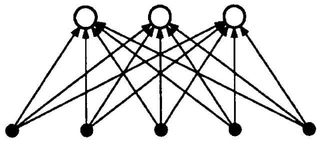
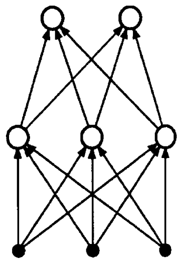
CSE 473

Lecture 26
(Chapter 18)

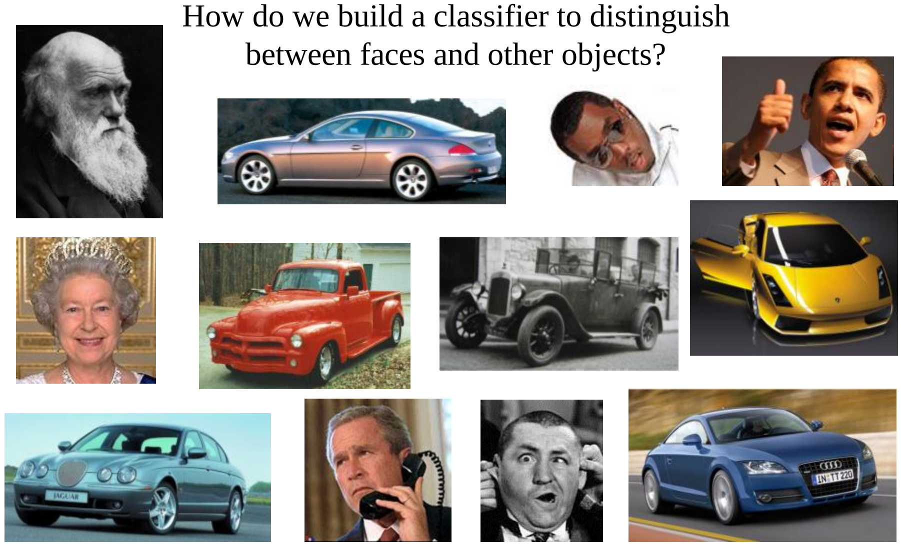
Neural Networks





Recall: Classification Problem

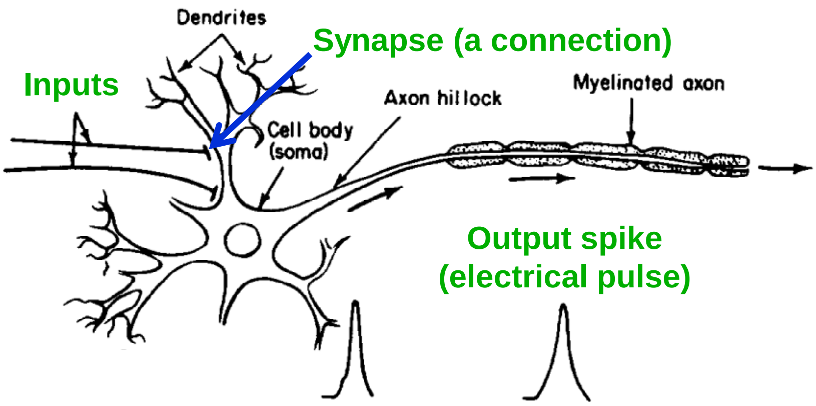
How do we build a classifier to distinguish between faces and other objects?



The human brain is extremely good at classifying images

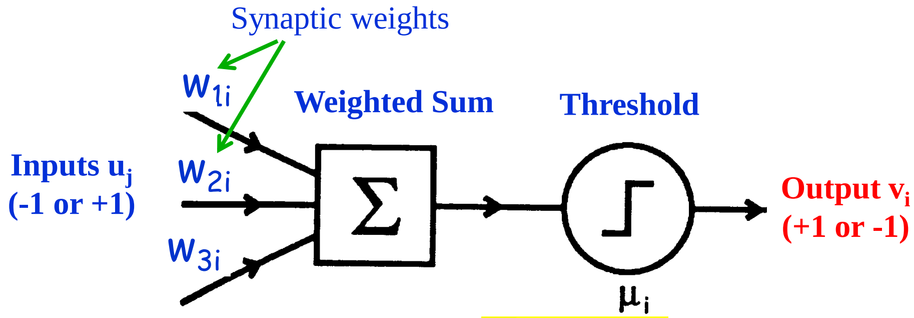
Can we develop classification methods by emulating the brain?

