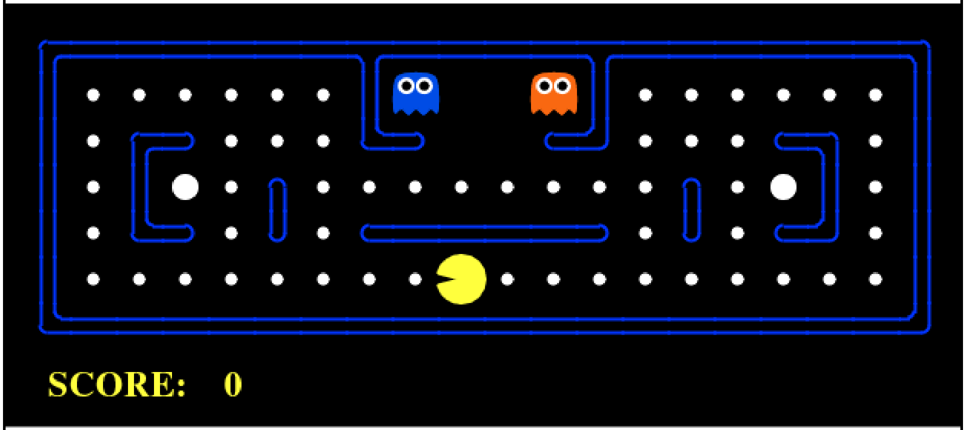
CSE 473

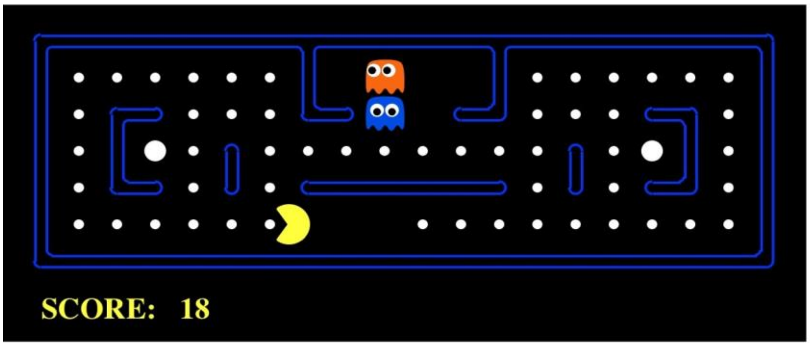
Chapter 3
Problem Solving using Search

"First, they do an on-line search"

Pac-Man as an Agent



The CSE 473 Pac-Man Projects



Originally developed at UC Berkeley:
http://www-inst.eecs.berkeley.edu/~cs188/pacman/pacman.html

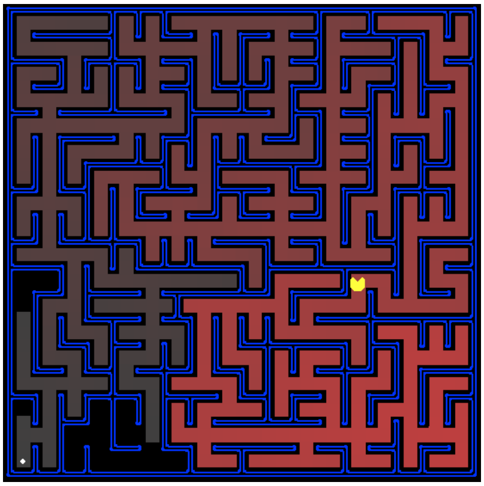
Project 1: Search

Goal:

- Help Pac-man find its way through the maze

Techniques:

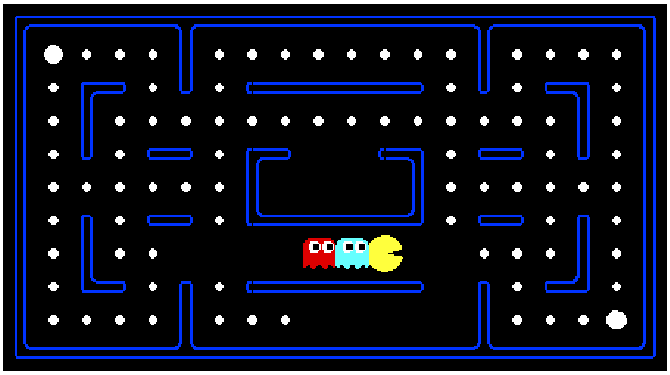
- Search: breadth-first, depth-first, etc.
- Heuristic Search: Best-first, A*, etc.



Project 2: Game Playing

Goal:
Build a rational
Pac-Man agent!

