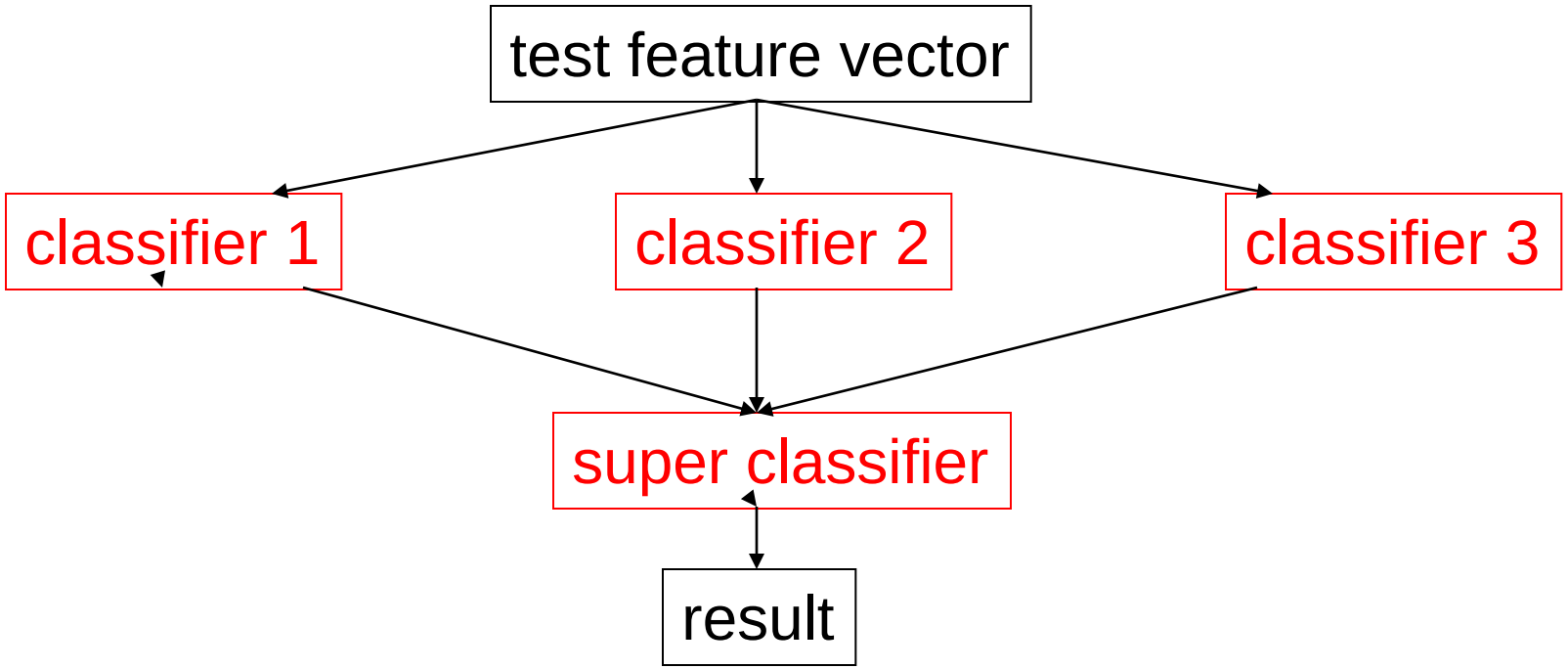
Ensembles

- An ensemble is a set of classifiers whose combined results give the final decision.

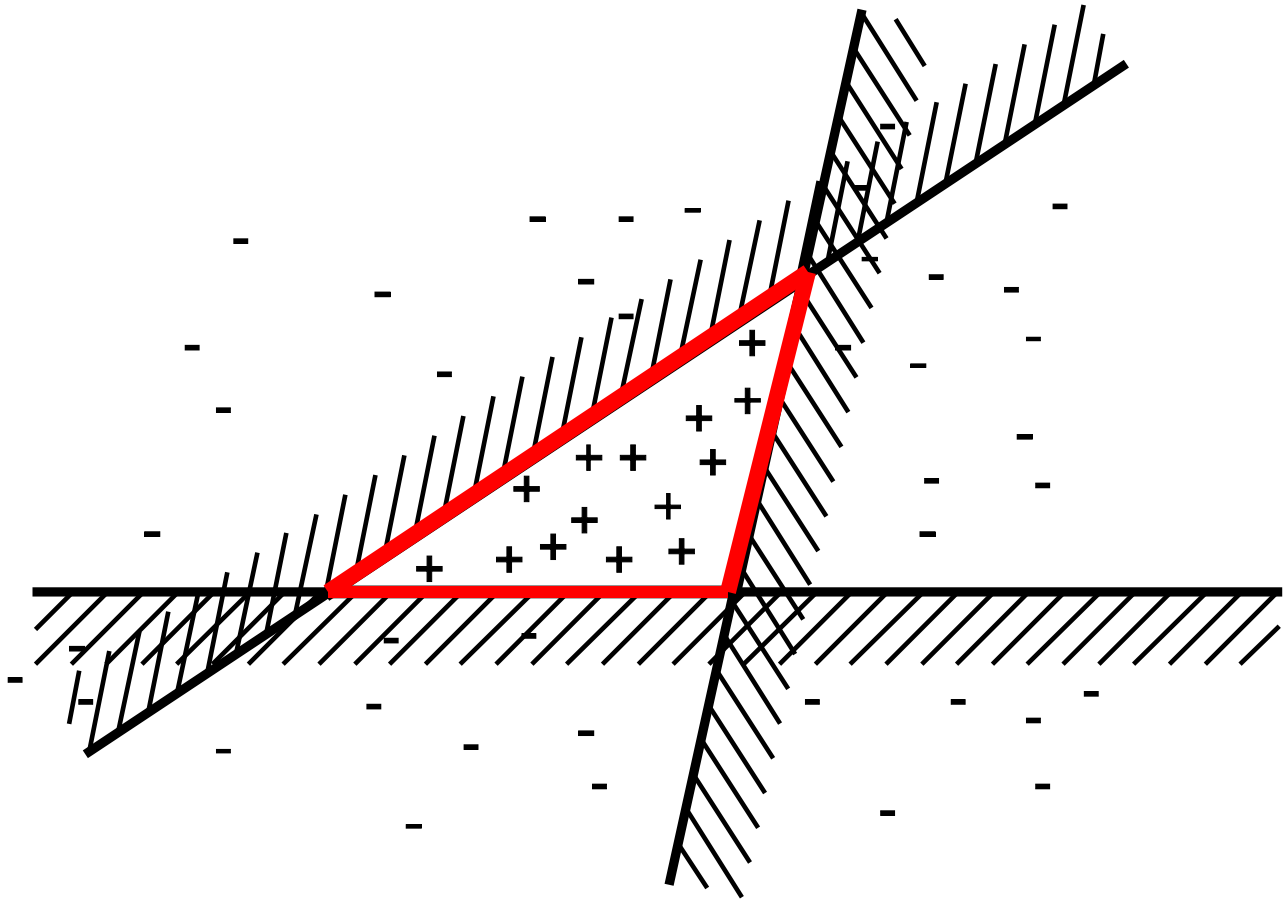


Model* Ensembles

- **Basic idea:**
  Instead of learning one model,
  Learn several and combine them
- Typically improves accuracy, often by a lot
- **Many methods:**
  - Bagging
  - Boosting
  - ECOC (error-correcting output coding)
  - Stacking
  - Etc.