Neurons (Brain Cells)



Output spike roughly dependent on whether weighted sum of inputs reaches a threshold

Neurons as "Threshold Units"

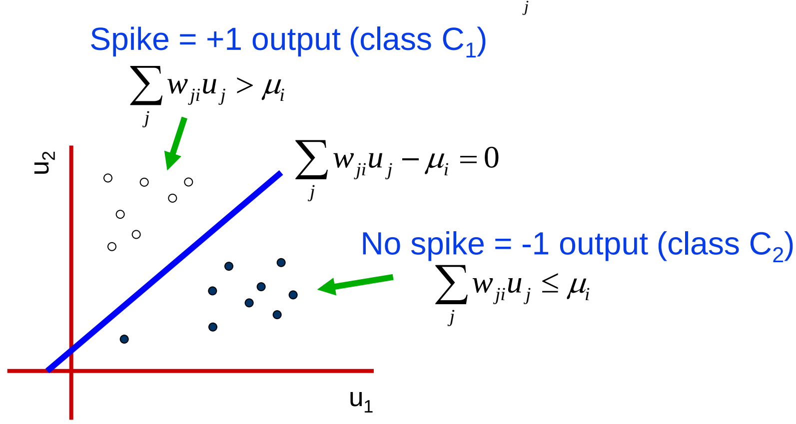


Output +1 if $\sum_j w_{ji} u_j > \mu_i$ and -1 otherwise

Artificial neuron "spikes" (output = +1) if weighted sum exceeds threshold

Neurons are Classifiers!

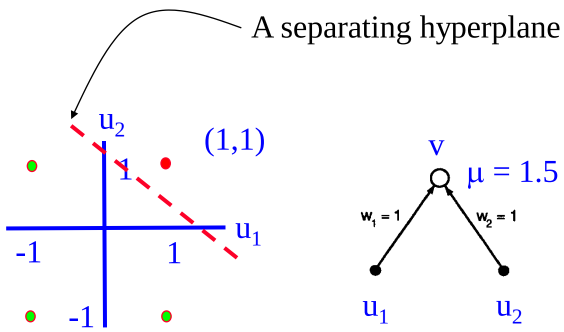
Each "neuron" defines a *hyperplane* $\sum_j w_{ji} u_j - \mu_i = 0$



Neurons can compute functions

Example: AND function

u_1	u_2	AND
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	1

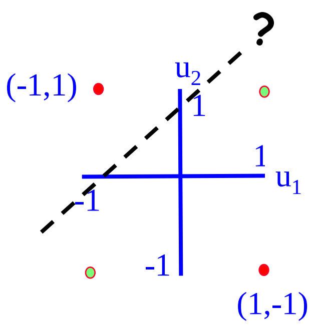


$v = 1$ iff $u_1 + u_2 - 1.5 > 0$

Similarly for OR and NOT

What about the XOR function?

u_1	u_2	XOR
-1	-1	-1
1	-1	1
-1	1	1
1	1	-1



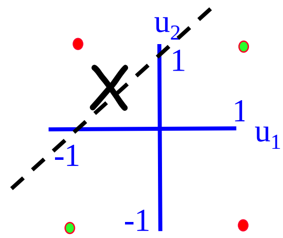
Can a neuron separate the +1 outputs from the -1 outputs?

Linear Inseparability

Artificial neuron with threshold fails if classification task is not linearly separable

- Example: XOR
- No single line can separate the "yes" (+1) outputs from the "no" (-1) outputs!

