Local Search and Optimization
Chapter 4

Mausam

(Based on slides of Padhraic Smyth, Stuart Russell, Rao Kambhampati, Raj Rao, Dan Weld…)

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Outline

- Local search techniques and optimization
  – Hill-climbing
  – Gradient methods
  – Simulated annealing
  – Genetic algorithms
  – Issues with local search

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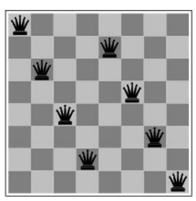
Local search and optimization

- Previous lecture: path to goal is solution to problem
  – systematic exploration of search space.
- This lecture: a state is solution to problem
  – for some problems path is irrelevant.
  – E.g., 8-queens
- Different algorithms can be used
  – Depth First Branch and Bound
  – Local search



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Goal Satisfaction

reach the goal node
Constraint satisfaction

Optimization

optimize(objective fn)
Constraint Optimization

You can go back and forth between the two problems
Typically in the same complexity class

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Local search and optimization

- Local search
  – Keep track of single current state
  – Move only to neighboring states
  – Ignore paths
- Advantages:
  – Use very little memory
  – Can often find reasonable solutions in large or infinite (continuous) state spaces.
- "Pure optimization" problems
  – All states have an objective function
  – Goal is to find state with max (or min) objective value
  – Does not quite fit into path-cost/goal-state formulation
  – Local search can do quite well on these problems.

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Trivial Algorithms

- Random Sampling
  – Generate a state randomly
- Random Walk
  – Randomly pick a neighbor of the current state
- Both algorithms asymptotically complete.

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Hill-climbing (Greedy Local Search) max version

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum
  input: *problem*, a problem
  local variables: *current*, **a node.**
    neighbor, **a node.**

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  loop do
    neighbor ← a highest valued successor of *current*
    if VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]
    current ← *neighbor*

min version will reverse inequalities and look for lowest valued successor

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Hill-climbing search

- "a loop that continuously moves towards increasing value"
  - terminates when a peak is reached
  - Aka greedy local search
- Value can be either
  - Objective function value
  - Heuristic function value (minimized)
- Hill climbing does not look ahead of the immediate neighbors
- Can randomly choose among the set of best successors
  - if multiple have the best value
- "climbing Mount Everest in a thick fog with amnesia"

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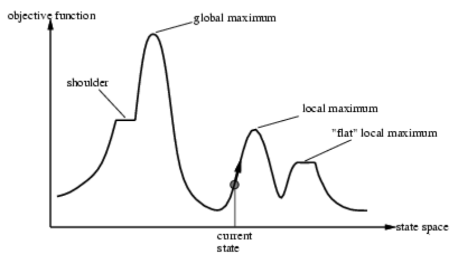
"Landscape" of search



Hill Climbing gets stuck in local minima
depending on?

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Example: *n*-queens

- Put *n* queens on an *n* x *n* board with no two queens on the same row, column, or diagonal

- Is it a satisfaction problem or optimization?

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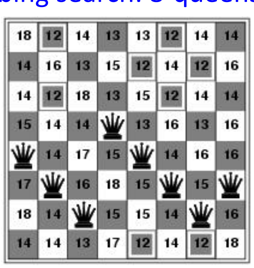
Hill-climbing search: 8-queens problem



- Need to convert to an optimization problem
- *h* = number of pairs of queens that are attacking each other
- *h = 17* for the above state

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Search Space

- State
  - All 8 queens on the board in some configuration
- Successor function
  - move a single queen to another square in the same column.
- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other
  - (so we want to minimize this)

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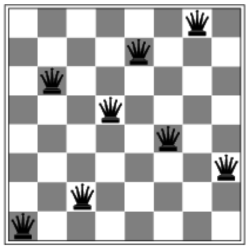
Hill-climbing search: 8-queens problem



- Is this a solution?
- What is h?

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Hill-climbing on 8-queens

- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum

- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with 8^8 =~17 million states)

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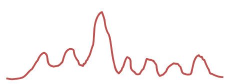
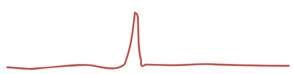
Hill Climbing Drawbacks

- Local maxima



- Plateaus



- Diagonal ridges

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Escaping Shoulders: Sideways Move

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Need to place a limit on the possible number of sideways moves to avoid infinite loops
- For 8-queens
  - Now allow sideways moves with a limit of 100
  - Raises percentage of problem instances solved from 14 to 94%

  - However….
    - 21 steps for every successful solution
    - 64 for each failure

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Tabu Search

- prevent returning quickly to the same state
- Keep fixed length queue ("tabu list")
- add most recent state to queue; drop oldest
- Never make the step that is currently tabu'ed