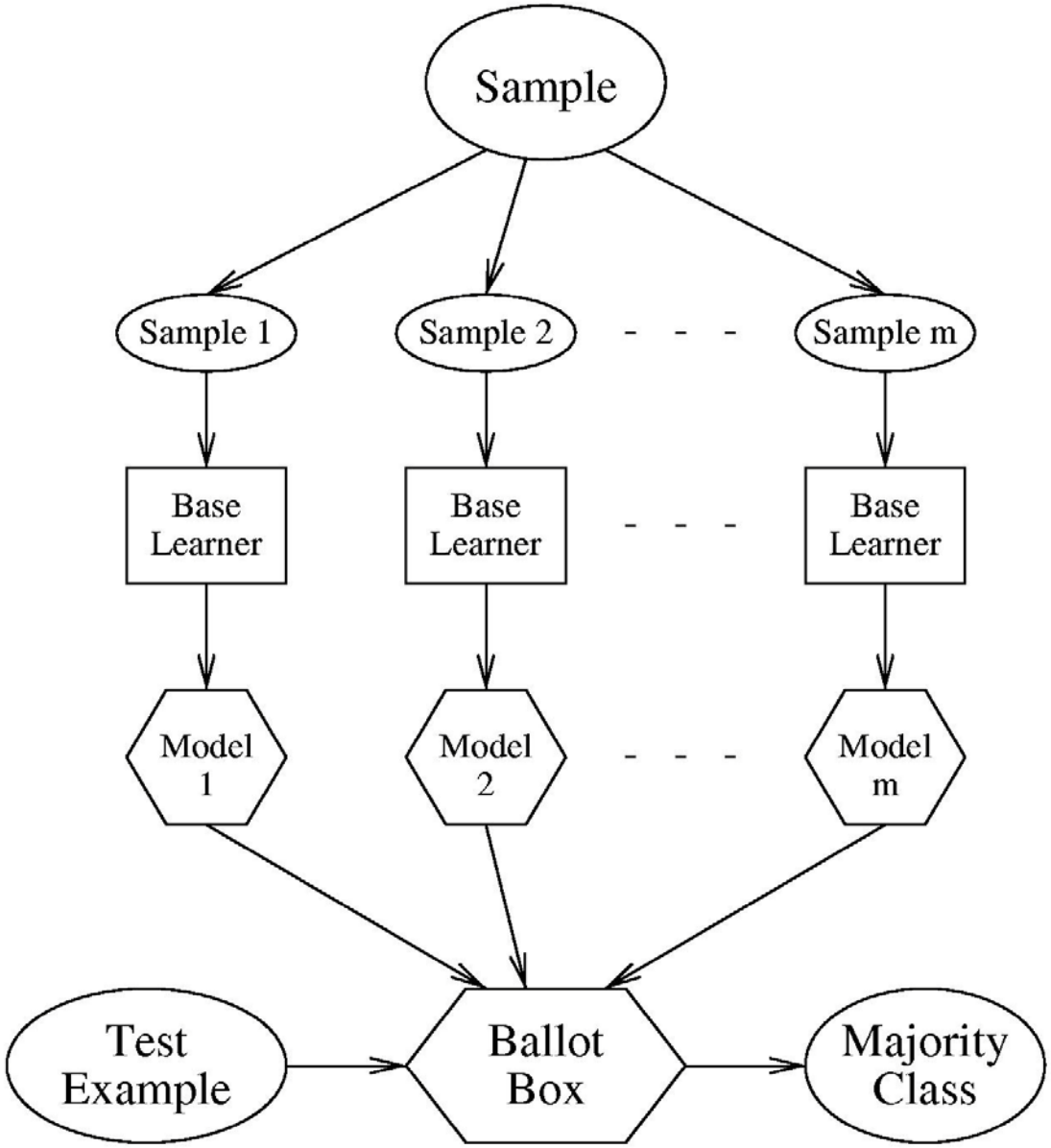
*A model is the learned decision rule. It can be as simple as a hyperplane in n-space (ie. a line in 2D or plane in 3D) or in the form of a decision tree or other modern classifier.

Majority Vote for Several Linear Models



Bagging

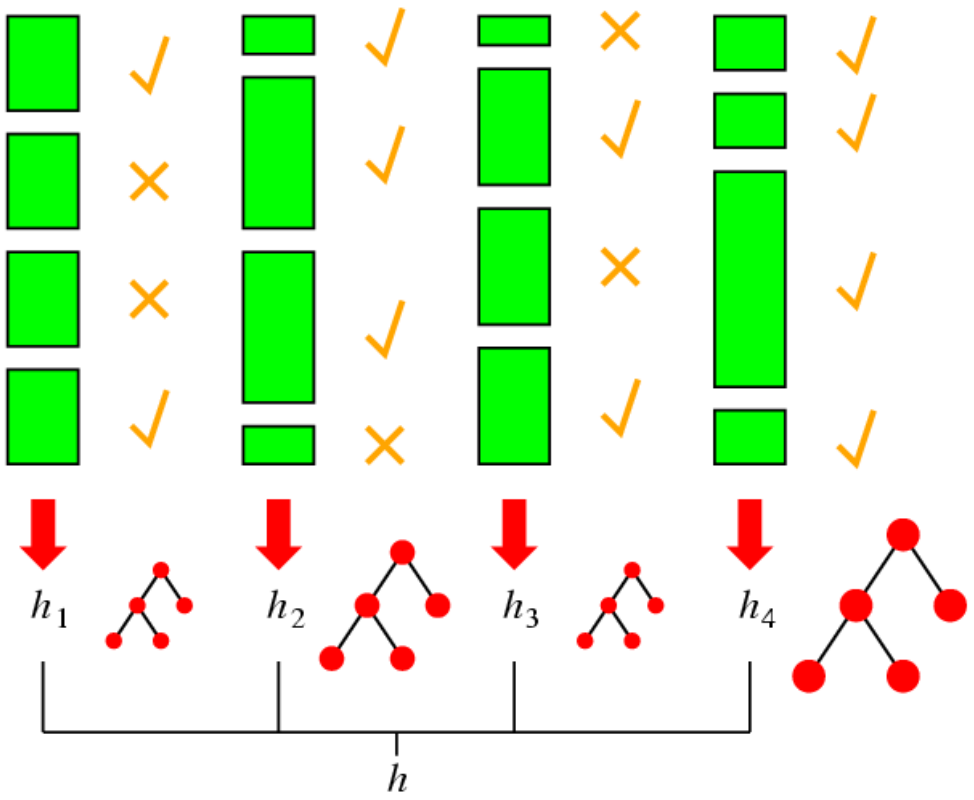
- Generate "bootstrap" replicates of training set by sampling with replacement
- Learn one model on each replicate
- Combine by uniform voting


