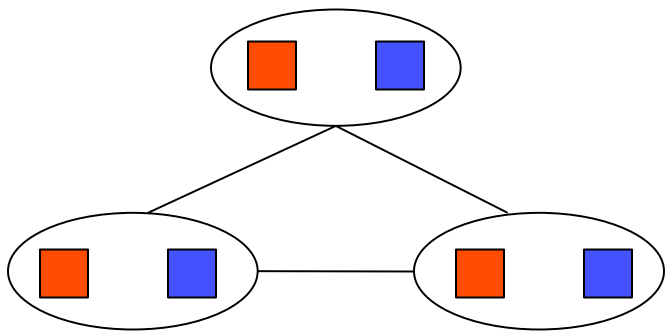
Revisiting and Reviewing

- Uninformed Search for Constraint Satisfaction Problems
- Backtracking Search
- Forward Checking
- *k*-Consistency
- Ordering Heuristics
  - Minimum Remaining Values
  - Least Constraining Values
- Tree- and almost-tree CSPs

Limitations of Arc Consistency

- After running arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)
- How can we fix it?

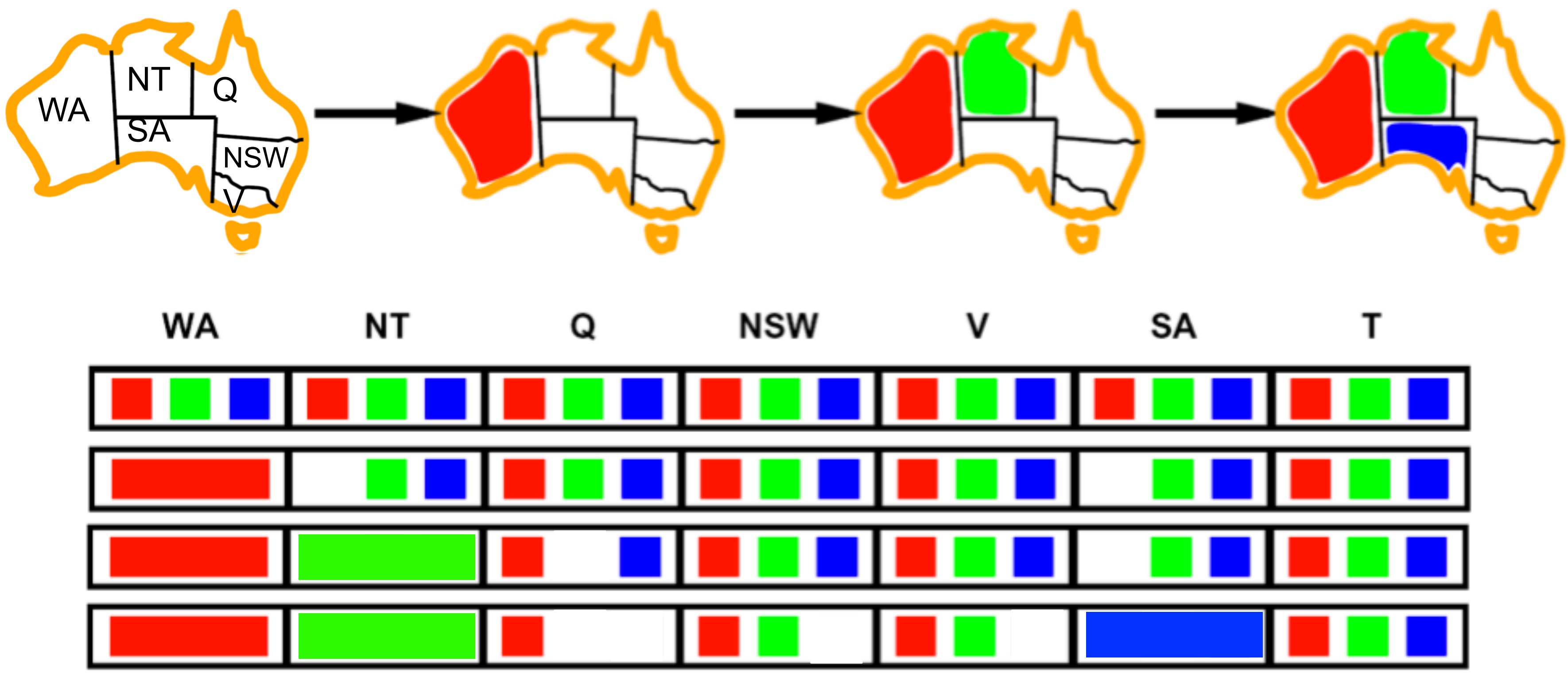




What went wrong here?