Minsky and Papert's book showing such negative results put a damper on neural networks research for over a decade!



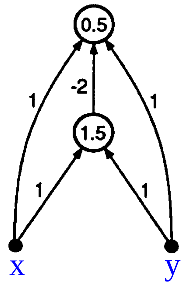
How do we deal with linear inseparability?

Multilayer Networks

Removes limitations of single-layer networks

- **Can solve XOR**

Example: Two-layer network that computes XOR



Output is +1 if and only if x + y – 2*(x + y > 1.5?) – 0.5 > 0

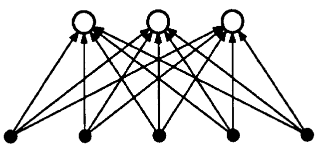
Perceptron

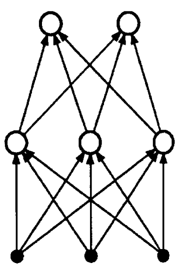
Fancy name for layered "feed-forward" network (no loops)

Network of artificial neurons ("units") with binary inputs and binary outputs (+1 or -1)

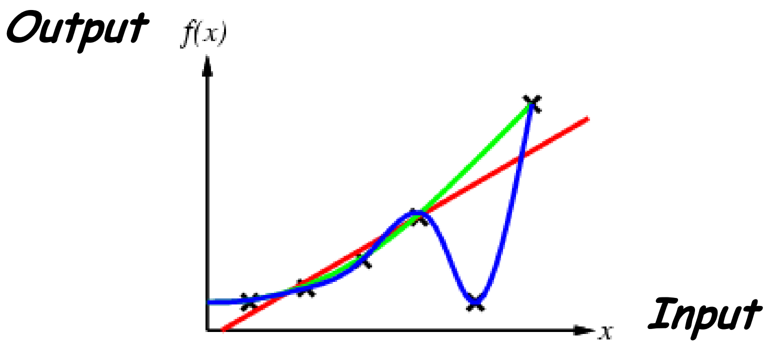
Multilayer

Single-layer





What if we want to learn continuous-valued functions?



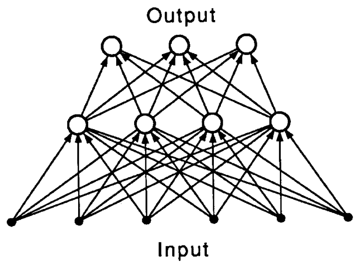
This is called "regression" (or curve fitting) in statistics

- E.g., Linear regression = fitting a line to a set of points

Regression using Neural Networks

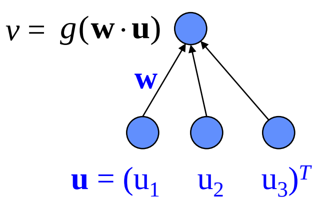
We want networks that can learn a function

- Network maps real-valued inputs to real-valued output



Continuous output values → Can't use binary threshold units anymore

Sigmoid Neurons

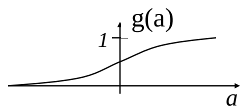


Output

Input nodes

Sigmoid output function:

$$g(a) = \frac{1}{1 + e^{-\beta a}}$$



Non-linear "squashing" function: Squashes input to be between 0 and 1. Parameter β controls the slope..

Learning the weights

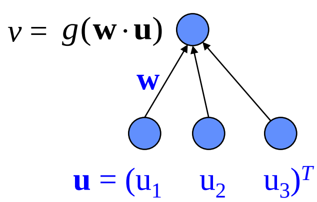
Given: Training data (input **u**, desired output **d**)

Problem: How do we learn the weights **w**?

Idea: *Minimize squared error* between network's output and desired output:

$$E(\mathbf{w}) = (d - v)^2$$

where $v = g(\mathbf{w} \cdot \mathbf{u})$



Starting from random values for **w**, want to change **w** so that *E*(**w**) is minimized – How?