Sample
Sample 1
Sample 2
Sample m
Base Learner
Base Learner
Base Learner
Model 1
Model 2
Model m
Test Example
Ballot Box
Majority Class

Boosting

- Maintain vector of weights for examples
- Initialize with uniform weights
- Loop:
  - Apply learner to weighted examples (or sample)
  - Increase weights of misclassified examples
- Combine models by weighted voting

Idea of Boosting



Boosting In More Detail
(Pedro Domingos' Algorithm)

1. Set all E weights to 1, and learn H1.
2. Repeat m times: increase the weights of misclassified Es, and learn H2,…Hm.
3. H1..Hm have "weighted majority" vote when classifying each test
   Weight(H)=accuracy of H on the training data

ADABoost

- ADABoost boosts the accuracy of the original learning algorithm.

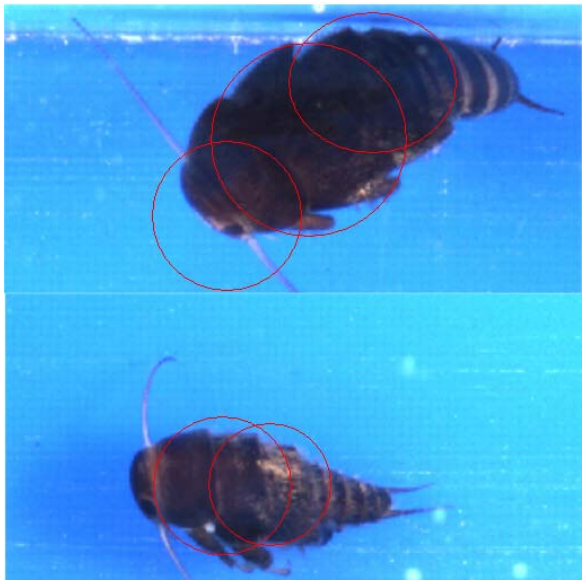
- If the original learning algorithm does slightly better than 50% accuracy, ADABoost with a large enough number of classifiers is guaranteed to classify the training data perfectly.

ADABoost Weight Updating

```
for j = 1 to N do /* go through training samples */
   if h[m](xj) <> yj then error <- error + wj

for j = 1 to N do
   if h[m](xj) = yj then w[j] <- w[j] * error/(1-error)
```

Sample Application: Insect Recognition



Using circular regions of interest selected by an interest operator, train a classifier to recognize the different classes of insects.

Boosting Comparison

- **ADTree classifier only** **(alternating decision tree)**

- Correctly Classified Instances 268 70.1571 %
- Incorrectly Classified Instances 114 29.8429 %
- Mean absolute error 0.3855
- Relative absolute error 77.2229 %

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	167	28
Real Doroneuria	51	136

Boosting Comparison

AdaboostM1 with ADTree classifier

- Correctly Classified Instances 303 79.3194 %
- Incorrectly Classified Instances 79 20.6806 %
- Mean absolute error 0.2277
- Relative absolute error 45.6144 %

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	167	28
Real Doroneuria	51	136

Boosting Comparison

- **RepTree classifier only** **(reduced error pruning)**

- Correctly Classified Instances 294 75.3846 %
- Incorrectly Classified Instances 96 24.6154 %
- Mean absolute error 0.3012
- Relative absolute error 60.606 %

Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	169	41
Real Doroneuria	55	125

Boosting Comparison

AdaboostM1 with RepTree classifier

- Correctly Classified Instances 324 83.0769 %
- Incorrectly Classified Instances 66 16.9231 %
- Mean absolute error 0.1978
- Relative absolute error 39.7848 %

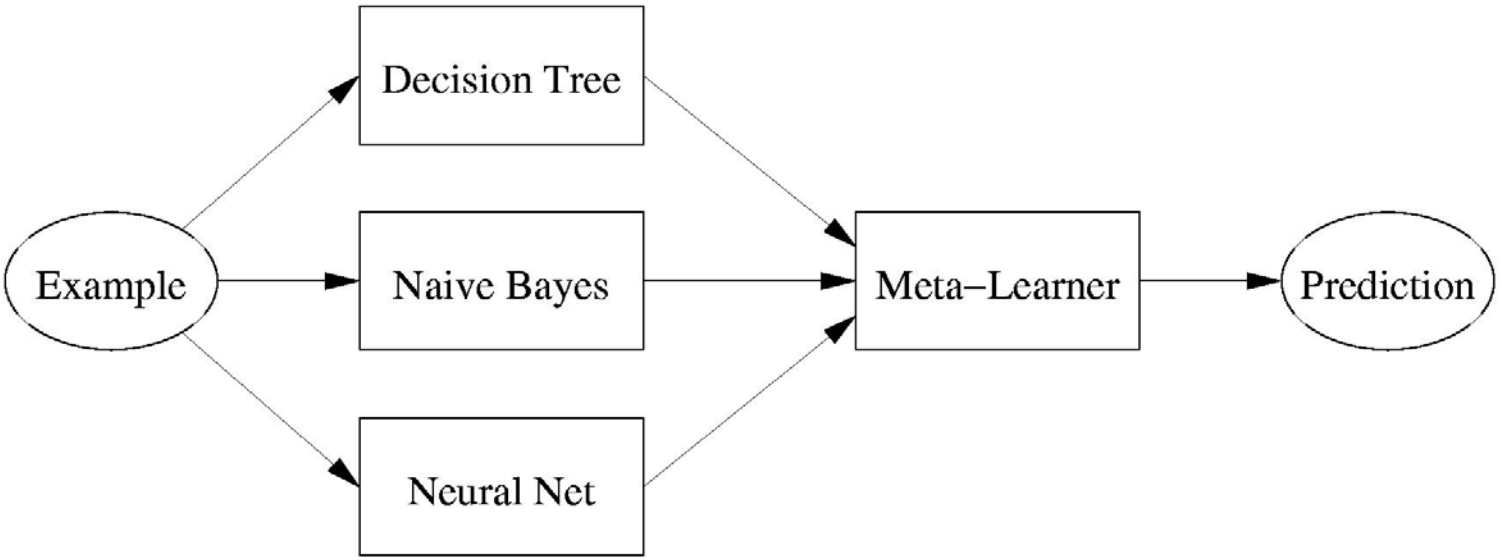
Classified as ->	Hesperperla	Doroneuria
Real Hesperperlas	180	30
Real Doroneuria	36	144

References

- **AdaboostM1**: Yoav Freund and Robert E. Schapire (1996). "Experiments with a new boosting algorithm". Proc International Conference on Machine Learning, pages 148-156, Morgan Kaufmann, San Francisco.
- **ADTree**: Freund, Y., Mason, L.: "The alternating decision tree learning algorithm". Proceeding of the Sixteenth International Conference on Machine Learning, Bled, Slovenia, (1999) 124-133.