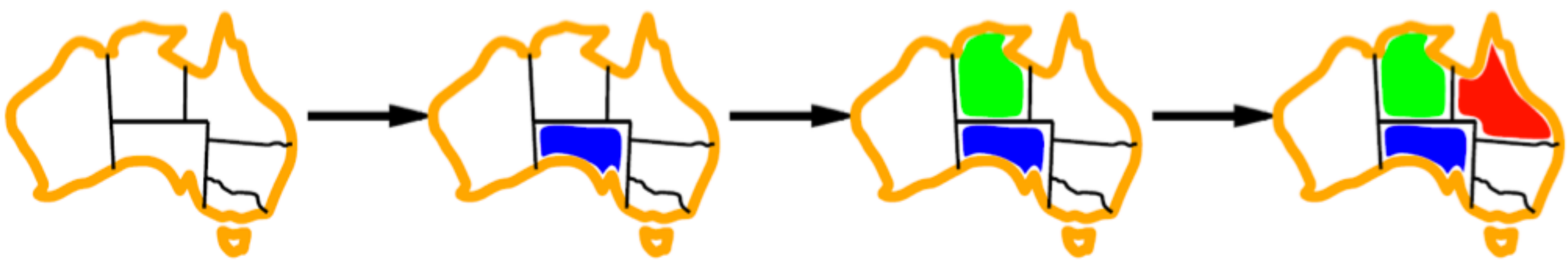
Variable Choice: Minimum Remaining Values

- Minimum remaining values (MRV):
  - Choose the variable with the fewest legal values



Ordering: Degree Heuristic

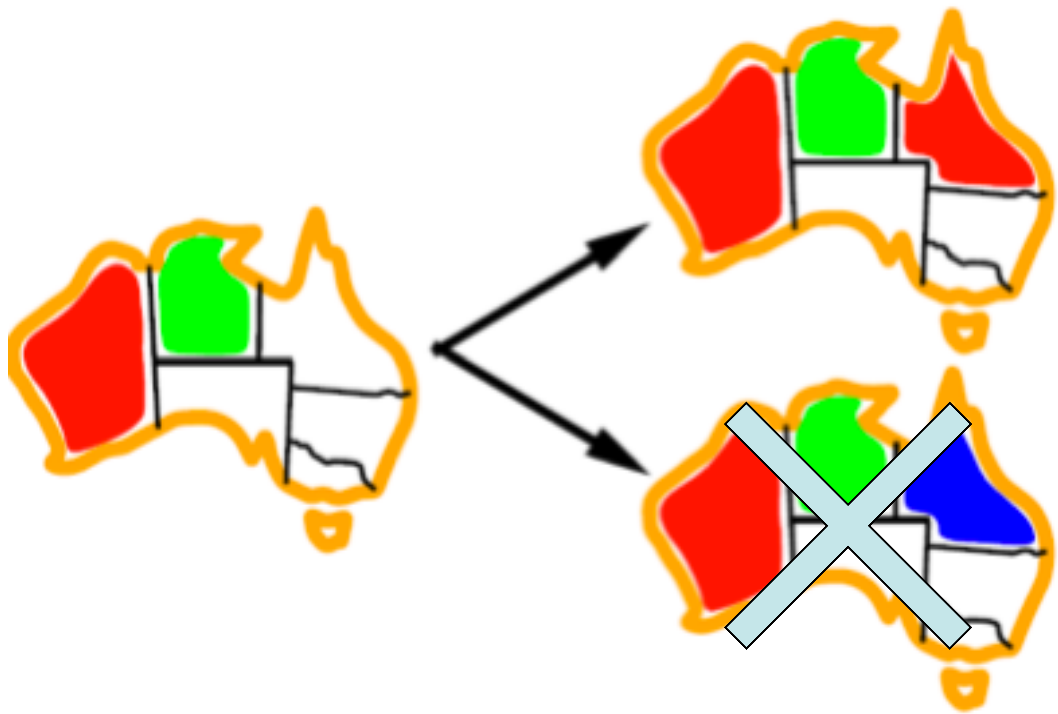
- Tie-breaker among MRV variables
- Degree heuristic:
  - Choose the variable participating in the most constraints on remaining variables (has the most arcs)



- Why most rather than fewest constraints?