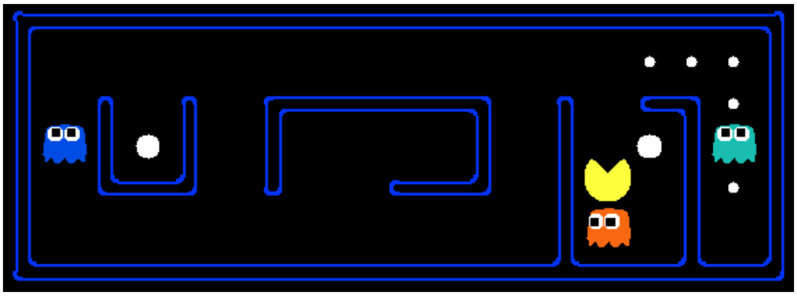
Techniques:
Adversarial Search: minimax,
alpha-beta, expectimax, etc.



Project 3: Planning and Learning

Goal:
Help Pac-Man
learn about its
world

Techniques:
- Planning: MDPs, Value Iteration
- Learning: Reinforcement Learning



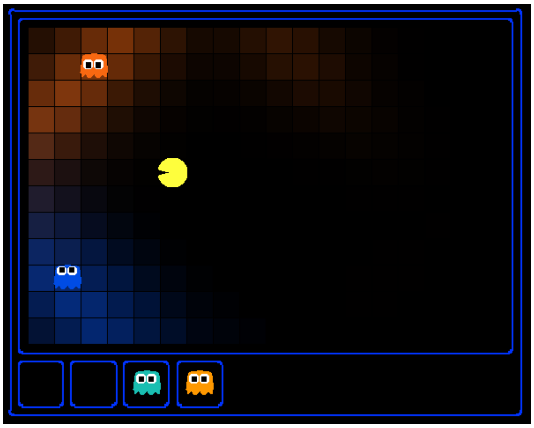
Project 4: Ghostbusters

Goal:

Help Pac-man hunt down the ghosts

Techniques:

- Probabilistic models: HMMs, Bayes Nets
- Inference: State estimation and particle filtering



Problem Solving using Search

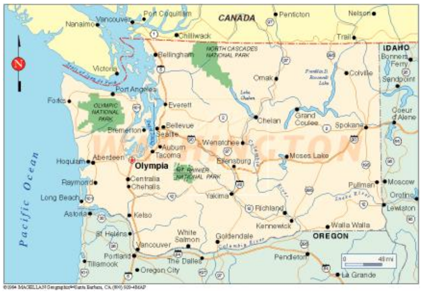
Example 1: The 8-puzzle

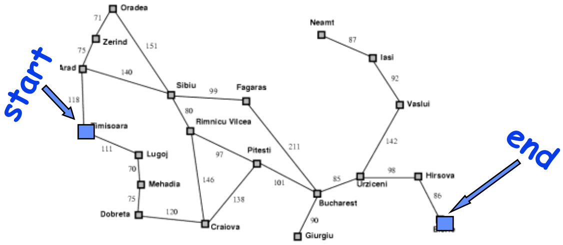
Start → **Goal**

1	2	3
8		4
7	6	5

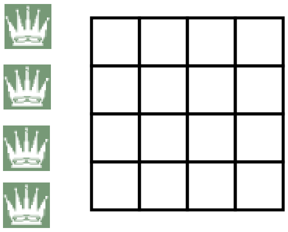
1	2	3
4	5	6
7	8	

Example 2: Route Planning





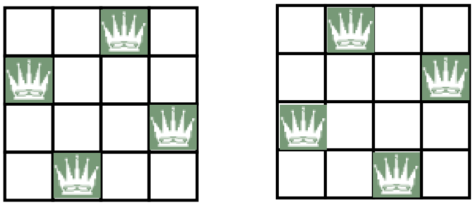
Example 3: N Queens



4 Queens problem

(Place queens such that no queen attacks any other)

Example: N Queens



4 Queens

State-Space Search Problems

General problem:

Find a path from a *start state* to a *goal state* given:

- A goal test: Tests if a given state is a goal state
- A successor function (transition model): Given a state and action, generate *successor* state

Variants:

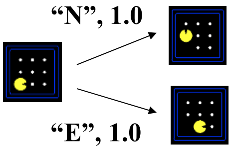
- Find any path *vs.* a least-cost path (if each step has a different cost i.e. a “step-cost”)
- Goal is completely specified, task is to find a path or least-cost path
  - Route planning
- Path doesn’t matter, only finding the goal state
  - 8 puzzle, N queens, Rubik’s cube

