CSE 473

Lecture 11

Chapter 7

Inference in Propositional Logic

Logic: another thing that penguins aren't very good at.

Recall: Propositional Logic Terminology

Literal

= proposition symbol or its negation

E.g., A, $\neg A$, B, $\neg B$, etc. (positive vs. negative)

Clause

= disjunction of literals

E.g., $(B \vee \neg C \vee \neg D)$

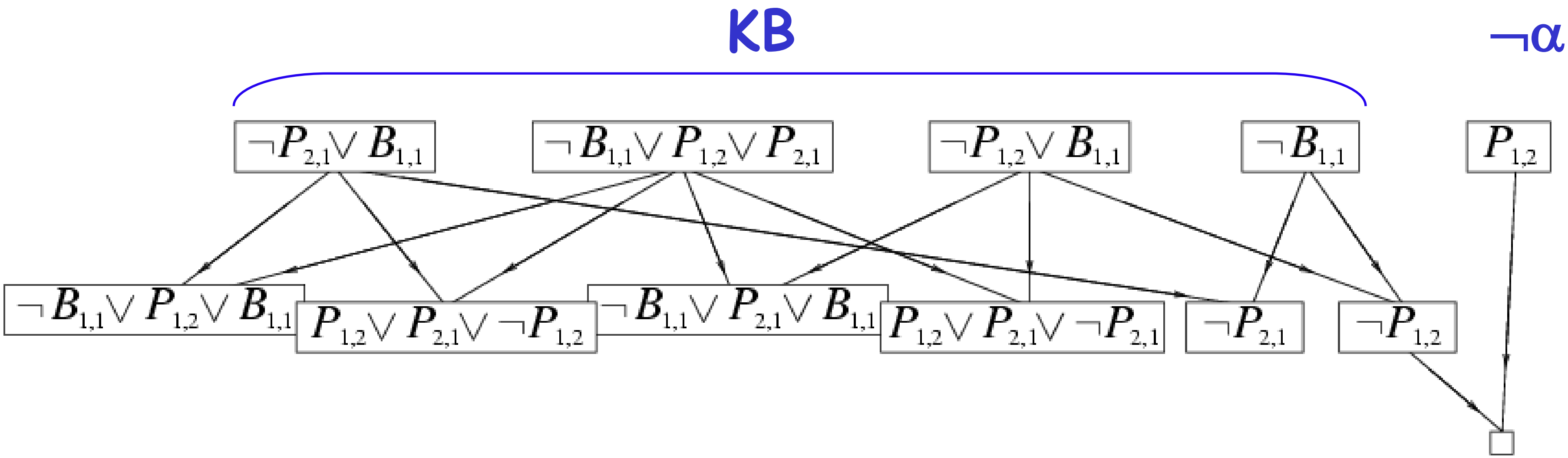
Conjunctive Normal Form (CNF):

sentence = conjunction of clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Can think of **KB** as a **conjunction of clauses**, i.e. one long sentence

Review: Inference Technique I: Resolution



You got a literal and its negation

Empty clause

What does this (empty clause) mean?

Recall that KB is a *conjunction* of all these clauses

Is $P_{1,2} \wedge \neg P_{1,2}$ satisfiable? No!

Therefore, KB $\wedge \neg \alpha$ is unsatisfiable, i.e., KB $\models \alpha$

Inference Technique II: Forward/Backward Chaining

- Requirement: Sentences need to be in **Horn Form**:

  KB = conjunction of Horn clauses

  $\neg P_{2,1} \vee B_{1,1}$

  Horn clause =

  - proposition symbol or
  - "(conjunction of symbols) $\Rightarrow$ symbol"

    (i.e. clause with at most 1 positive literal)

  E.g., KB = $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

- F/B chaining based on "Modus Ponens" rule:

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

  Complete for Horn clauses

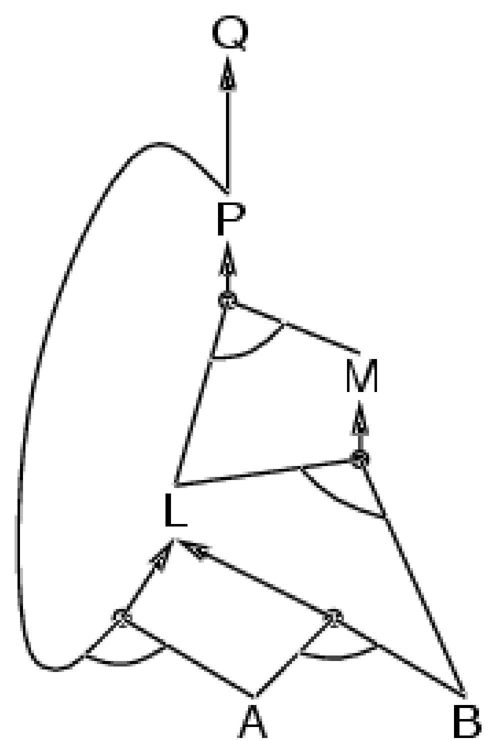
- Very natural and linear time complexity in size of KB

Forward chaining

- Idea: fire any rule whose premises are satisfied in *KB*, add its conclusion to *KB*, until query q is found

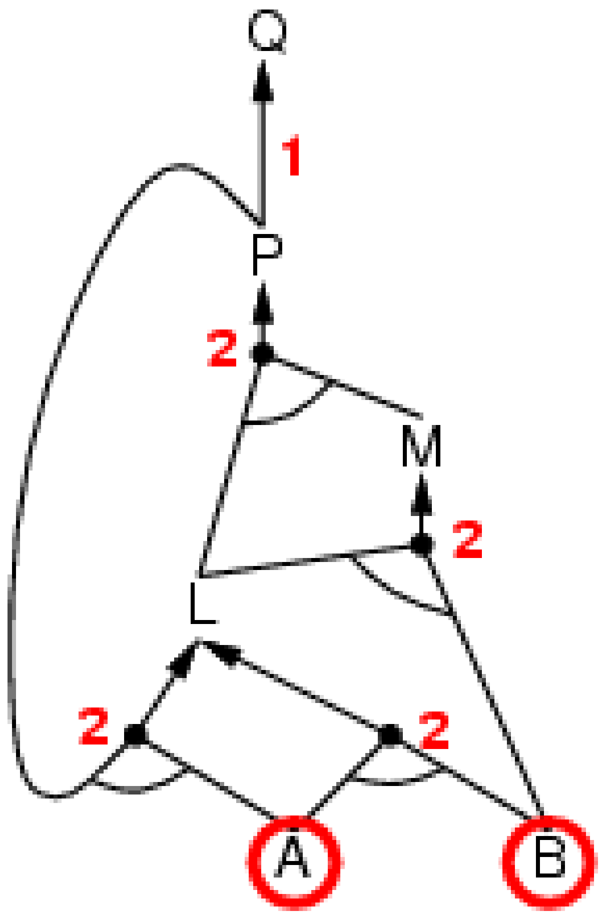
$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

Query = "Is Q true?"



