First-Order Logic

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Overview

Propositional. Logic *vs.* First Order

	Propositional	First Order
Ontology	Facts (P, Q)	Objects, Properties, Relations
Syntax	Atomic sentences Connectives	Variables & quantification Sentences have structure: terms father-of(mother-of(X)))
Semantics	Truth Tables	Interpretations *(Much more complicated)*
Inference Algorithm	DPLL, GSAT Fast in practice	Unification Forward, Backward chaining Prolog, theorem proving
Complexity	NP-Complete	Semi-decidable

FOL Definitions

- ***Constants***: a,b, dog33.
  Name a specific object.
- ***Variables***: X, Y.
  Refer to an object without naming it.
- ***Functions***: dad-of
  Mapping from objects to objects.
- ***Terms***: dad-of(dog33)
  Refer to objects
- ***Atomic Sentences***: in(dad-of(dog33), food6)
  Can be true or false
  Correspond to propositional symbols P, Q

More Definitions

Logical connectives: and, or, not, =>

male(dan) $\wedge$ male(father-of(dan))

P $\wedge$ Q

male(dan) $\wedge$ male(son-of(father-of(dan)))

More Definitions

- **Quantifiers**:
  $\forall$ Forall
  $\exists$ There exists
- **Examples**
  Dumbo is grey

  Elephants are grey

  There is a grey elephant

More Definitions

- **Quantifiers**:
  ∀ Forall
  ∃ There exists
- **Examples**
  Dumbo is grey
  grey(dumbo)

  Elephants are grey

  There is a grey elephant

More Definitions

- **Quantifiers**:
  ∀ Forall
  ∃ There exists
- **Examples**
  Dumbo is grey
  grey(dumbo)

  Elephants are grey
  $\forall$ elephant(x) $\Rightarrow$ grey(x)

  There is a grey elephant

More Definitions

- **Quantifiers**:
  ∀ Forall
  ∃ There exists
- **Examples**
  Dumbo is grey
  grey(dumbo)

  Elephants are grey
  $\forall$ x elephant(x) $\Rightarrow$ grey(x)

  There is a grey elephant
  $\exists$ x elephant(x) $\wedge$ grey(x)

Quantifier / Connective Interaction

E(x) == "x is an elephant"
G(x) == "x has the color grey"

1. $\forall x\ E(x) \wedge G(x)$
2. $\forall x\ E(x) \Rightarrow G(x)$
3. $\exists x\ E(x) \wedge G(x)$
4. $\exists x\ E(x) \Rightarrow G(x)$

Nested Quantifiers: Order matters!

$$\forall x \exists y\ P(x,y) \neq \exists y \forall x\ P(x,y)$$

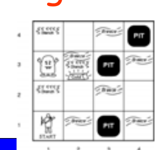
- Examples
  Every dog has a tail
  $\forall d \exists t\ has(d,t)$ **?**

  Every dog ***shares*** a tail!
  $\exists t \forall d\ has(d,t)$

  Someone is loved by everyone
  $\exists x \forall y\ loves(y, x)$

Wumpus world in prop logic

Let $P_{i,j}$ be true if there is a pit in [i, j].
Let $B_{i,j}$ be true if there is a breeze in [i, j].

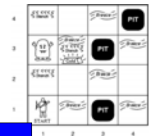


KB:
$\neg P_{1,1}$
$\neg B_{1,1}$

"Pits cause breezes in adjacent squares"
$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

Wumpus world in prop logic

Let pit(i,j) be true if there is a pit in [i, j].
Let breeze(i,j) be true if breezy in [i, j].

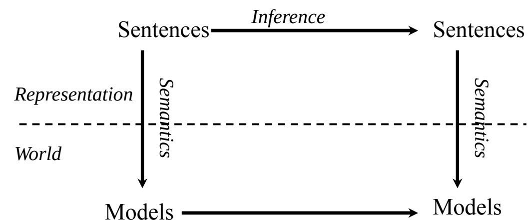


KB:
¬ pit(1,1)
¬ breeze(1,1)

"Pits cause breezes in adjacent squares"
∀i,j breeze(i,j) ⇔ pit(i, add(j,1)) ∨ pit(i, add(j, -1)) ∨ …

Semantics

- ***Syntax***: a description of the *legal* arrangements of symbols
  (Def "sentences")
- ***Semantics***: what the arrangement of symbols *means* in the world

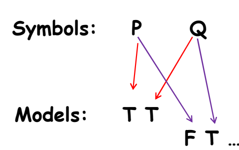


Satisfiability, Validity, & Entailment

- S is **valid** if it is true in all interpretations
- S is **satisfiable** if it is true in some interp
- S is **unsatisfiable** if it is false all interps
- S1 **entails** S2 if (|=)
  forall interps where S1 is true,
  S2 is also true