Stacking

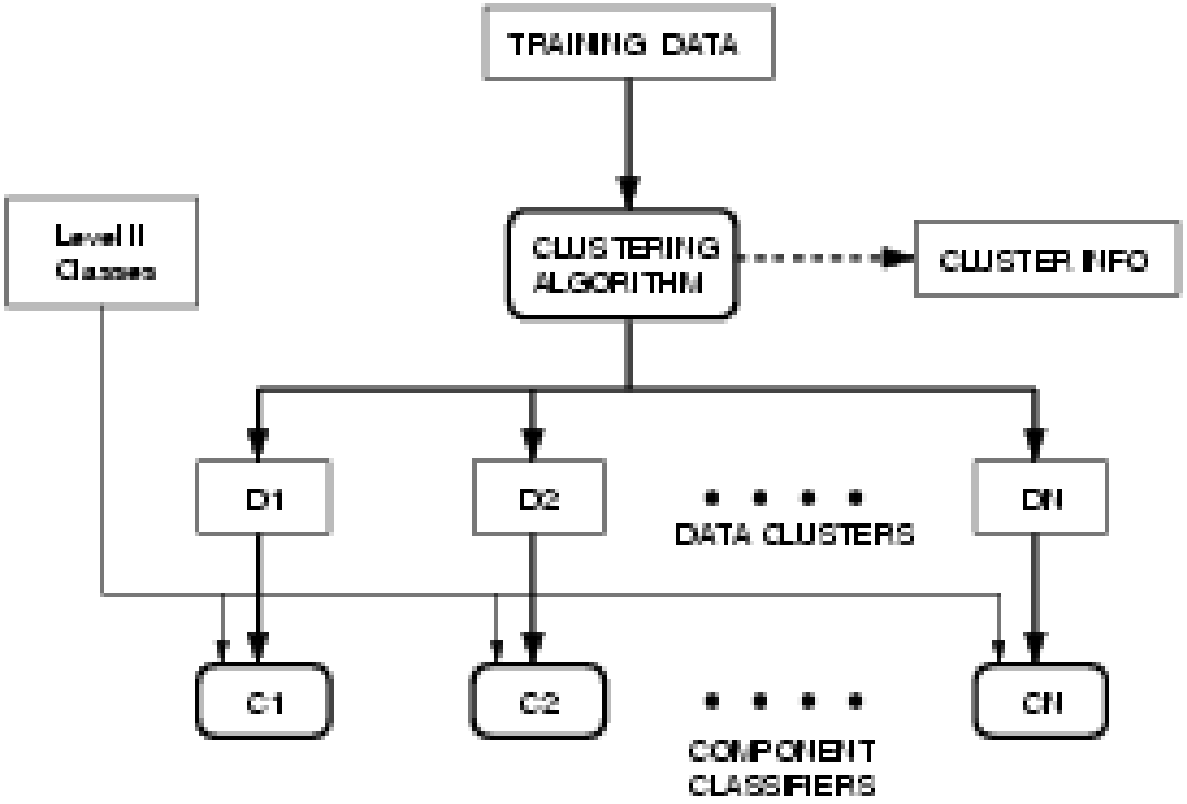
- Apply multiple base learners
  (e.g.: decision trees, naive Bayes, neural nets)
- Meta-learner: Inputs = Base learner predictions
- Training by leave-one-out cross-validation:
  Meta-L. inputs = Predictions on left-out examples



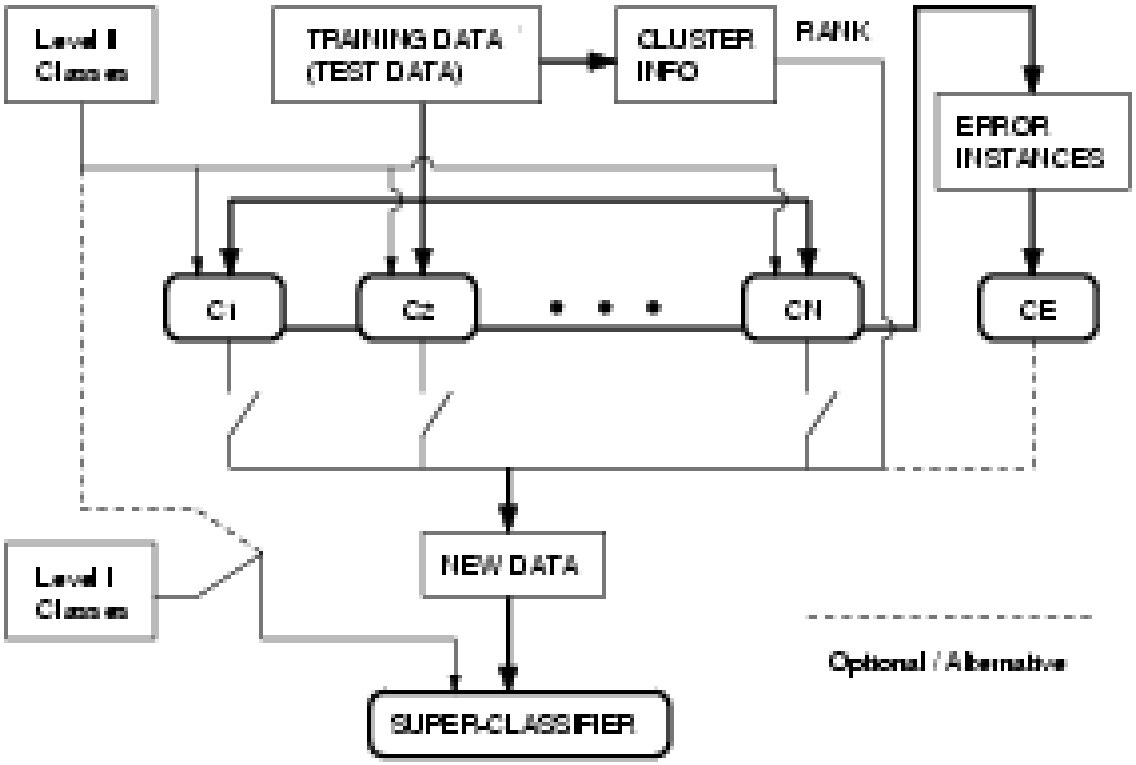
Yu-Yu Chou's Hierarchical Classifiers

- Developed for pap smear analysis in which the categories were normal, abnormal (cancer), and artifact plus subclasses of each
- More than 300 attributes per feature vector and little or no knowledge of what they were.
- Large amount of training data making classifier construction slow or impossible.

