CSE 473

Lecture 12

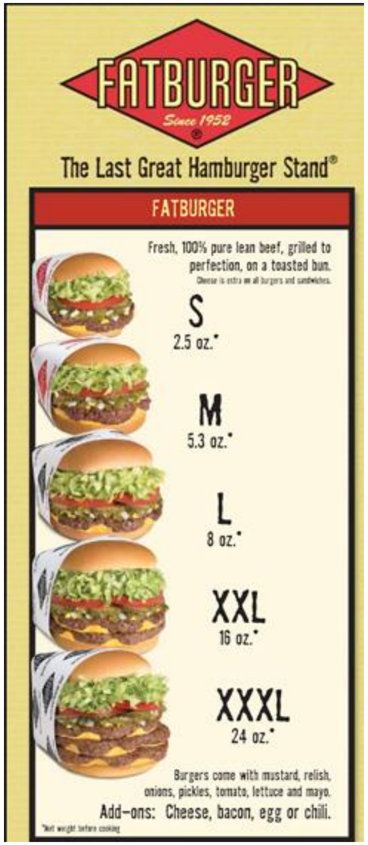
Chapter 8

First-Order Logic

What's on our menu today?

First-Order Logic

- Definitions
- Universal and Existential Quantifiers
- Skolemization
- Unification



Propositional vs. First-Order

Propositional logic: Deals with facts and propositions (can be true or false):

$P_{1,1}$ "there is a pit in (1,1)"

George_Monkey "George is a monkey"

George_Curious "George is curious"

473student1_Monkey

(George_Monkey $\wedge$ $\neg$473student1_Monkey) $\vee$ …

Propositional vs. First-Order

First-order logic: Deals with objects and relations

Objects: George, 473Student1, Miley, Raj, …

Relations: Monkey(George), Curious(George),
CanTwerk(Miley), WillNotTwerk(Raj)
Smarter(473Student1, Monkey2)
Smarter(Monkey2, Raj)
Stooges(Larry, Moe, Curly)
PokesInTheEyes(Moe, Curly)
PokesInTheEyes(473Student1, Raj)

FOL Definitions

Constants: Name a specific object.

George, Monkey2, Larry, ...

Variables: Refer to an object without naming it.

X, Y, ...

Relations (predicates): Properties of or relationships between objects.

Curious, CanTwerk, PokesInTheEyes, ...

FOL Definitions

Functions: Mapping from objects to objects.

banana-of, grade-of, bad-song-of

Terms: Logical expressions referring to objects

banana-of(George)

grade-of(stdnt1)

bad-song-of(JayZ)

bad-song-of(Raj)

More Definitions

Logical connectives: and, or, not, $\Rightarrow$, $\Leftrightarrow$

Quantifiers:

- $\forall$ For all (Universal quantifier)
- $\exists$ There exists (Existential quantifier)

Examples

- George is a monkey and he is curious
  Monkey(George) $\wedge$ Curious(George)
- All monkeys are curious
  $\forall$x: Monkey(x) $\Rightarrow$ Curious(x)
- There is a curious monkey
  $\exists$x: Monkey(x) $\wedge$ Curious(x)

Quantifier / Connective Interaction

M(x) == "x is a monkey"
C(x) == "x is curious"

$\forall x$: $M(x) \land C(x)$

"Everything is a curious monkey"

$\forall x$: $M(x) \Rightarrow C(x)$

"All monkeys are curious"

$\exists x$: $M(x) \land C(x)$

"There exists a curious monkey"

$\exists x$: $M(x) \Rightarrow C(x)$

"There exists an object that is *either* a curious monkey, *or* not a monkey at all"

Nested Quantifiers: Order matters!

$$\forall x \exists y\ P(x,y) \neq \exists y \forall x P(x,y)$$

Example

Every monkey has a tail

$$\forall m \exists t\ has(m,t)$$

Every monkey ***shares*** a tail!

$$\exists t \forall m\ has(m,t)$$

Try:

Everybody loves somebody *vs.* Someone is loved by everyone

$$\forall x \exists y\ loves(x, y) \qquad \exists y \forall x\ loves(x, y)$$

Semantics

Semantics = **what the arrangement of symbols *means* in the world**

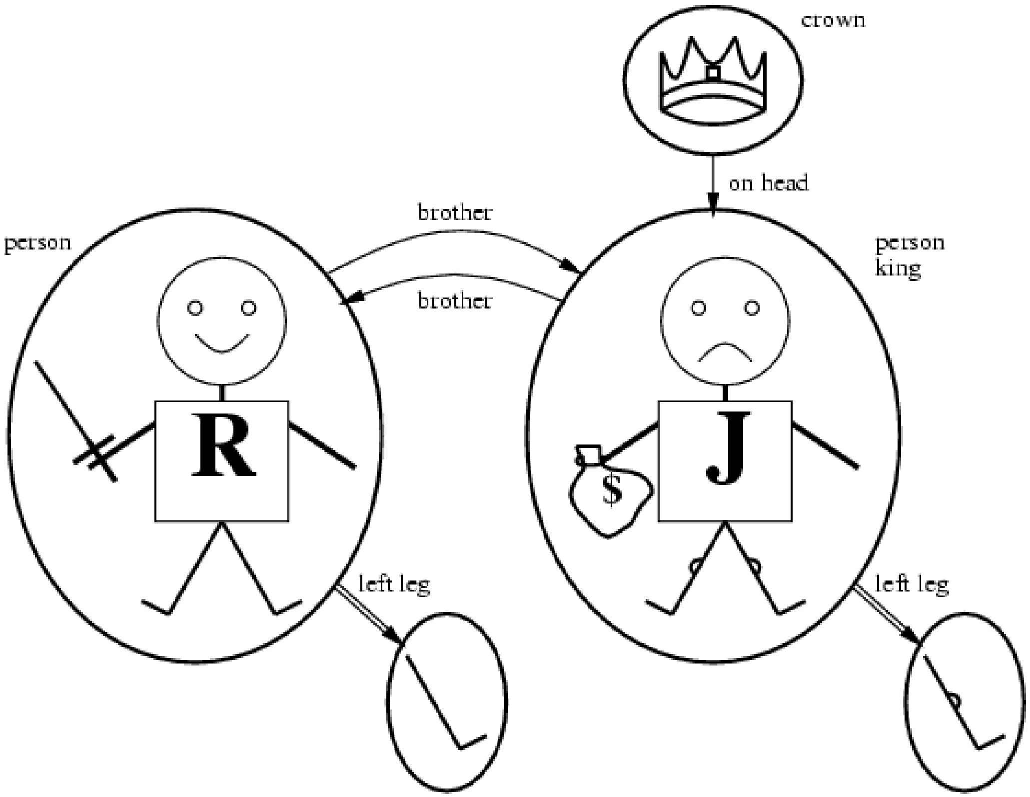
Propositional logic

- **Basic elements are *propositional variables*** e.g., $P_{1,1}$
  (refer to facts about the world)
- **Possible worlds: mappings from variables to T/F**

First-order logic

- **Basic elements are *terms*, e.g.,** George, banana-of(George), bad-song-of(dad-of(Miley))
  (logical expressions that refer to objects)
- Interpretations: **mappings from terms to real-world elements**

Example: A World of Kings and Legs



Syntactic elements:

Constants:	Functions:	Relations:
Richard John	LeftLeg(p)	On(x,y) King(p)

