Assignment 3
CSE 473 Autumn 2010

November 5, 2010

The assignment is graded out of 100 points and is due November 24 on paper in class. Please type your answers or print clearly.

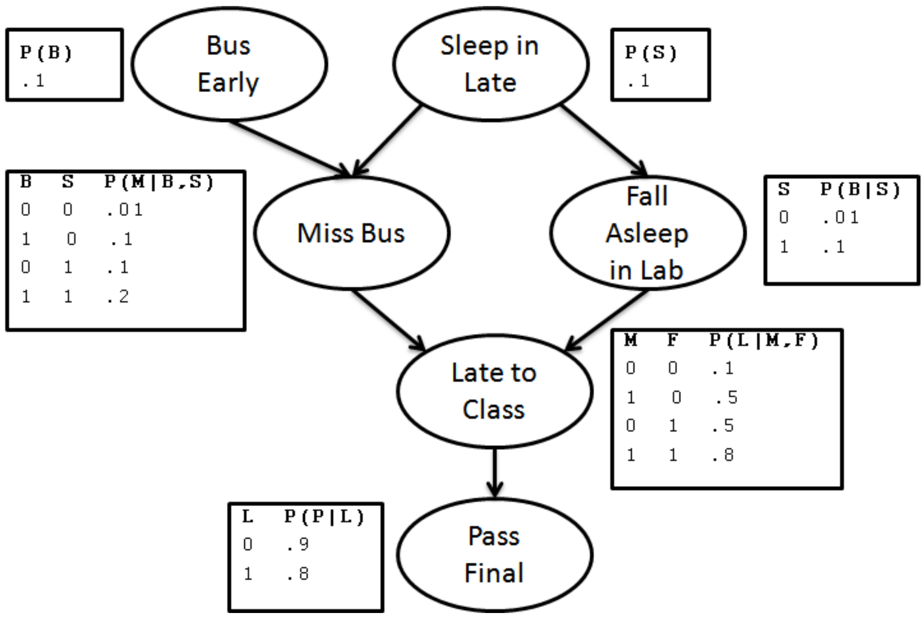
Uncertainty (40 Points)

Problem 1. (20 Points) According to extremely reliable sources (Wikipedia), 78% of email is spam. According to experiments conducted on my own inbox, 11% of spam email messages contain the word "Pills". In comparison, only 1% of non-spam email messages contain this word.
A. (5 Points) What is the probability that a message contains the word "Pills" and is spam?
B. (5 Points) What is the probability that a message does not contain the word "Pills"?
C. (5 Points) What is the probability that a message does not contain the word "Pills" or is spam?
D. (5 Points) Given that a message contains the word "Pills", what is the probability that it is spam?

Problem 2. (10 Points) One definition of independence is that a and b are independent if $P(a|b) = P(a)$. Another is that $P(a \wedge b) = P(a)P(b)$. Show these two definitions are equivalent.

Problem 3. (10 Points) (From R&N 13.21, adapted from Pearl 1988.) Suppose you are a witness to a night-time hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? If it is possible, what is the most likely color and how did you determine this? If it is not possible, why?

Bayesian Networks (50 points)



Problem 4. (25 Points) Use the Bayesian network above
A. (5 Points) Is “Bus Early”independent of “Pass Final” given “Late to Class”?
B. (5 Points) Is “Fall Asleep in Lab” independent of “Miss Bus”?
C. (5 Points) Is “Fall Asleep in Lab” conditionally independent of “Miss Bus” given “Sleep in Late”?
D. (5 Points) Is “Sleep in Late” independent of “Bus Early”?
E. (5 Points) Is “Sleep in Late” conditionally independent of “Bus Early” given “Miss Bus”?

Problem 5. (15 points) The first step in converting a directed Bayesian network to an undirected Markov network is creating the moral graph. The moral graph is an undirected graph in which nodes in the original Bayesian network are connected to every node in their Markov blanket.
A. (5 points) Draw the moral graph for the Bayesian network above.
B. (10 points) Is there a one-to-one mapping between Bayesian networks and their moral graph counterparts? Why or why not? What does this imply about converting a Bayesian network into a Markov network?

Problem 6. (5 points) Let’s say we are using Likelihood-Weighting to compute the probability distribution for “Late to Class” given that “Fall Asleep in Lab” is $True$ and “Bus Early” is $False$. What would be the weight be for a sample with the following values:

$$\begin{aligned} \text{Bus Early} &= False \\ \text{Sleep in Late} &= True \\ \text{Miss Bus} &= False \\ \text{Fall Asleep in Lab} &= True \\ \text{Late to Class} &= False \\ \text{Pass Final} &= True \end{aligned}$$

Problem 7. (5 points) Let's say we are using Gibbs sampling to compute the probability distribution for "Fall Asleep in Lab" given that "Bus Early" is $True$. Our current sample has the following values

$$\begin{aligned} \text{Bus Early} &= True \\ \text{Sleep in Late} &= False \\ \text{Miss Bus} &= True \\ \text{Fall Asleep in Lab} &= False \\ \text{Late to Class} &= False \\ \text{Pass Final} &= True \end{aligned}$$

Let's say our sampler selects "Miss Bus". What is the probability we flip the value for this variable? *Hint:* the text book has a formula that will help you out.

Markov Logic (10 points)

Problem 8. (10 points) Consider the Markov logic network given by the following formulas, each with a weight of 1

$$\begin{aligned} \forall x, y\; Mother(x, y) &\Rightarrow Parent(x, y) \\ \forall x, y\; Mother(x, y) &\Rightarrow Female(x) \\ \forall x, y\; Parent(x, y) \wedge \neg Mother(x, y) &\Rightarrow Father(x, y) \end{aligned}$$

Draw the Markov network defined by this Markov logic network for the ground set $\{Tom, Susan\}$. When grounding the predicates $Mother(x, y)$, $Parent(x, y)$ and $Father(x, y)$, skip all groundings for which $x = y$.