- Properties:
  - As the size of the tabu list grows, hill-climbing will asymptotically become "non-redundant" (won't look at the same state twice)
  - In practice, a reasonable sized tabu list (say 100 or so) improves the performance of hill climbing in many problems

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Escaping Shoulders/local Optima Enforced Hill Climbing

- Perform breadth first search from a local optima
  - to find the next state with better h function

- Typically,
  - prolonged periods of exhaustive search
  - bridged by relatively quick periods of hill-climbing

- Middle ground b/w local and systematic search

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Hill-climbing: stochastic variations

- Stochastic hill-climbing
  - Random selection among the uphill moves.
  - The selection probability can vary with the steepness of the uphill move.
- To avoid getting stuck in local minima
  - Random-walk hill-climbing
  - Random-restart hill-climbing
  - Hill-climbing with both

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Hill Climbing: stochastic variations

→When the state-space landscape has local minima, any search that moves only in the greedy direction cannot be complete

→Random walk, on the other hand, is asymptotically complete

Idea: Put random walk into greedy hill-climbing

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Hill-climbing with random restarts

- If at first you don't succeed, try, try again!
- Different variations
  - For each restart: run until termination vs. run for a fixed time
  - Run a fixed number of restarts or run indefinitely
- Analysis
  - Say each search has probability p of success
    - E.g., for 8-queens, p = 0.14 with no sideways moves
  - Expected number of restarts?
  - Expected number of steps taken?
- If you want to pick one local search algorithm, learn this one!!

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Hill-climbing with random walk

- At each step do one of the two
  - Greedy: With prob p move to the neighbor with largest value
  - Random: With prob 1-p move to a random neighbor

Hill-climbing with both

- At each step do one of the three
  - Greedy: move to the neighbor with largest value
  - Random Walk: move to a random neighbor
  - Random Restart: Resample a new current state

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Simulated Annealing

- Simulated Annealing = physics inspired twist on random walk
- Basic ideas:
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the change in objective function is δ
  - if δ is positive, then move to that state
  - otherwise:
    - move to this state with probability proportional to δ
    - thus: worse moves (very large negative δ) are executed less often
  - however, there is always a chance of escaping from local maxima
  - over time, make it less likely to accept locally bad moves
  - (Can also make the size of the move random as well, i.e., allow "large" steps in state space)

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Physical Interpretation of Simulated Annealing

- A Physical Analogy:
  - imagine letting a ball roll downhill on the function surface
    - this is like hill-climbing (for minimization)
  - now imagine shaking the surface, while the ball rolls, gradually reducing the amount of shaking
    - this is like simulated annealing
- Annealing = physical process of cooling a liquid or metal until particles achieve a certain frozen crystal state
  - simulated annealing:
    - free variables are like particles
    - seek "low energy" (high quality) configuration
    - slowly reducing temp. T with particles moving around randomly

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Simulated annealing

function SIMULATED-ANNEALING(*problem, schedule*) **return** a solution state
  input: *problem*, a problem
    schedule, a mapping from time to temperature
  local variables: *current***,** a node.
    *next***,** a node.
    *T***,** a “temperature” controlling the prob. of downward steps

current ← MAKE-NODE(INITIAL-STATE[*problem*])
for t ← 1 to ∞ do
  T ← *schedule*[*t*]
  if $T = 0$ **then return** *current*
  next ← a randomly selected successor of *current*
  ΔE ← VALUE[*next*] - VALUE[*current*]
  if $\Delta E > 0$ **then** *current* ← *next*
  else *current* ← *next* only with probability $e^{\Delta E / T}$

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Temperature T

- high T: probability of “locally bad” move is higher
- low T: probability of “locally bad” move is lower
- typically, T is decreased as the algorithm runs longer
- i.e., there is a “temperature schedule”

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Simulated Annealing in Practice

- method proposed in 1983 by IBM researchers for solving VLSI layout problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).
  - theoretically will always find the global optimum
- Other applications: Traveling salesman, Graph partitioning, Graph coloring, Scheduling, Facility Layout, Image Processing, …
- useful for some problems, but can be very slow
  - slowness comes about because T must be decreased very gradually to retain optimality

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Local beam search

- Idea: Keeping only one node in memory is an extreme reaction to memory problems.
- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal → finished
  - Else select *k* best from successors and repeat

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Local Beam Search (contd)

- Not the same as *k random-start searches run in parallel!*
- Searches that find good states recruit other searches to join them
- Problem: quite often, all *k states end up on same local hill*
- Idea: Stochastic beam search
  - Choose *k successors randomly, biased towards good ones*
- Observe the close analogy to natural selection!