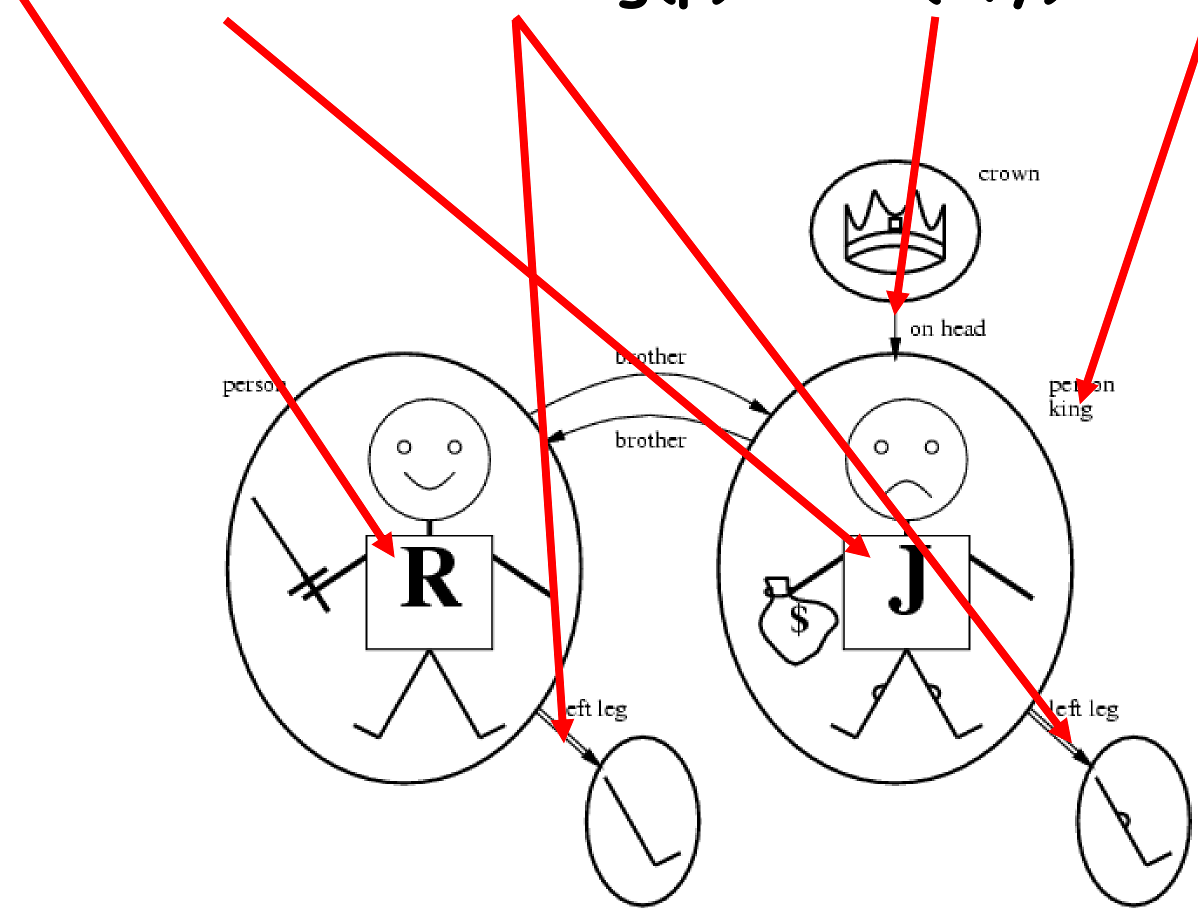
Interpretation I

Interpretations map syntactic tokens to model elements

Constants: Richard John

Functions: LeftLeg(p)

Relations: On(x,y) King(p)