Training


TRAINING DATA
Level II Classes
CLUSTERING ALGORITHM
CLUSTER INFO
D1
D2
DN
DATA CLUSTERS
C1
C2
CN
COMPONENT CLASSIFIERS

Classification



Results

- Our classifier was able to beat the handcrafted decision tree classifier that had taken Neopath years to develop.
- It was tested successfully on another pap smear data set and a forest cover data set.
- It was tested against bagging and boosting. It was better at detecting abnormal pap smears than both, and not as good at classifying normal ones as normal. It was slightly higher than both in overall classification rate.

Bayesian Learning

- Bayes' Rule provides a way to calculate probability of a hypothesis based on
  - its prior probability
  - the probability of observing the data, given that hypothesis
  - the observed data (feature vector)

Bayes' Rule

$$P(h \mid X) = \frac{P(X \mid h)\ P(h)}{P(X)}$$

Often assumed constant and left out.

- h is the hypothesis (such as the class).
- X is the feature vector to be classified.
- P(X | h) is the prior probability that this feature vector occurs, given that h is true.
- P(h) is the prior probability of hypothesis h.
- P(X) = the prior probability of the feature vector X.
- These priors are usually calculated from frequencies in the training data set.

Example

x1	x2	x3	y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

- Suppose we want to know the probabilty of class 1 for feature vector [0,1,0].

- P(1 | [0,1,0]) = P([0,1,0] | 1) P(1) / P([0,1,0])
  = (0.25) (0.5) / (.125)
  = 1.0

Of course the training set would be much bigger and for real data could include multiple instances of a given feature vector.

MAP

- Suppose H is a set of candidate hypotheses.
- We would like to find the most probable h in H.
- h_{MAP} is a MAP (maxiumum a posteriori) hypothesis if

$$h_{MAP} = \underset{h \in H}{\text{argmax}} \; P(h \mid X)$$

- This just says to calculate P(h | X) by Bayes' rule for each possible class h and take the one that gets the highest score.

Cancer Test Example

P(cancer) = .008
P(not cancer) = .992
P(positive | cancer) = .98
P(positive | not cancer) = .03
P(negative | cancer) = .02
P(negative | not cancer) =.97

Priors

New patient's test comes back positive.

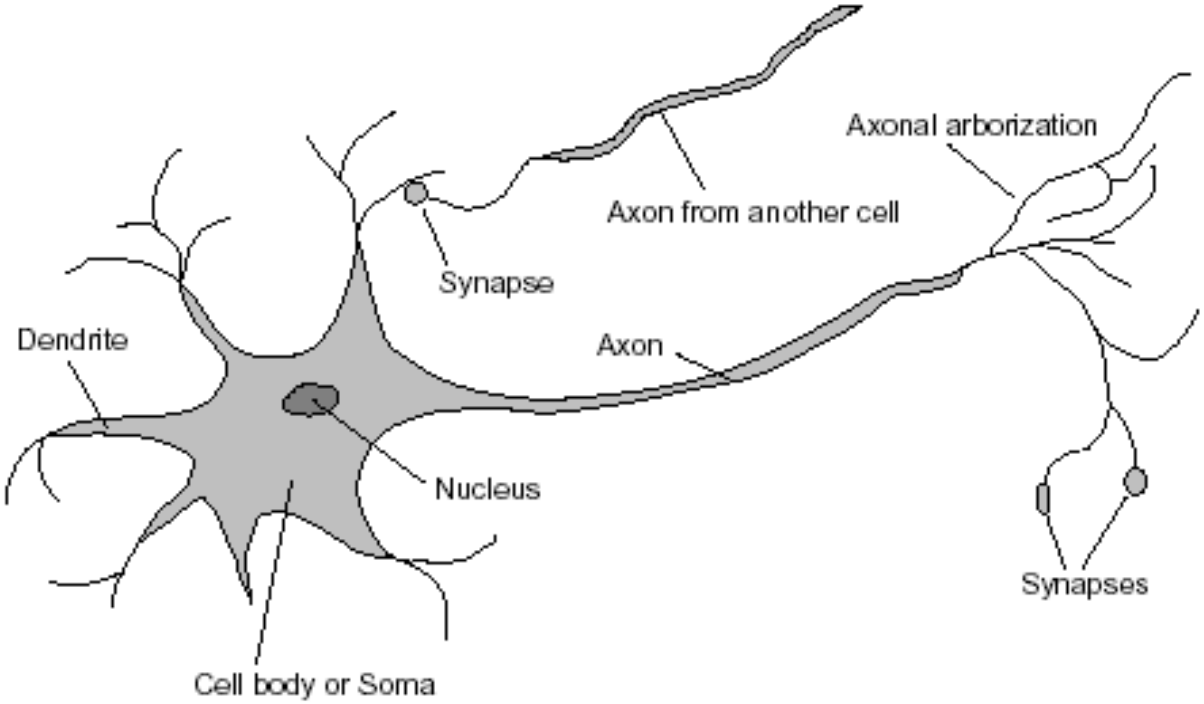
P(cancer | positive) = P(positive | cancer) P(cancer)
= (.98) (.008) = .0078
P(not cancer | positive = P(positive | not cancer) P(not cancer)
= (.03) (.992) = .0298
h_{MAP} would say it's not cancer. Depends strongly on priors!

Neural Net Learning

- Motivated by studies of the brain.
- A network of “artificial neurons” that learns a function.
- Doesn’t have clear decision rules like decision trees, but highly successful in many different applications. (e.g. face detection)
- Our hierarchical classifier used neural net classifiers as its components.