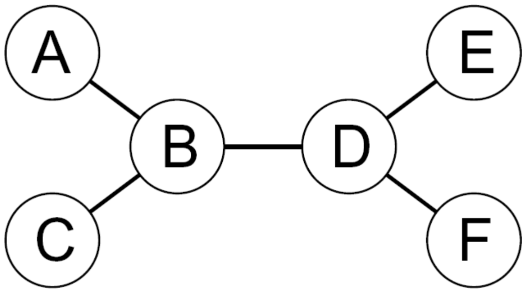
Ordering: Least Constraining Value

- Given a choice of variable:
  - Choose the one that rules out the fewest values in the remaining variables
    - Why?
  - Computationally expensive (sometimes)

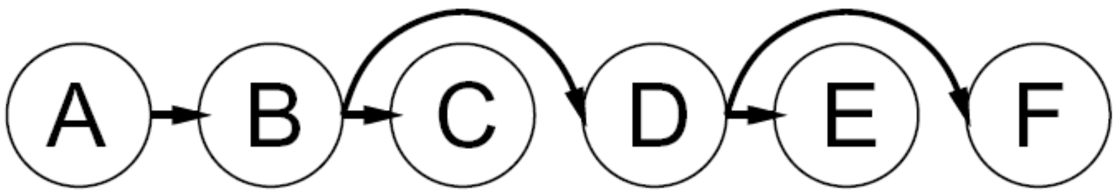


Tree-Structured CSPs

- Choose a variable as root, order variables from root to leaves such that every node's parent precedes it



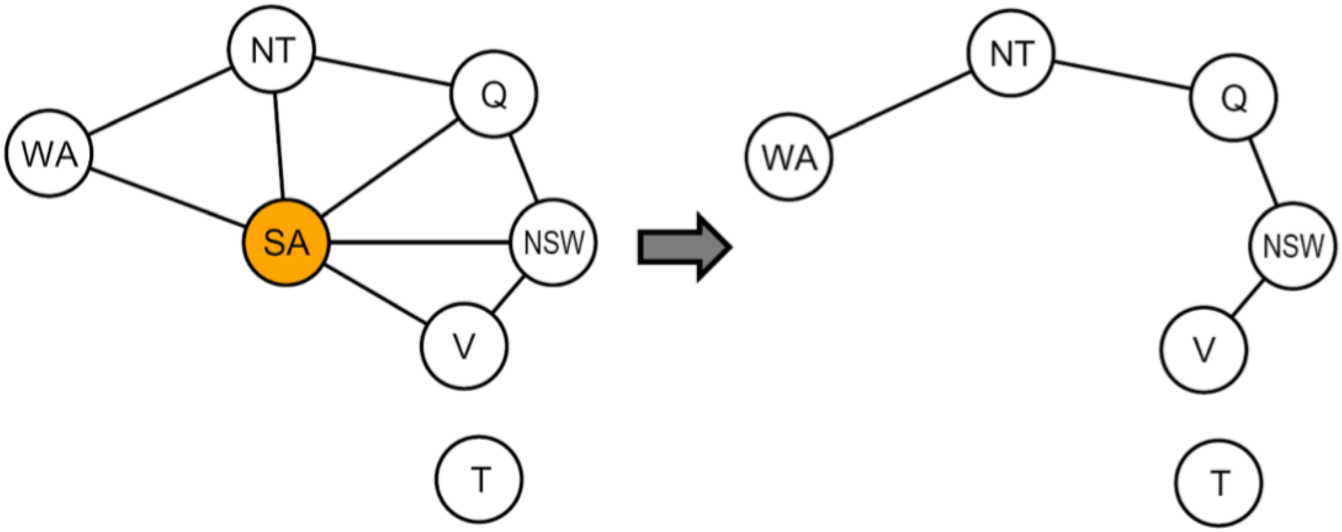
For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
For i = 1 : n, assign X_i consistently with Parent(X_i)



- Runtime: O(n d^2)

- Takeaway: tree-structured CSPs can be solved very efficiently

Nearly Tree-Structured CSPs



- Cutset conditioning:
  - Choose variable to instantiate that makes everything *left* into a tree
- Instantiate a variable every possible way
  - Here, you now have 3 tree-search problems
- Takeaway: you can turn some CSPs into trees (which can still be solved very efficiently)