Example: Simplified Pac-Man

Input:

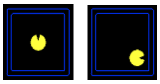
- State space

- Successor function

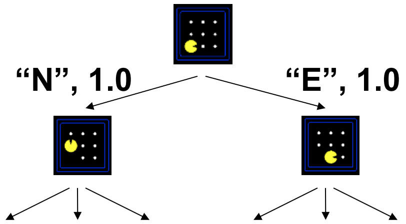


- Start state

- Goal test



Search Trees



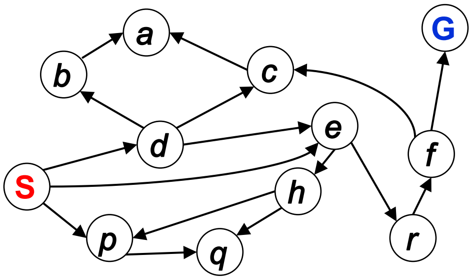
A search tree:

- Root contains Start state
- Children = successor states
- Edges = actions and step-costs
- Path from Root to a node is a "plan" to get to that state
- For most problems, we can never actually build the whole tree (why?)

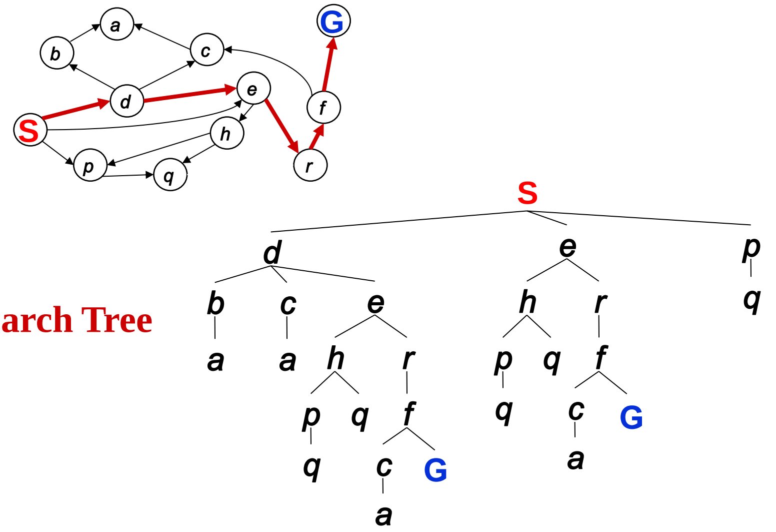
State Space Graph versus Search Trees

State Space Graph

(graph of states with arrows pointing to successors)

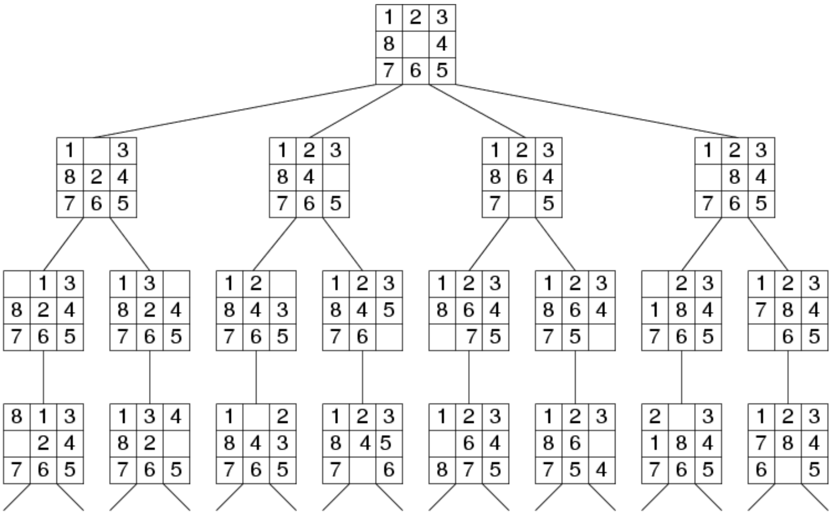


State Space Graph versus Search Trees



Search Tree

Search Tree for 8-Puzzle

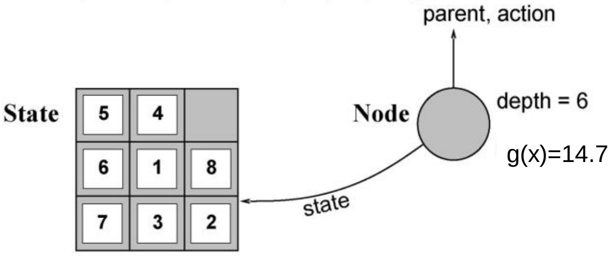


Implementation: states vs. nodes

A *state* is a (representation of) a physical configuration

A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$

States do not have parents, children, depth, or path cost!