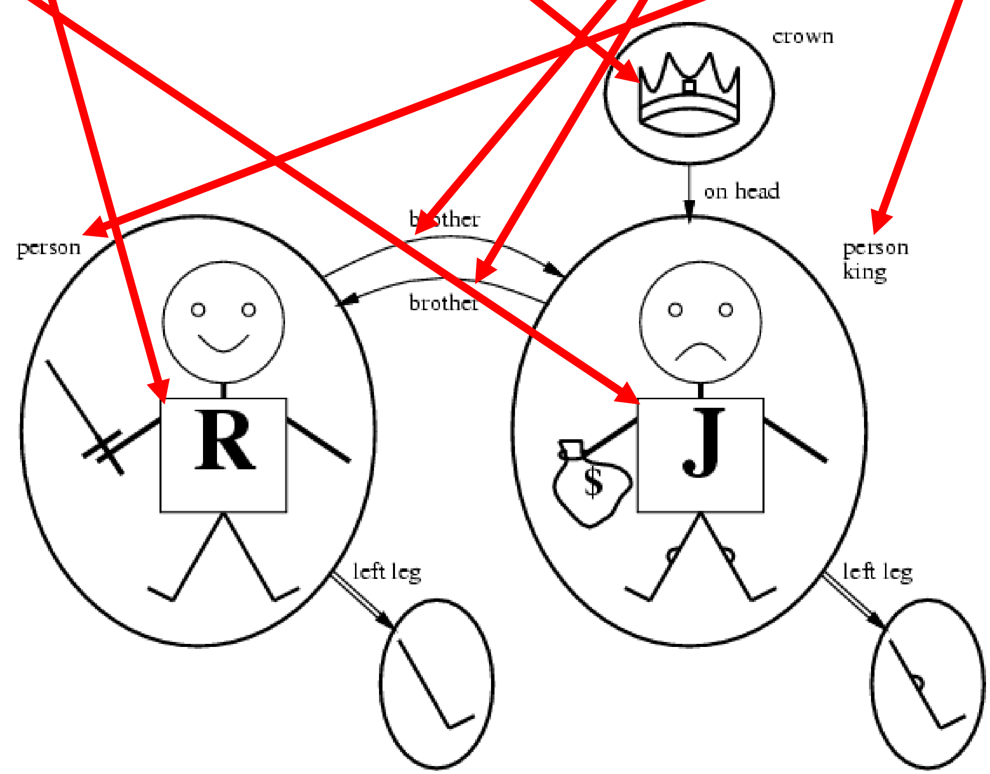


Interpretation II

Constants: Richard John

Functions: LeftLeg(p)

Relations: On(x,y) King(p)



How Many Interpretations?

Two constants (and 5 objects in world): # possible mappings?

- Richard, John (objects: R, J, crown, RL, JL)

  $5^2 = 25$ object mappings

One unary relation

King(x)

Infinite number of values for x → infinite mappings

If we restricted x to R, J, crown, RL, JL:

$2^5 = 32$ unary truth mappings

Two binary relations

Leg(x, y); On(x, y)

Infinite. If we restrict x, y to five objects each?

Still yields 2^{25} mappings *for each* binary relation

Satisfiability, Validity, & Entailment

S is valid if it is true in all interpretations

S is satisfiable if it is true in some interp

S is unsatisfiable if it is false in all interps

S1 $\models$ S2 (S1 entails S2) if

for all interps where S1 is true,

S2 is also true

Propositional. Logic *vs.* First Order

Ontology	Facts (P, Q,…)	Objects, Properties, Relations
Syntax	Atomic sentences Connectives	Variables & quantification Sentences have structure: terms father-of(mother-of(X)))
Semantics	Truth Tables	Interpretations (Much more complicated)
Inference Algorithm	WalkSAT, DPLL Fast in practice	Unification Forward, Backward chaining Prolog, theorem proving
Complexity	NP-Complete	Semi-decidable May run forever if $KB \not\models \alpha$

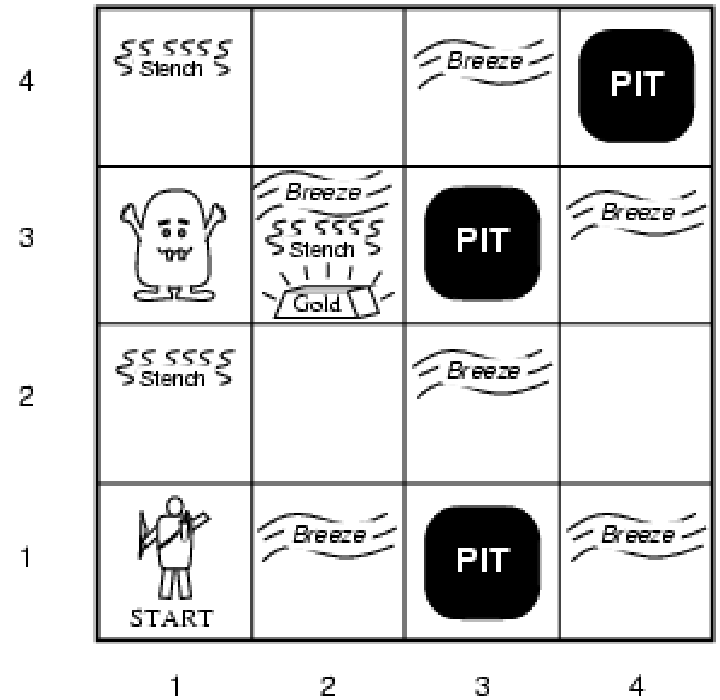
First-Order Wumpus World

Objects

- Squares, wumpuses, agents,
- gold, pits, stinkiness, breezes

Relations

- Square topology (adjacency),
- Pits/breezes,
- Wumpus/stinkiness



Wumpus World: Squares

- Each square as an object:

  $Square_{1,1}$, $Square_{1,2}$, …,
  $Square_{3,4}$, $Square_{4,4}$

- Square topology relations?