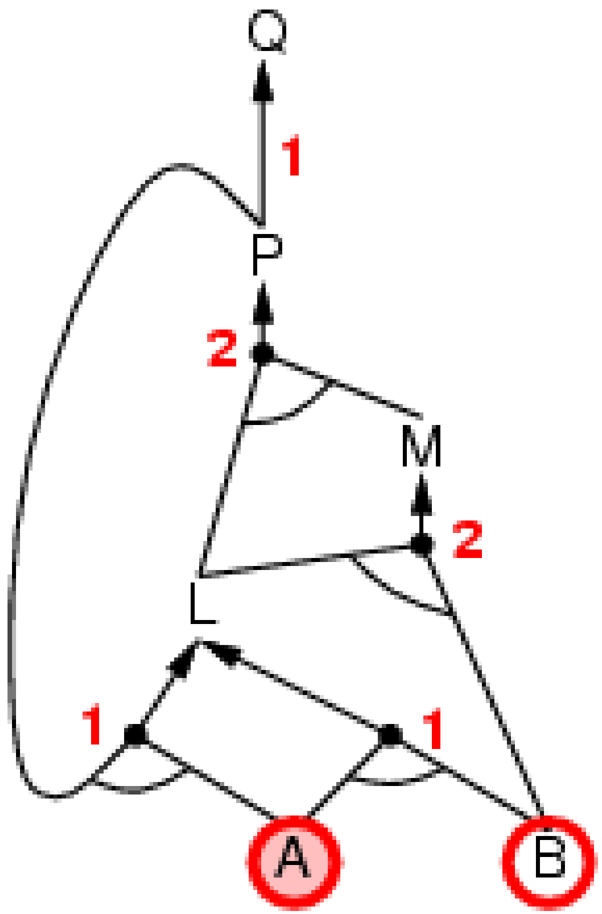
AND-OR Graph

Forward chaining example

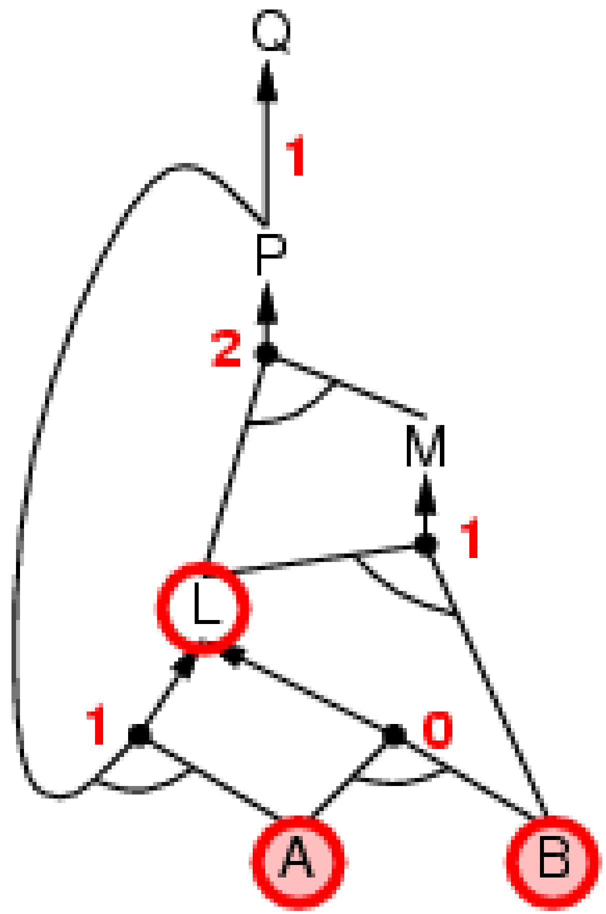


Query = Q
(i.e. "Is Q true?")