Learning by Gradient-Descent
(opposite of "Hill-Climbing")

Change w so that *E*(w) is minimized

- Use Gradient Descent: Change w in proportion to −d*E*/dw (why?)

$$\mathbf{w} \rightarrow \mathbf{w} - \varepsilon \frac{dE}{d\mathbf{w}}$$

Derivative of sigmoid

$$\frac{dE}{d\mathbf{w}} = -2(d-v)\frac{dv}{d\mathbf{w}} = -2\underbrace{(d-v)}_{\text{delta = error}}g'(\mathbf{w}\cdot\mathbf{u})\mathbf{u}$$

Also known as the "delta rule" or "LMS (least mean square) rule"

But wait!

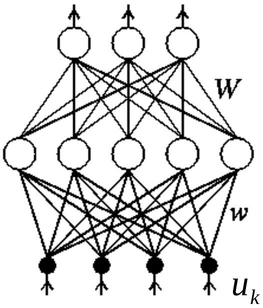
This rule is for a one layer network

- **One layer networks are not that interesting!! (remember XOR?)**

What if we have multiple layers?

Learning Multilayer Networks

$$v_i = g(\sum_j W_{ji} g(\sum_k w_{kj} u_k))$$



Start with random weights **W**, **w**

Given input vector **u**, network produces output vector **v**

Use gradient descent to find **W** and **w** that minimize total error over all output units (labeled *i*):

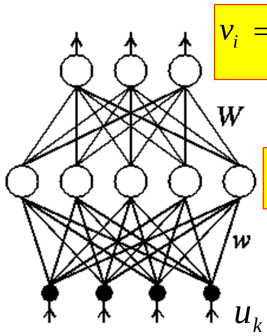
$$E(\mathbf{W},\mathbf{w}) = \frac{1}{2}\sum_i (d_i - v_i)^2$$

This leads to the famous "Backpropagation" learning rule

Backpropagation: Output Weights

$$E(\mathbf{W},\mathbf{w}) = \frac{1}{2}\sum_i (d_i - v_i)^2$$

$$\boxed{v_i = g(\sum_j W_{ji} x_j)}$$



Learning rule for <u>hidden-output weights W</u>:

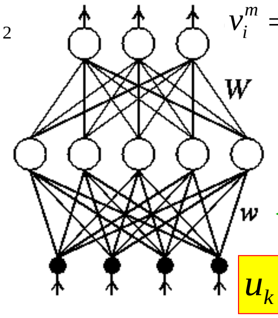
$$W_{ji} \rightarrow W_{ji} - \varepsilon \frac{dE}{dW_{ji}} \qquad \{\text{gradient descent}\}$$

$$\frac{dE}{dW_{ji}} = -(d_i - v_i) g'(\sum_j W_{ji} x_j) x_j \qquad \{\text{delta rule}\}$$

Backpropagation: Hidden Weights

$$E(\mathbf{W},\mathbf{w}) = \frac{1}{2}\sum_i (d_i - v_i)^2$$

$$v_i^m = g(\sum_j W_{ji} x_j)$$



$$\boxed{x_j = g(\sum_k w_{kj} u_k)}$$

Learning rule for input-hidden weights w:

$$w_{kj} \rightarrow w_{kj} - \varepsilon \frac{dE}{dw_{kj}} \quad \text{But}: \frac{dE}{dw_{kj}} = \frac{dE}{dx_j} \cdot \frac{dx_j}{dw_{kj}} \quad \{\text{chain rule}\}$$

$$\frac{dE}{dw_{kj}} = \left[-\sum_i (d_i - v_i) g'(\sum_j W_{ji} x_j) W_{ji} \right] \cdot \left[g'(\sum_k w_{kj} u_k) u_k \right]$$

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Next Time

- Wrap up of machine learning
  - Learning to drive using neural networks
  - Ensemble learning
- To Do:
  - Project 4 due this Wednesday!
  - Read Chapter 18

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