  Adjacent($Square_{1,1}$, $Square_{2,1}$)
  …
  Adjacent($Square_{3,4}$, $Square_{4,4}$)

Better: **Squares as lists:**

[1, 1], [1,2], …, [4, 4]

Square topology relations:

∀x, y, a, b: Adjacent([x, y], [a, b]) ⇔
[a, b] ∈ {[x+1, y], [x-1, y], [x, y+1], [x, y-1]}

Wumpus World: Pits

- Each pit as an object:

  $Pit_{1,1}$, $Pit_{1,2}$, …,
  $Pit_{3,4}$, $Pit_{4,4}$

- Problem?

  Not all squares have pits

List only the pits we have?

$Pit_{3,1}$, $Pit_{3,3}$, $Pit_{4,4}$

Problem?

No reason to distinguish pits (same properties)

Better: **pit as unary predicate**

Pit(x)

Pit([3,1]), Pit([3,3]), Pit([4,4]) will be true

Wumpus World: Breezes

- Represent breezes like pits, as unary predicates:
  Breezy(x)

"Squares next to pits are breezy":

$\forall$a, b, c, d:

Pit([a, b]) $\wedge$ Adjacent([a, b], [c, d]) $\Rightarrow$ Breezy([c, d])

Wumpus World: Wumpuses

- Wumpus as object:
  Wumpus

- Wumpus home as unary predicate:
  WumpusIn(x)

Better: **Wumpus's home as a function:**

Home(Wumpus) references the wumpus's home square.

FOL Reasoning: Outline

Basics of FOL reasoning

Classes of FOL reasoning methods

- Forward & Backward Chaining
- Resolution
- Compilation to SAT

Basics: Universal Instantiation

Universally quantified sentence:

- $\forall x$: Monkey(x) $\Rightarrow$ Curious(x)

Intutively, x can be anything:

- Monkey(George) $\Rightarrow$ Curious(George)
- Monkey(473Student1) $\Rightarrow$ Curious(473Student1)
- Monkey(DJof(Miley)) $\Rightarrow$ Curious(DJof(Miley))

Formally:

$$\frac{\forall x \quad S}{\text{Subst}(\{x/p\}, S)}$$

x is replaced with p in S, and the quantifier removed

Example:

$$\frac{\forall x \quad \text{Monkey}(x) \rightarrow \text{Curious}(x)}{\text{Monkey}(\text{George}) \rightarrow \text{Curious}(\text{George})}$$

x is replaced with George in S, and the quantifier removed

Basics: Existential Instantiation

Existentially quantified sentence:

$\exists x$: **Monkey(x) ∧ ¬Curious(x)**

Intutively, x must name something. But what?

Can we conclude:

Monkey(George) ∧ ¬Curious(George) ???

No! Sentence might not be true for George!

Use a *Skolem Constant* and draw the conclusion:

Monkey(K) ∧ ¬Curious(K)

where K is a completely new symbol you created **(stands for the monkey for which the statement is true)**

Formally:

$$\frac{\exists x \;\; S}{\text{Subst}(\{x/K\}, S)}$$

K is called a Skolem constant

Basics: Generalized Skolemization

What if our existential variable is nested?

$\forall x\ \exists y$: Monkey(x) $\Rightarrow$ HasTail(x, y)

Can we conclude:

$\forall x$: Monkey(x) $\Rightarrow$ HasTail(x, K_Tail) ???

Nested existential variables can be replaced by **Skolem functions** that you create

- Args to function are all surrounding $\forall$ vars

$\forall x$: Monkey(x) $\Rightarrow$ HasTail(x, f(x))

"tail-of" function

Next Time

- Reasoning with FOL
  - Chaining
  - Resolution
  - Compilation to SAT

- To Do:
  - Project #2
  - Read Chapters 8-9