Propositional Logic: **SEMANTICS**

- "Interpretation" (or "possible world")
- Specifically, TRUTH ASSIGNMENTS
  - Assignment to each variable either T or F
  - Assignment of T or F to each connective



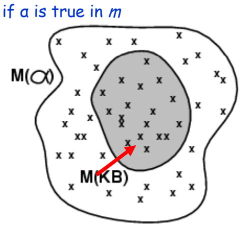
		Q	
		T	F
P	T		
	F		

P ∧ Q

Models

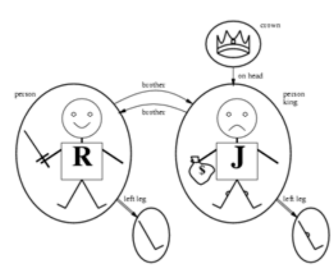
- Logicians often think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
  - In propositional case, each model = truth assignment
  - Set of models can be enumerated in a truth table
- We say *m* is a model *of* a sentence α if α is true in *m*
  (Equivalently "*m* satisfies α")
- *M(α)* is the set of all models *of* α
- Then KB ╞ α iff *M(KB)* ⊆ *M*(α)
-

  E.g. *KB* = (P∨Q) ∧(¬P ∨ R)
  α = (P∨R)



First Order Logic: Models

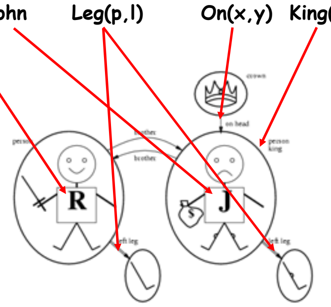
- Depiction of one possible "real-world" model



Interpretations=Mappings syntactic tokens → model elements

Depiction of one possible interpretation, assuming

Constants:	Functions:	Relations:
Richard John	Leg(p,l)	On(x,y) King(p)



Interpretations=Mappings syntactic tokens → model elements

Another interpretation, same assumptions

Constants:	Functions:	Relations:
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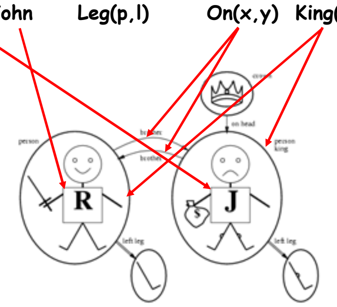

FOL Reasoning

- FO Forward & Backward Chaining
- FO Resolution
- Many other types of theorem proving
- Restricted representations
  - Description logics
  - Horn Clauses
- Compilation to SAT

Forward Chaining

- Given
  - ∀?x lifeform(?x) => mortal(?x)
  - ∀?x mammal(?x) => lifeform(?x)
  - ∀?x dog(?x) => mammal(?x)
  - dog(fido)
- Prove
  - mortal(fido)

∀?x dog(?x) => mammal(?x)
dog(fido)
―――――――――――――――― ?
mammal(fido)

Unification

- Emphasize variables with ?
- Useful for FO inference (modus ponens, …)
  - Also for compilation of FOPC -> propositional
- Unify(Φ, Ψ) returns "mgu"
  - Unify(city(?a), city(kent)) returns ?a/kent
- Substitute(expr, mapping) returns new expr
  - Substitute(connected(?a, ?b), {?a/kent}) returns connected(kent, ?b)

Unification Examples

- Unify(road(?a, kent), road(seattle, ?b))
- Unify(road(?a, ?a), road(seattle, kent))
- Unify(f(g(?x, dog), ?y)), f(g(cat, ?y), dog)
- Unify(f(g(?x)), f(?x))

Compilation to Prop. Logic I

- Typed Logic
  $\forall_{city}\ a,b$ connected(a,b)
- Universe
  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula:

Compilation to Prop. Logic II

- Universe
  - Cities: seattle, tacoma, enumclaw
  - Firms: IBM, Microsoft, Boeing
- First-Order formula
  $\forall_{city}\ c\ \exists_{firm}\ f$ hasHQ(c, f)
- Equivalent propositional formula

Hey!

- You said FO Inference is semi-decidable
- But you compiled it to SAT
  Which is NP Complete
- So now we can always do the inference?!?
  Tho it might take exponential time…
- Something seems wrong here….????

Restricted Forms of FO Logic

- Known, Finite Universes
  Compile to SAT
- Frame Systems
  Ban certain types of expressions
- Horn Clauses
  Aka Prolog
- Function-Free Horn Clauses
  Aka Datalog