Searching with Search Trees

Search:

- Expand out possible nodes
- Maintain a fringe or frontier of as yet unexpanded nodes
- Try to expand as few tree nodes as possible

Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← REMOVE-FRONT(fringe)
       if GOAL-TEST[problem] applied to STATE(node) succeeds return node
       fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Implementation: general tree search

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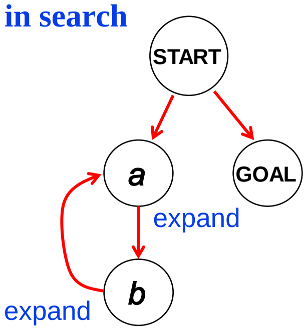
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;   ACTION[s] ← action;   STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Handling Repeated States

Failure to detect repeated states (e.g., in 8 puzzle) can cause infinite loops in search



Graph Search algorithm: Augment Tree-Search to store expanded nodes in a set called *explored set (or closed set)* and only add *new* nodes not in the explored set to the fringe

Search strategies

A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

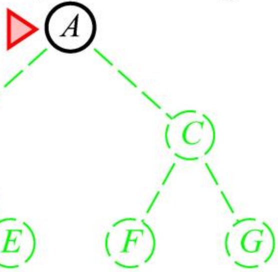
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end

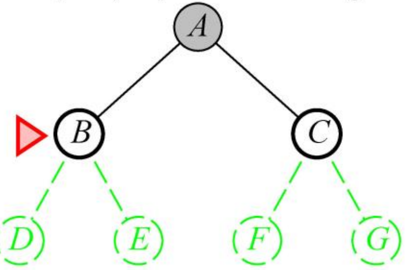


Breadth-first search

Expand shallowest unexpanded node

Implementation:

fringe is a FIFO queue, i.e., new successors go at end



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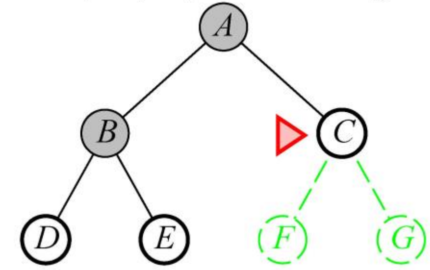

Breadth-first search

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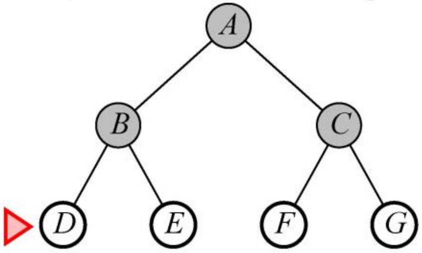

Properties of breadth-first search

Complete??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \cdots + b^d = O(b^d)$ i.e. exp in d

Space??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \cdots + b^d = O(b^d)$ i.e. exp in d

Space?? $O(b^d)$

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

Time?? $b + b^2 + b^3 + \cdots + b^d = O(b^d)$ i.e. exp in d

Space?? $O(b^d)$

Optimal?? Yes if all step costs are equal. Not optimal in general.

Space and time are big problems for BFS.

Example: b = 10 with 1,000,000 nodes/sec, 1000 Bytes/node

d = 2 ➔ 110 nodes, 0.11 millisecs, 107KB

d = 4 ➔ 11,110 nodes, 11 millisecs, 10.6 MB

d = 8 ➔ 10^8 nodes, 2 minutes, 103 GB

d = 16 ➔ 10^{16} nodes, 350 years, 10 EB (1 billion GB)

What if the step costs are not equal?

Can we modify BFS to handle any step cost function?

Uniform-cost search

Expand least-cost unexpanded node

Implementation:
fringe = queue ordered by path cost $g(n)$ (Use **priority queue**)

Equivalent to breadth-first if step costs all equal

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lfloor C^*/\varepsilon \rfloor + 1})$
where C^* is the cost of the optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lfloor C^*/\varepsilon \rfloor + 1})$

Optimal?? Yes—nodes expanded in increasing order of $g(n)$

Can we do better?

Next time: depth first search, depth limited search, iterative deepening search, bidirectional search

All these methods are slow (because they are "blind")

Solution → use problem-specific knowledge to guide search ("heuristic function")
→ "informed search" (next lecture)

To Do

- **Start Project #1**
- **Read Chapter 3**