Brains

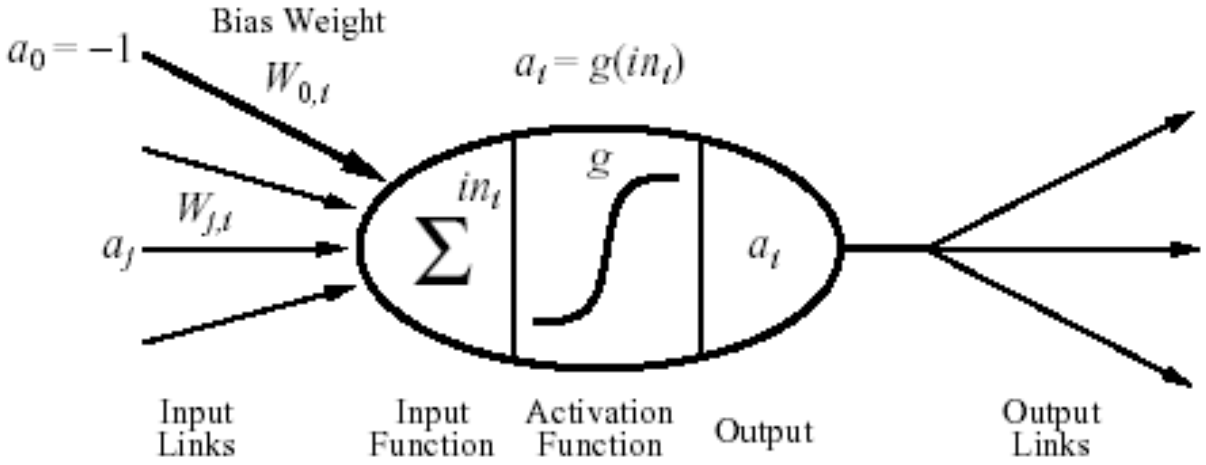
10^{11} neurons of > 20 types, 10^{14} synapses, 1ms–10ms cycle time
Signals are noisy "spike trains" of electrical potential



McCulloch–Pitts "unit"

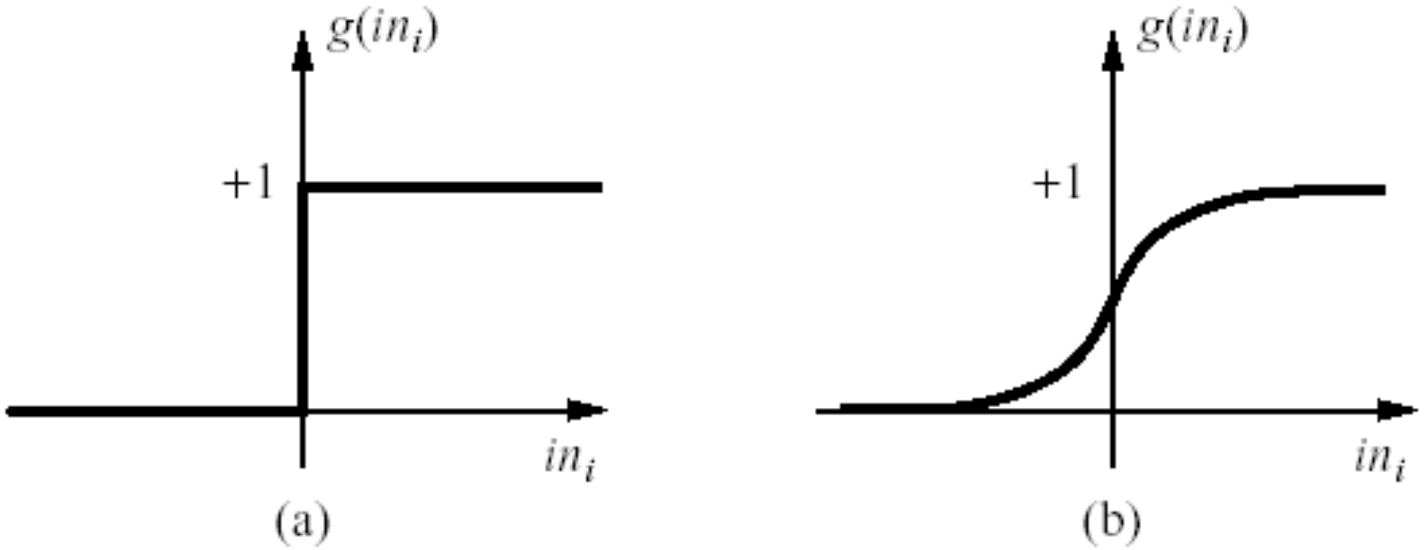
Output is a "squashed" linear function of the inputs:

$$a_i \leftarrow g(in_i) = g\left(\sum_j W_{j,i} a_j\right)$$



A gross oversimplification of real neurons, but its purpose is to develop understanding of what networks of simple units can do

Activation functions

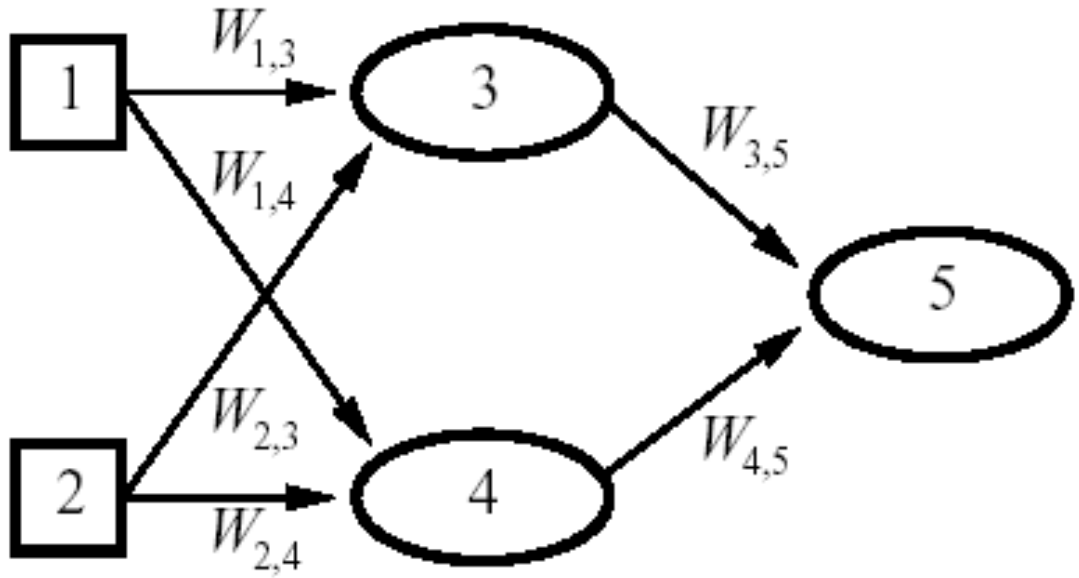


(a) is a step function or threshold function

(b) is a sigmoid function $1/(1+e^{-x})$

Changing the bias weight $W_{0,i}$ moves the threshold location

Feed-forward example



Feed-forward network = a parameterized family of nonlinear functions:

$$\begin{aligned} a_5 &= g(W_{3,5} \cdot a_3 + W_{4,5} \cdot a_4) \\ &= g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2)) \end{aligned}$$

Adjusting weights changes the function: do learning this way!