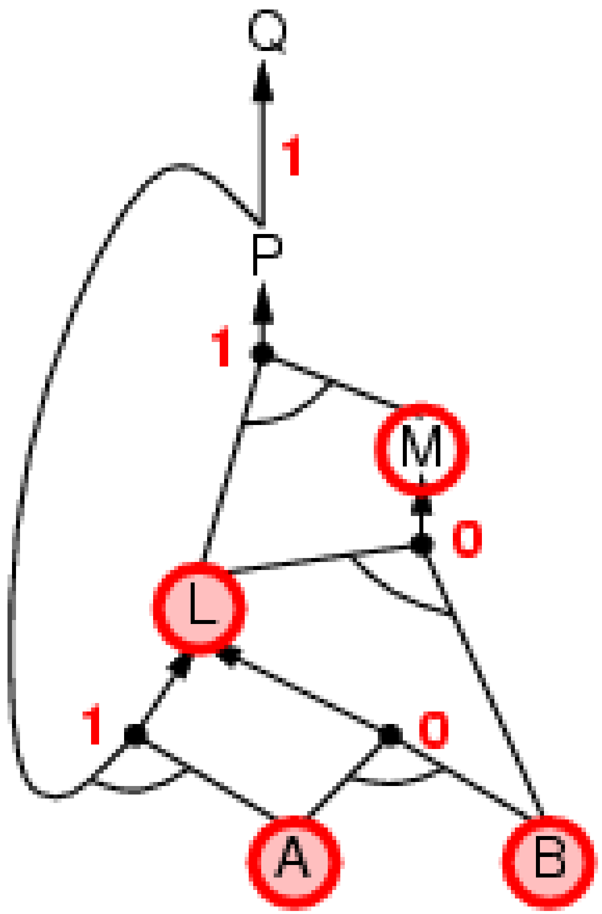
Forward chaining example



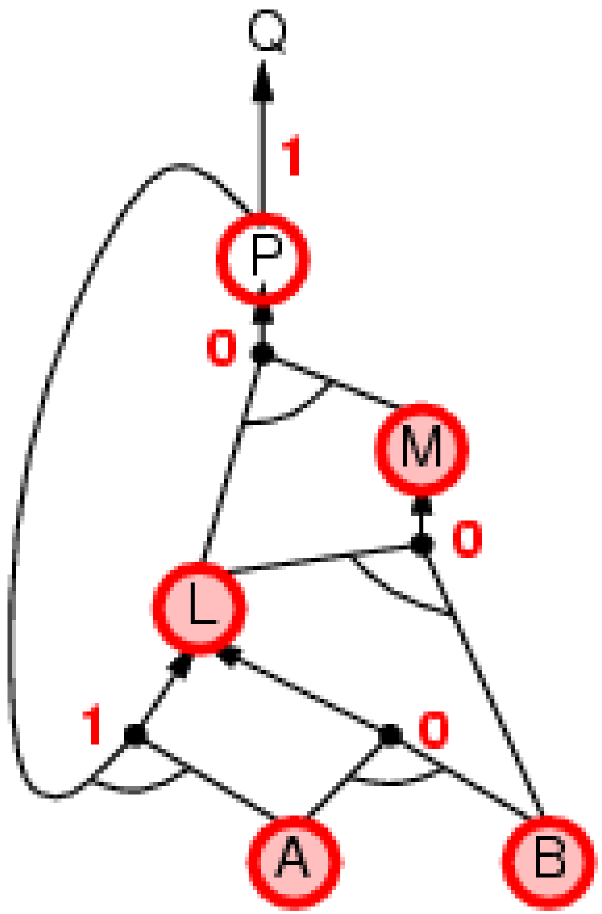
Forward chaining example



Forward chaining example



Forward chaining example



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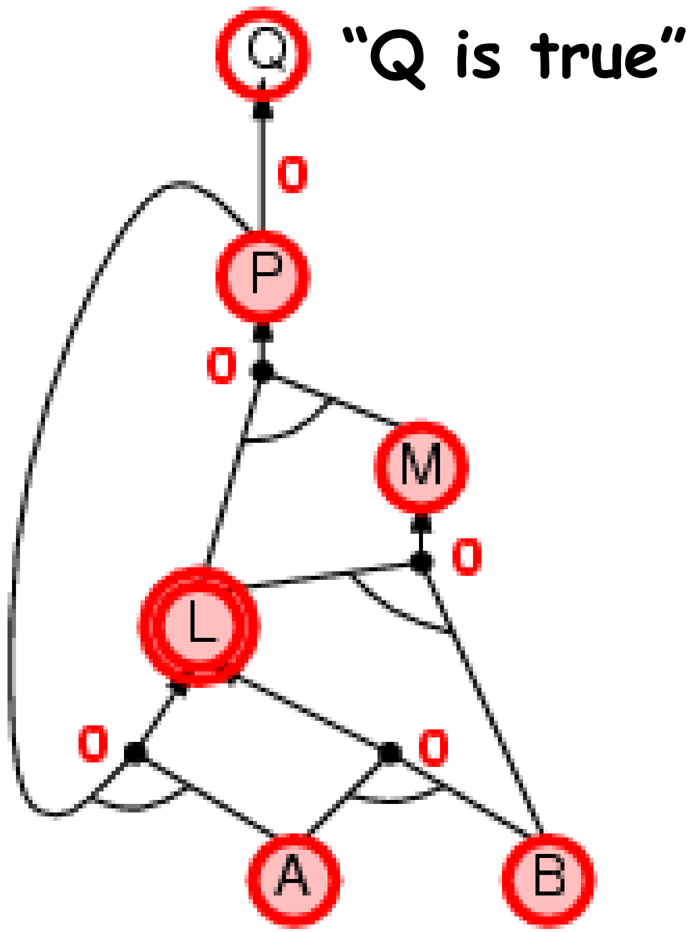


Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known to be true

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
                   PUSH(HEAD[c], agenda)
   return false
```

Forward chaining is sound & complete for Horn KB

Backward Chaining (BC)

Idea: work backwards from the query q:

To prove q by BC,

check if q is known to be true already, or

prove by BC all premises of some rule concluding q

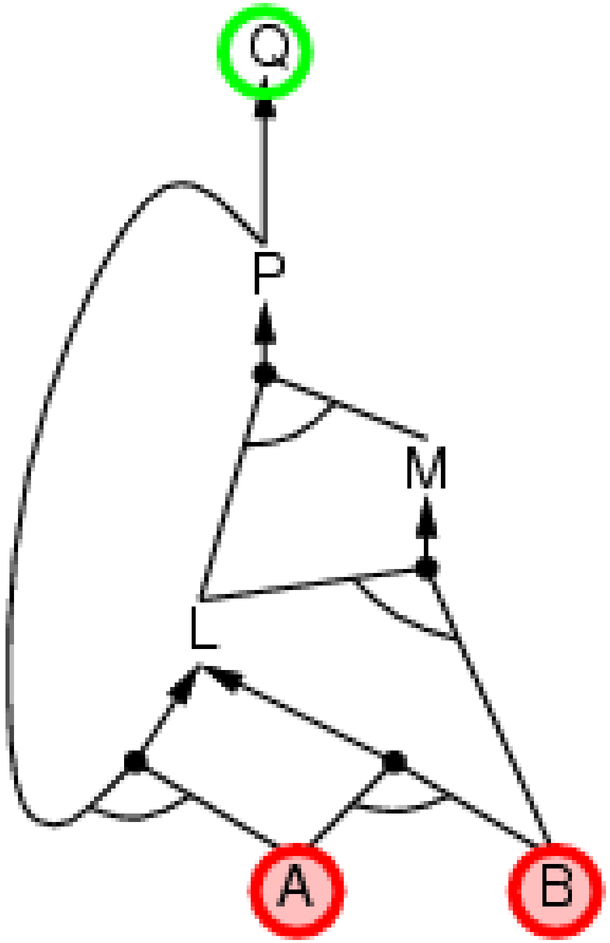
(each premise to be proved is a subgoal)

Avoid loops: check if new subgoal is already on goal stack

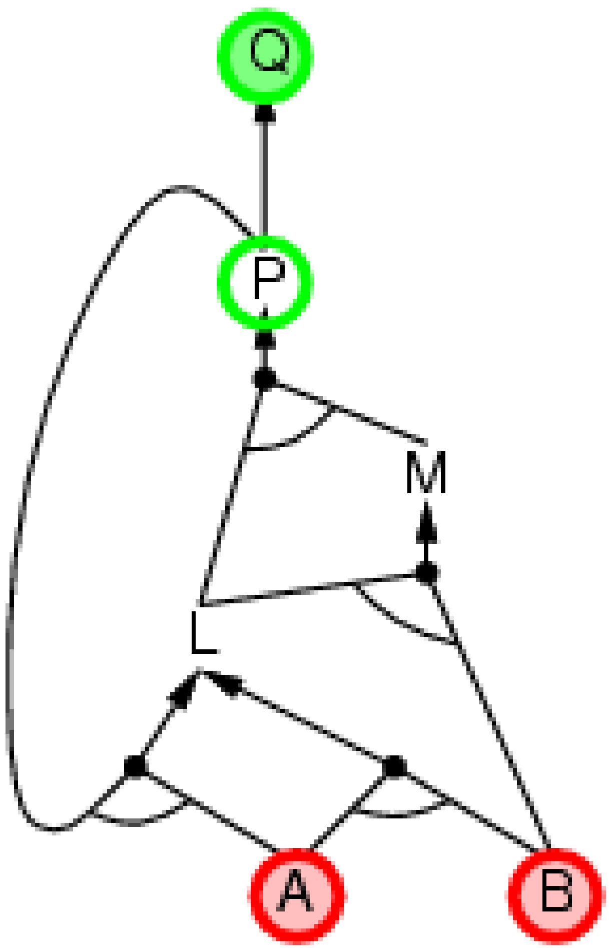
Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

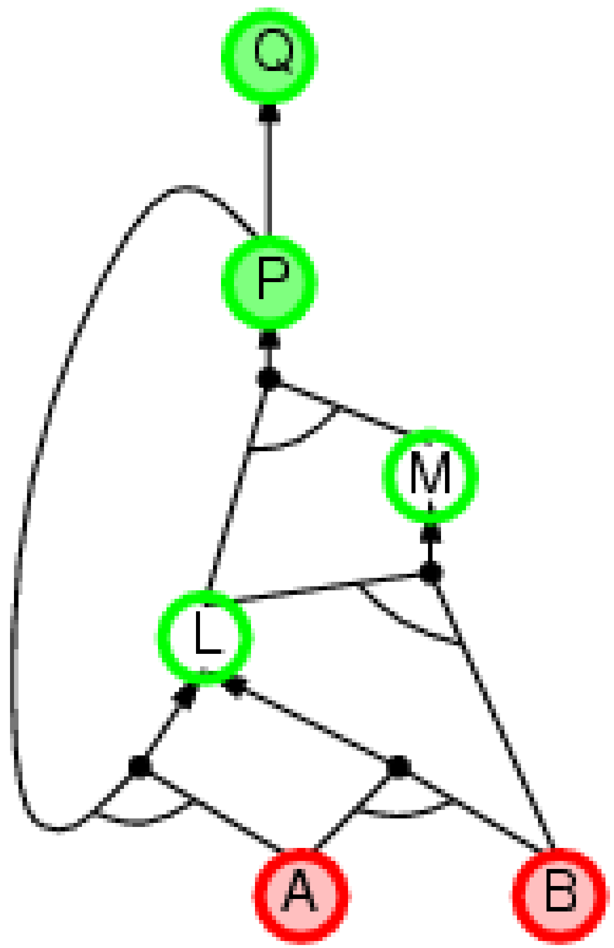
Backward chaining example



Backward chaining example



Backward chaining example



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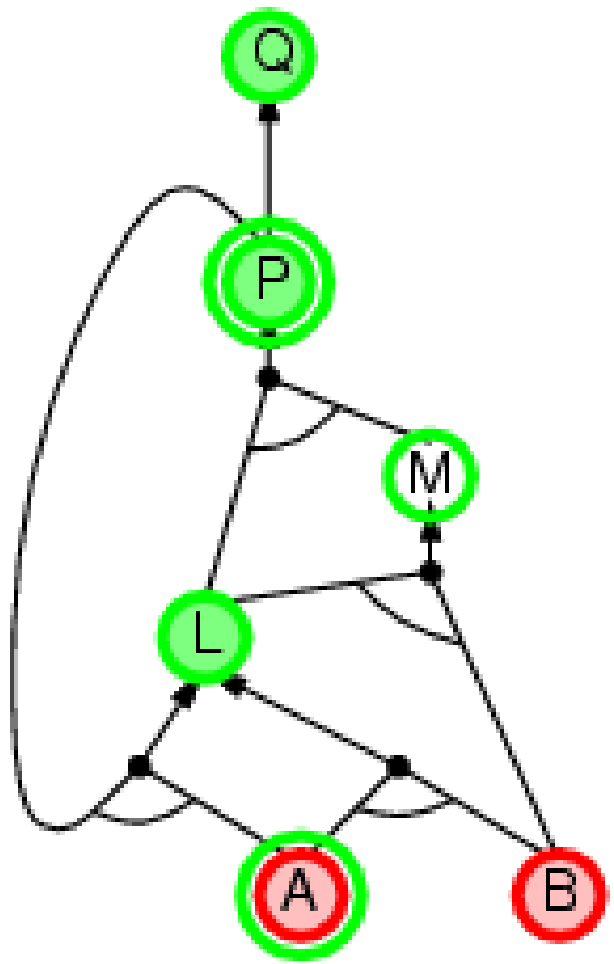
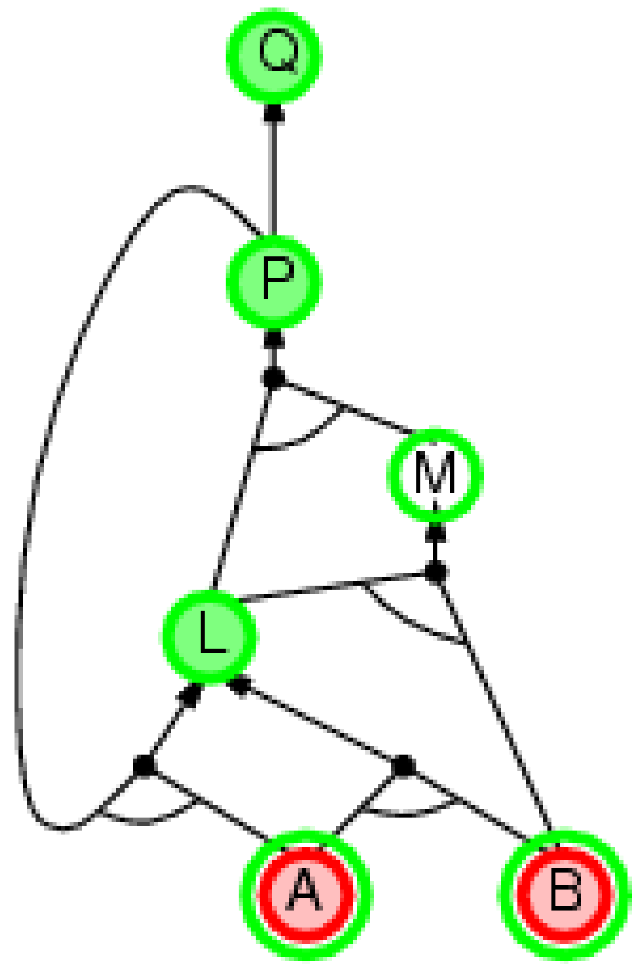


Backward chaining example



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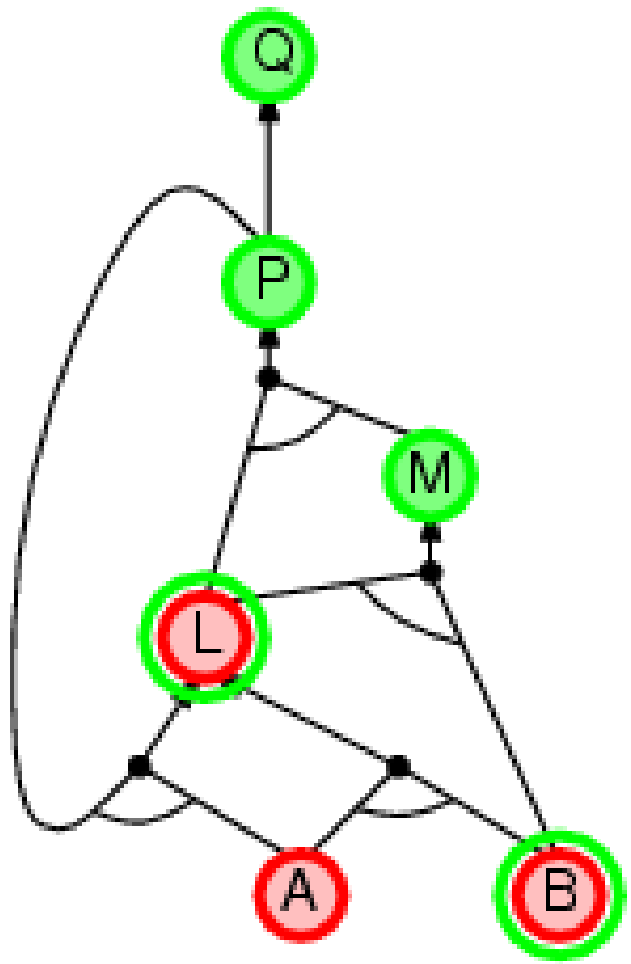

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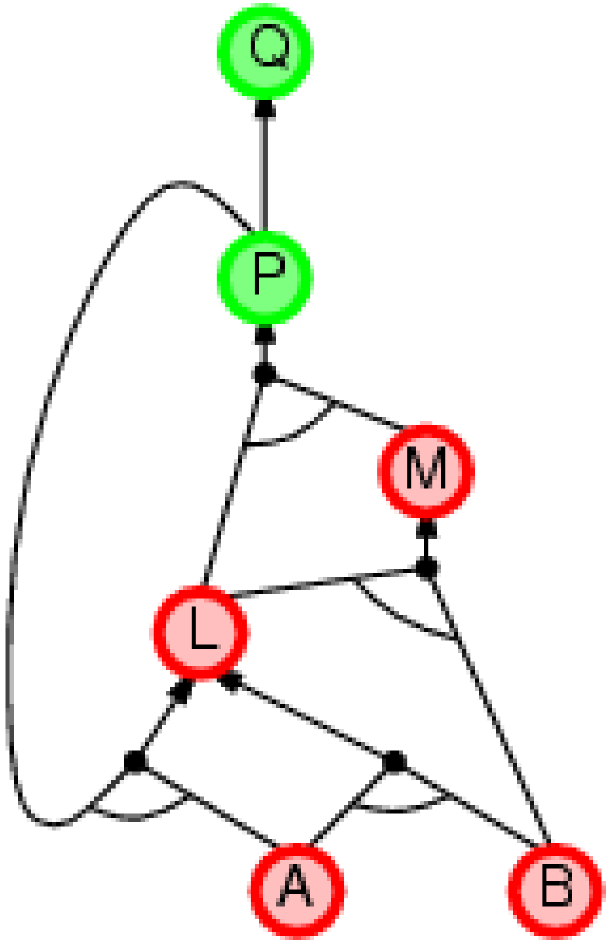