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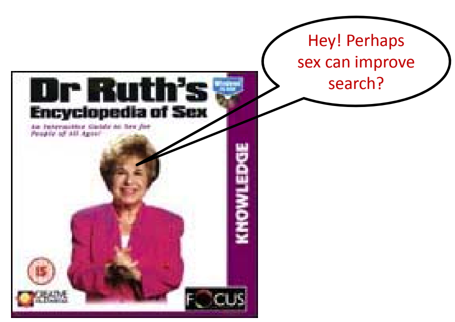

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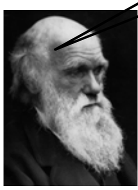
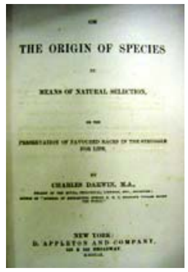
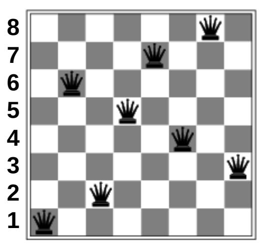
Sure! Check out ye book.





Genetic algorithms

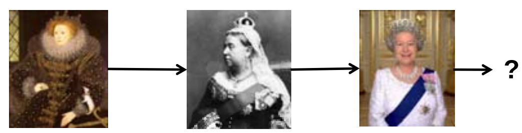
- Twist on Local Search: successor is generated by combining two parent states
- A state is represented as a string over a finite alphabet (e.g. binary)
  - 8-queens
    - State = position of 8 queens each in a column
- Start with *k* randomly generated states (population)
- Evaluation function (fitness function):
  - Higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
- Produce the next generation of states by "simulated evolution"
  - Random selection
  - Crossover
  - Random mutation



String representation
16257483

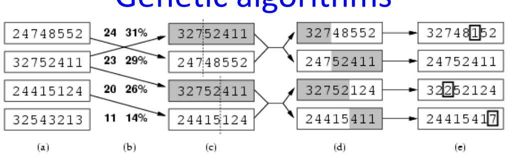
Can we evolve 8-queens through genetic algorithms?

Evolving 8-queens



Sorry!
Wrong queens

Genetic algorithms



4 states for 8-queens problem

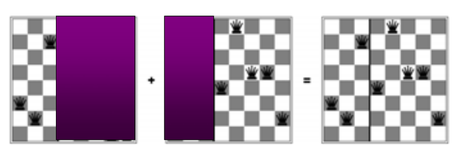
2 pairs of 2 states randomly selected based on fitness. Random crossover points selected

New states after crossover

Random mutation applied

- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms



Has the effect of "jumping" to a completely different new part of the search space (quite non-local)

Comments on Genetic Algorithms

- Genetic algorithm is a variant of "stochastic beam search"
- Positive points
  - Random exploration can find solutions that local search can't
    - (via crossover primarily)
  - Appealing connection to human evolution
    - "neural" networks, and "genetic" algorithms are **metaphors**!
- Negative points
  - Large number of "tunable" parameters
    - Difficult to replicate performance from one problem to another
  - Lack of good empirical studies comparing to simpler methods
  - Useful on some (small?) set of problems but no convincing evidence that GAs are better than hill-climbing w/random restarts in general

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Optimization of Continuous Functions

- Discretization
  - use hill-climbing
- Gradient descent
  - make a move in the direction of the gradient
    - gradients: closed form or empirical

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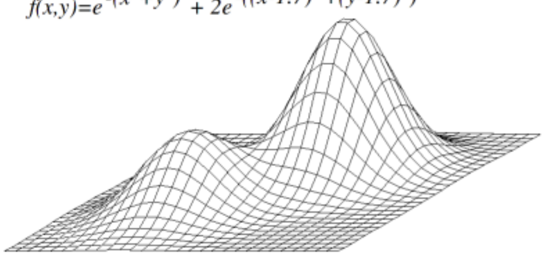
$$f(x,y)=e^{-(x^2+y^2)} + 2e^{-((x-1.7)^2+(y-1.7)^2)}$$



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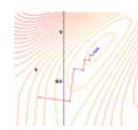
Gradient Descent

Assume we have a continuous function: $f(x_1,x_2,\ldots,x_N)$
and we want minimize over continuous variables X1,X2,..,Xn

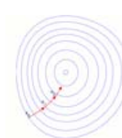


1. Compute the *gradients* for all i: $\partial f(x_1,x_2,\ldots,x_N)/\partial x_i$
2. Take a small step downhill in the direction of the gradient:

$$x_i \leftarrow x_i - \lambda \partial f(x_1,x_2,\ldots,x_N)/\partial x_i$$

3. Repeat.

- How to select λ
  - Line search: successively double
    - until f starts to increase again



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