Perceptron learning

Learn by adjusting weights to reduce error on training set

The squared error for an example with input $\mathbf{x}$ and true output y is

$$E = \frac{1}{2}Err^2 \equiv \frac{1}{2}(y - h_{\mathbf{W}}(\mathbf{x}))^2 \ ,$$

Perform optimization search by gradient descent:

$$\begin{aligned}\frac{\partial E}{\partial W_j} &= Err \times \frac{\partial Err}{\partial W_j} = Err \times \frac{\partial}{\partial W_j}\left(y - g(\Sigma_{j=0}^{n} W_j x_j)\right) \\ &= -Err \times g'(in) \times x_j\end{aligned}$$

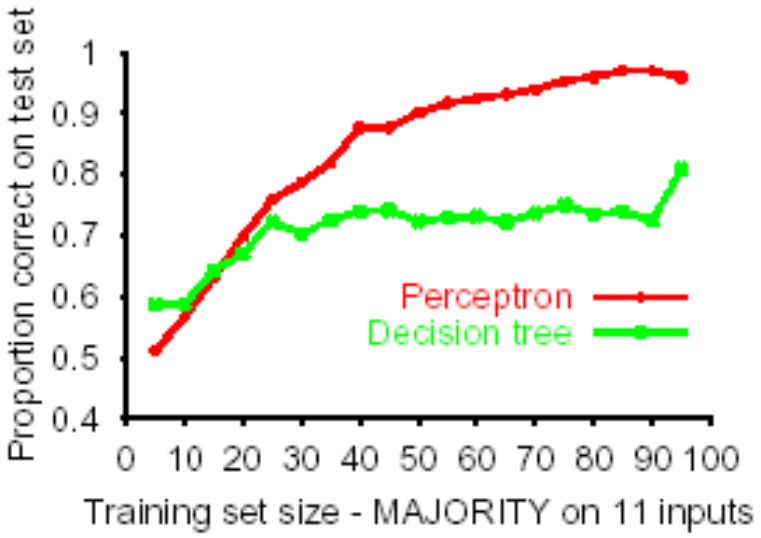
Simple weight update rule:

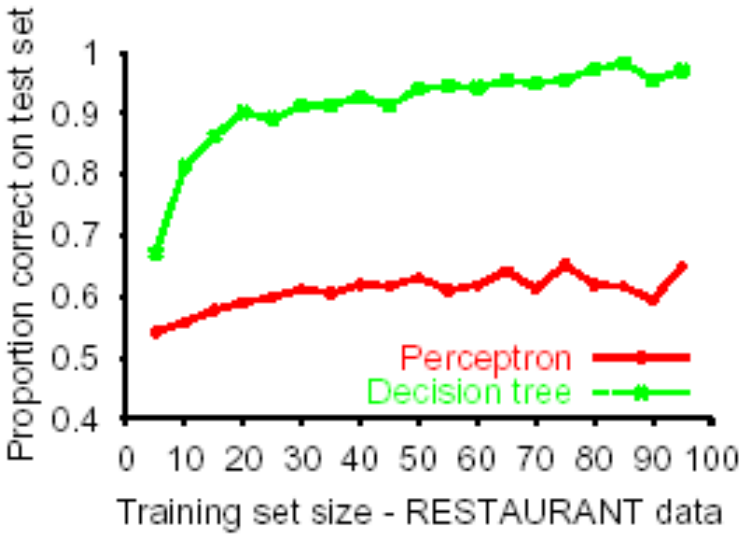
$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

E.g., +ve error $\Rightarrow$ increase network output
$\Rightarrow$ increase weights on +ve inputs, decrease on -ve inputs

Perceptron learning contd.

Perceptron learning rule converges to a consistent function
for any linearly separable data set



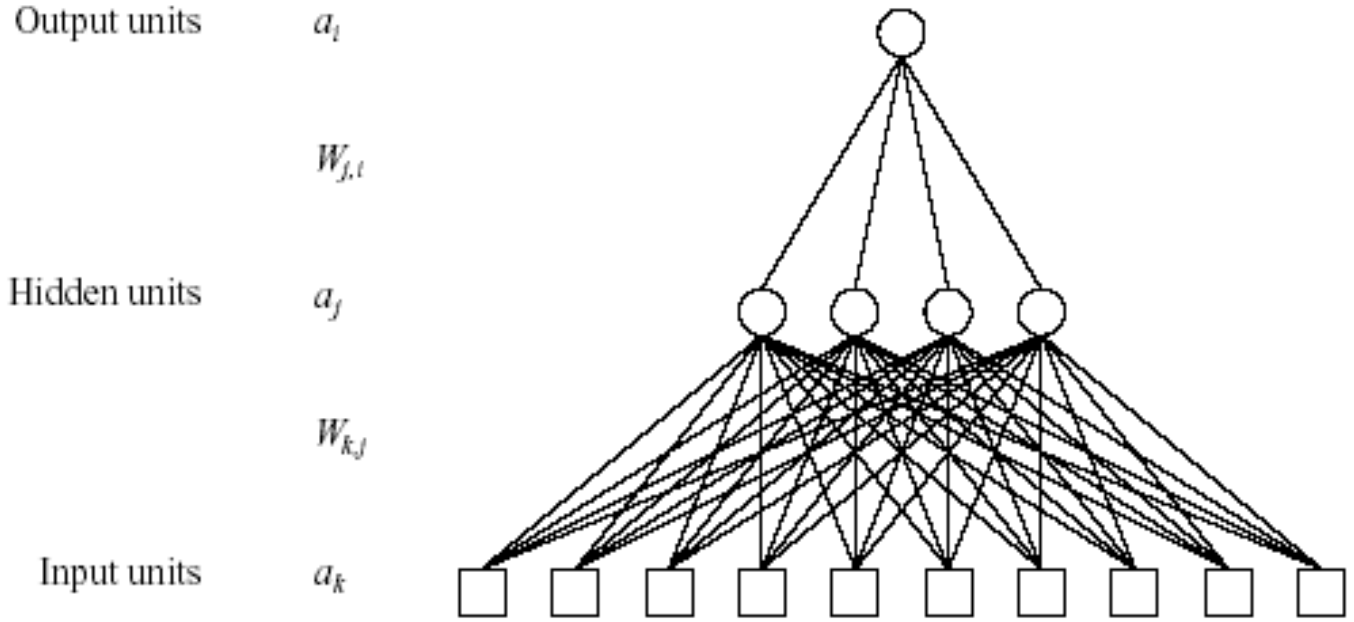


Perceptron learns majority function easily, DTL is hopeless

DTL learns restaurant function easily, perceptron cannot represent it

Multilayer perceptrons

Layers are usually fully connected;
numbers of hidden units typically chosen by hand



Back-propagation learning

Output layer: same as for single-layer perceptron,

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

where $\Delta_i = Err_i \times g'(in_i)$

Hidden layer: **back-propagate** the error from the output layer:

$$\Delta_j = g'(in_j) \sum_i W_{j,i} \Delta_i \ .$$

Update rule for weights in hidden layer:

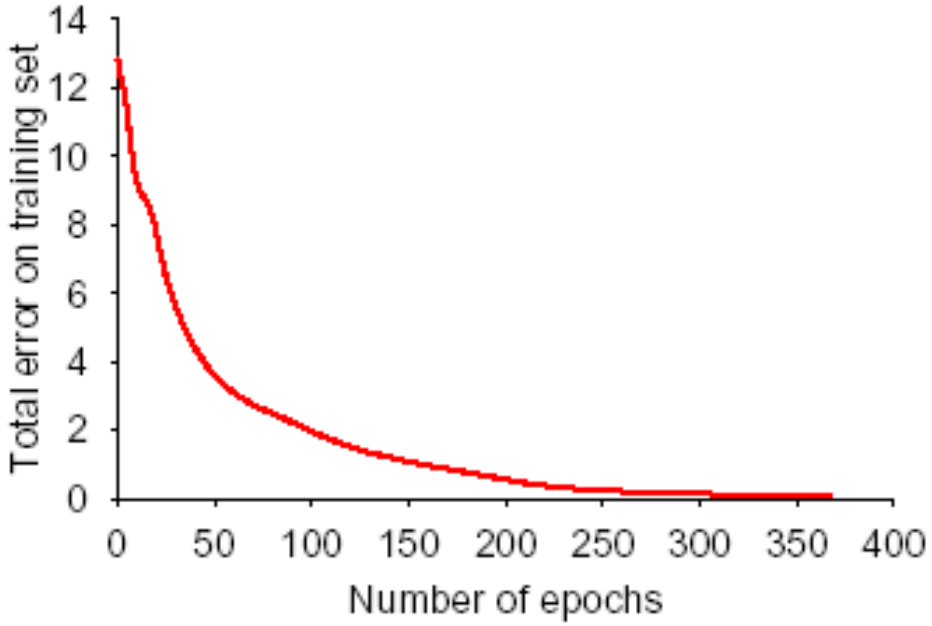
$$W_{k,j} \leftarrow W_{k,j} + \alpha \times a_k \times \Delta_j \ .$$

(Most neuroscientists deny that back-propagation occurs in the brain)

Back-propagation learning contd.

At each epoch, sum gradient updates for all examples and apply

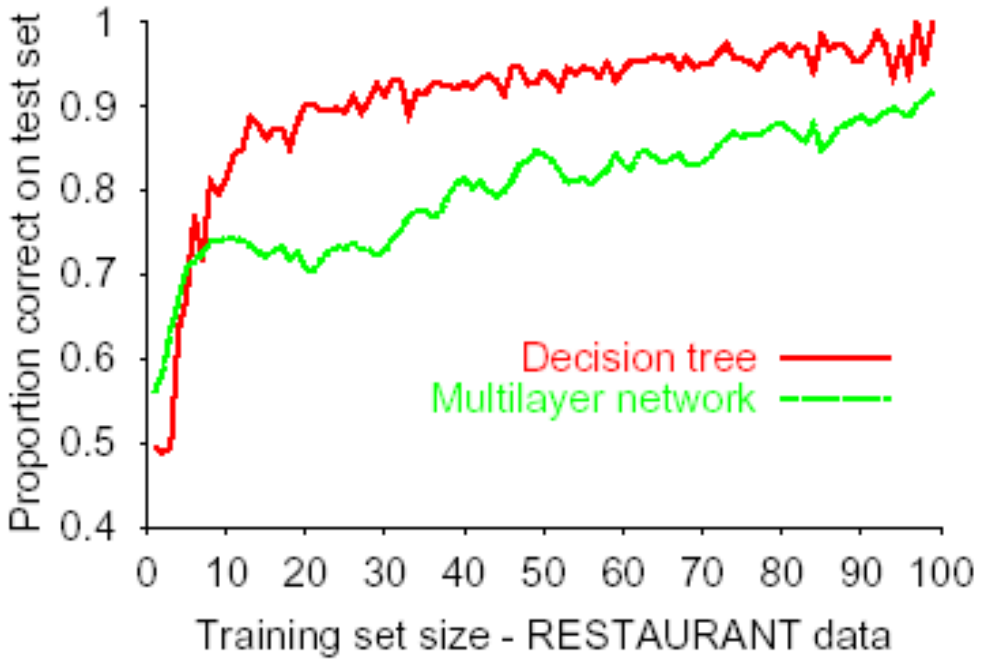
Training curve for 100 restaurant examples: finds exact fit



Typical problems: slow convergence, local minima

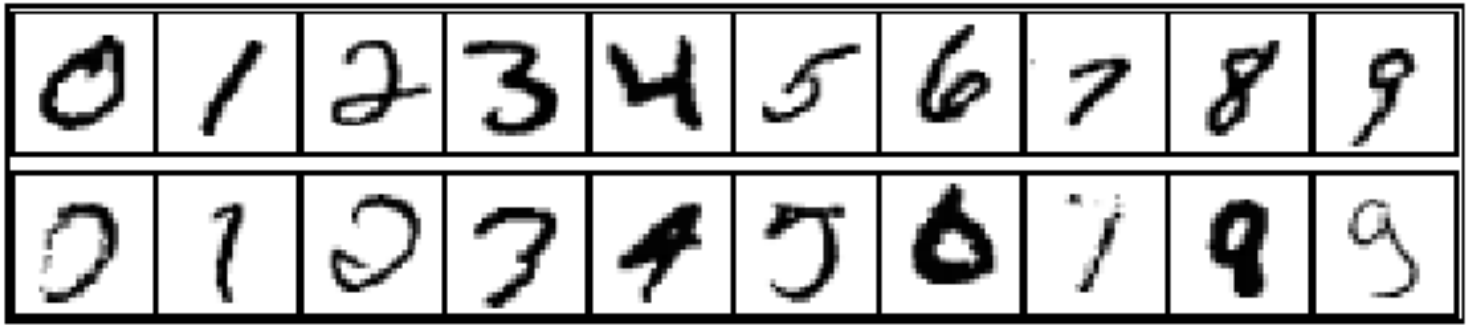
Back-propagation learning contd.

Learning curve for MLP with 4 hidden units:



MLPs are quite good for complex pattern recognition tasks,
but resulting hypotheses cannot be understood easily

Handwritten digit recognition



3-nearest-neighbor = 2.4% error

400–300–10 unit MLP = 1.6% error

LeNet: 768–192–30–10 unit MLP = 0.9% error

Current best (kernel machines, vision algorithms) $\approx$ 0.6% error

Summary

Most brains have lots of neurons; each neuron $\approx$ linear–threshold unit (?)

Perceptrons (one-layer networks) insufficiently expressive

Multi-layer networks are sufficiently expressive; can be trained by gradient descent, i.e., error back-propagation

Many applications: speech, driving, handwriting, fraud detection, etc.

Engineering, cognitive modelling, and neural system modelling
subfields have largely diverged