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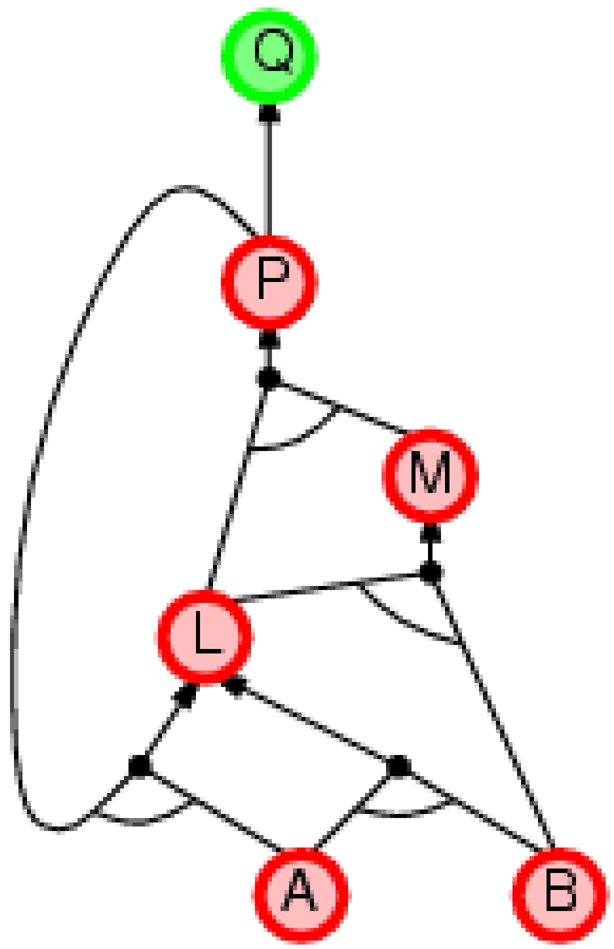

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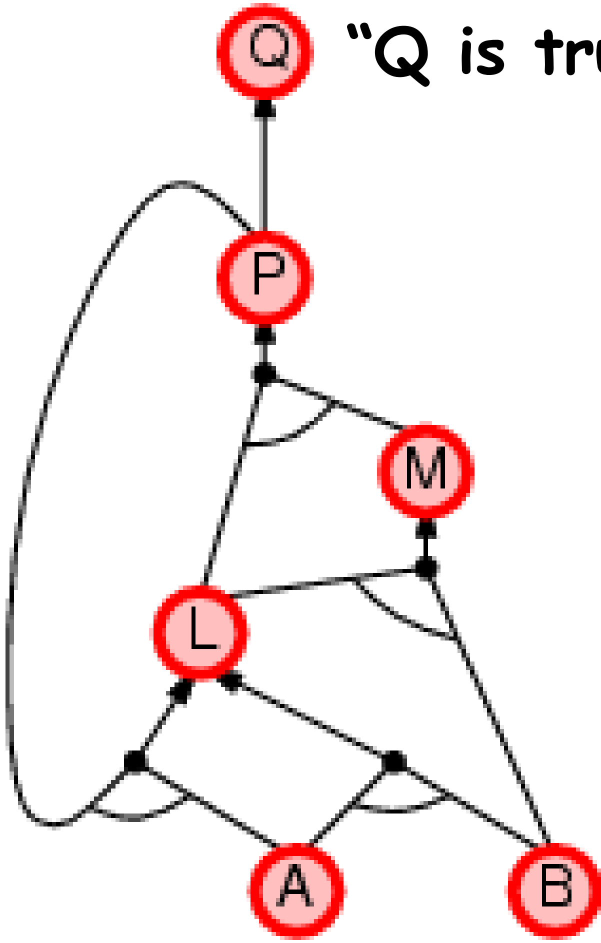


"Q is true"

Forward vs. backward chaining

- FC is data-driven, automatic, unconscious processing,
  e.g., object recognition, routine decisions

- FC may do lots of work that is irrelevant to the goal

- BC is goal-driven, appropriate for problem-solving,
  e.g., How do I get an A in this class?
  e.g., What is my best exit strategy out of the classroom?
  e.g., How can I impress my date tonight?

- Complexity of BC *can be* much less than linear in size of KB

Recall: Inference by Model Checking

Complete search algorithms

Truth table enumeration: Recursive depth-first enumeration of assignments to all symbols (*TT-entails*)

Heuristic search

DPLL algorithm (Davis, Putnam, Logemann, Loveland): Recursive depth-first enumeration of possible models *with heuristics* (see textbook if interested)

Incomplete *local search* algorithms

`WalkSAT` algorithm for checking satisfiability

Why Satisfiability?

Can't get
¬satisfaction

Why Satisfiability?

- Recall: $KB \models \alpha$ iff $KB \wedge \neg\alpha$ is unsatisfiable
  - Equivalent to proving sentence α by contradiction

- Thus, algorithms for satisfiability can be used for inference (entailment)

- However, determining if a sentence is satisfiable or not (the SAT problem) is NP-complete

  Finding a fast algorithm for SAT automatically yields fast algorithms for hundreds of difficult (NP-complete) problems

Satisfiability Examples

E.g. 2-CNF sentences (2 literals per clause):

$(\neg A \vee \neg B) \wedge (A \vee B) \wedge (A \vee \neg B)$

Satisfiable?

Yes (e.g., A = true, B = false)

$(\neg A \vee \neg B) \wedge (A \vee B) \wedge (A \vee \neg B) \wedge (\neg A \vee B)$

Satisfiable?

No

The WalkSAT algorithm

- Local search algorithm
  - Incomplete: may not always find a satisfying assignment even if one exists
- Evaluation function? ("fitness" function)
  = Number of satisfied clauses
  - WalkSAT tries to maximize this function
- Balance between greediness and randomness
  - Each iteration:
  - Randomly select a symbol for flipping (T to F or F to T)
  - OR select symbol that maximizes # satisfied clauses

The `WalkSAT` algorithm

function WALKSAT(*clauses*, *p*, *max-flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move
    max-flips, number of flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for i = 1 **to** *max-flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    with probability p flip the value in *model* of a randomly selected symbol
      from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*

Greed **Randomness**

Hard Satisfiability Problems

Consider random 3-CNF sentences. e.g.,

$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor A) \land (A \lor \neg D \lor B) \land (B \lor D \lor \neg C)$

Satisfiable?

(Yes, e.g., $A = B = C =$ *true*)

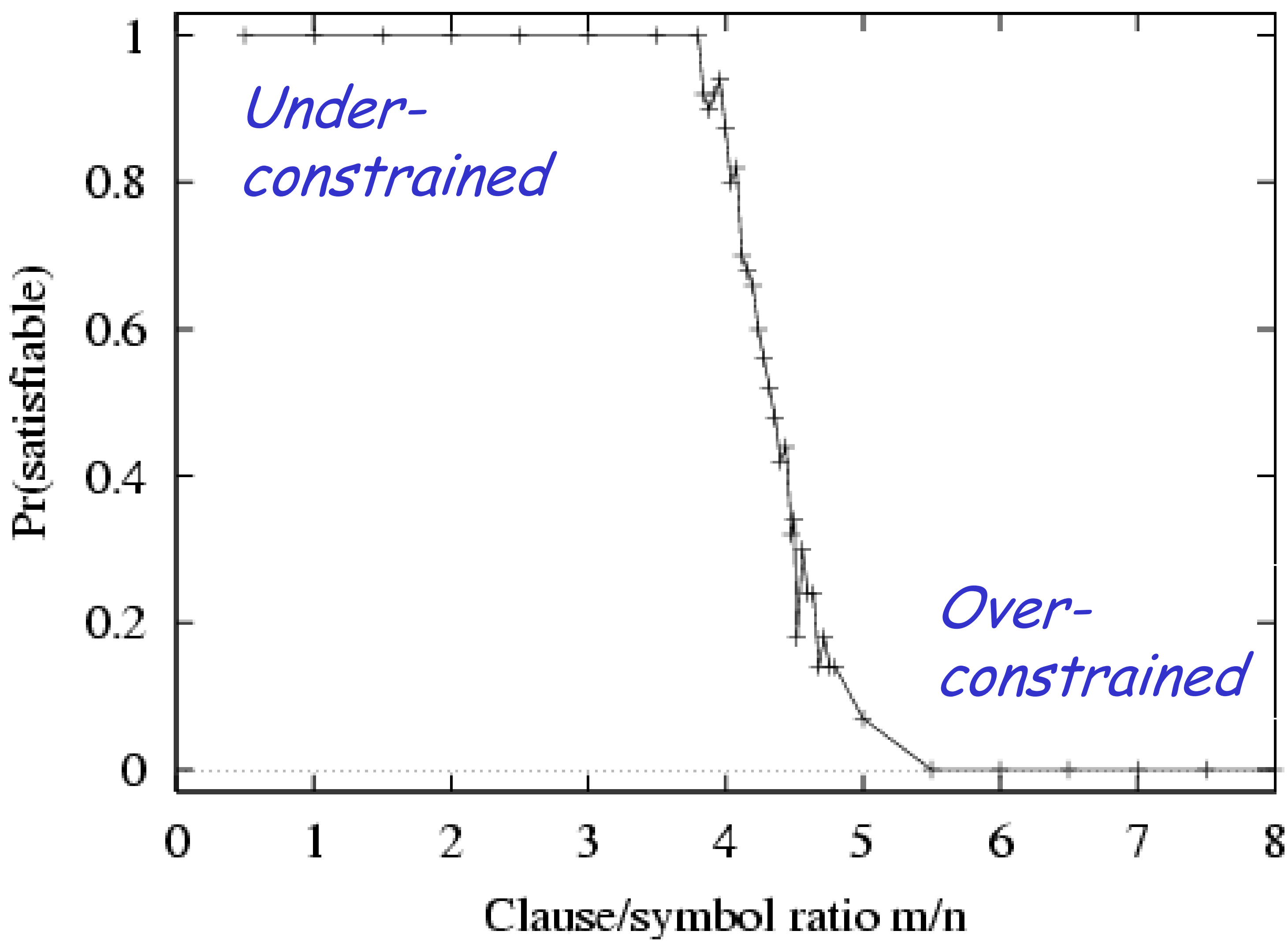
m = number of clauses (Here 5)

n = number of symbols (Here 4 – A, B, C, D)

m/n = 1.25 (enough symbols, usually satisfiable)

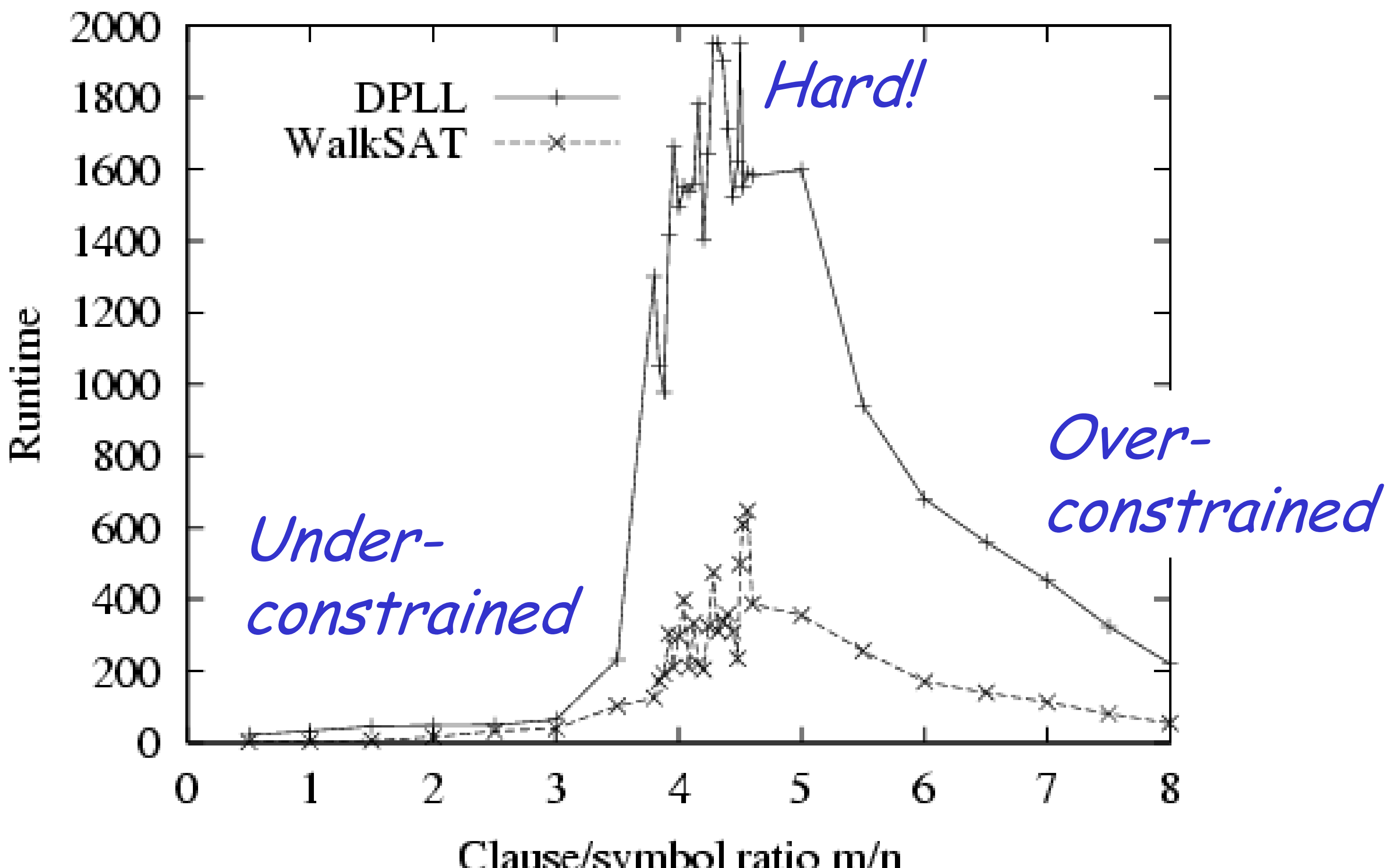
Hard instances of SAT seem to cluster near m/n = 4.3 (critical point)

Hard Satisfiability Problems

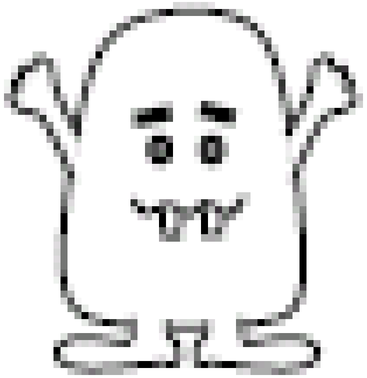


Hard Satisfiability Problems

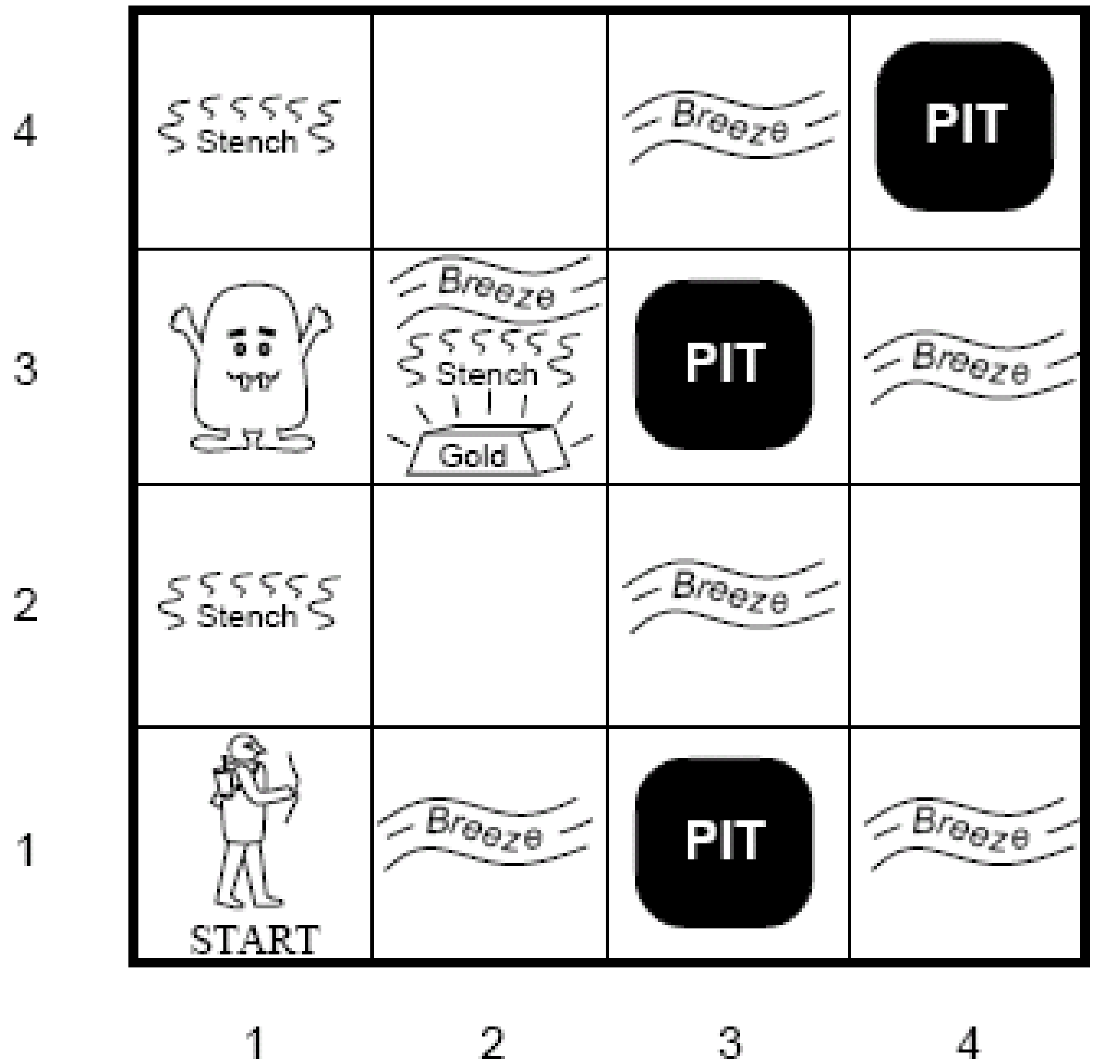
Median runtime for 100 satisfiable random 3-CNF sentences, $n = 50$



What about me?



Wumpus World



Putting it all together: Logical Wumpus Agents

A wumpus-world agent using propositional logic:

$\neg P_{1,1}$
$\neg W_{1,1}$
For x = 1, 2, 3, 4 and y = 1, 2, 3, 4, add (with appropriate boundary conditions):

$B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$
$S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$
$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$ At least 1 wumpus
$\neg(W_{1,1} \wedge W_{1,2})$
$\neg(W_{1,1} \wedge W_{1,3})$ At most 1 wumpus

...

$\Rightarrow$ 64 distinct proposition symbols, 155 sentences!

Limitations of propositional logic

- KB contains "physics" sentences for every single square

- For every time step *t* and every location [*x*,*y*], we need to add to the KB "physics" rules such as:

$$L^{t}_{x,y} \wedge FacingRight^{t} \wedge Forward^{t} \Rightarrow L^{t+1}_{x+1,y}$$

- Rapid proliferation of sentences…

What we'd like is a way to talk about *objects* and *groups* of objects, and to define relationships between them.

Enter: First-order logic
(aka "predicate logic")

Next Time

- First-Order Logic
- To Do:
  - Project #2
  - Read chapter 8