Constraint Satisfaction

CSE 473
University of Washington

Today: Constraint Satisfaction Problems

- Definition
  - Factoring state spaces
- Variable-ordering heuristics
- Backtracking policies
- Preprocessing algorithms

Constraint Satisfaction

- Kind of ***search*** in which
  - States are ***factored*** into sets of variables
  - Search = assigning values to these variables
  - Structure of space is encoded with constraints
- Backtracking-style algorithms work
  - E.g. DFS for SAT (i.e. DPLL)
- But other techniques add speed
  - Propagation
  - Variable ordering
  - Preprocessing

Chinese Food as Search

- **States?**
- **Operators?**
- **Start state?**
- **Goal states?**

Suppose every meal for n people has n dishes plus soup

Chinese Food as Search

- **States?**
  - Partially specified meals
- **Operators?**
  - Add, remove, change dishes
- **Start state?**
  - Null meal
- **Goal states?**
  - Meal meeting certain conditions

    Two non-peanut dishes, with at least one of the others non-spicy.

Suppose every meal for n people has n dishes plus soup

Factoring States

- Break the state into its independent components:

State = (dish1, … dishN, soup)

Becomes variables:

Soup =
Dish 1 =
Dish 2 =
…
Dish n =

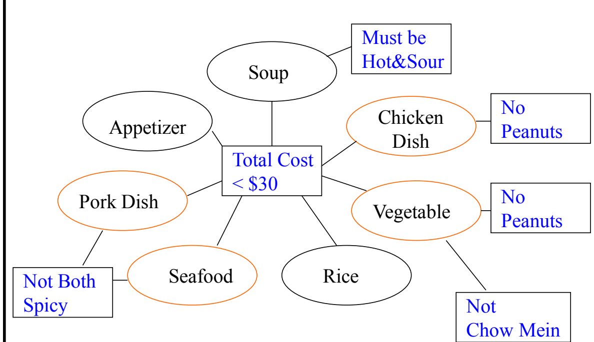
Factoring States

- Encode the structure of the problem as constraints
- Often this structure is the description of the goal state:

Goal condition = "Two peanut-free dishes;
of the others, at least one must be non-spicy"

How to express this as constraints?

Chinese Constraint Network

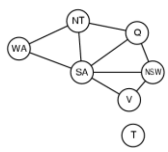


Binary Constraint Network

- Set of n ***variables***: $x_1 \dots x_n$
- Value ***domains*** for each variable: $D_1 \dots D_n$
- Set of binary ***constraints*** (also "relations")

  Each binary constraint specifies which pairs (x_i, x_j) are consistent

  $R_{ij} \subseteq D_i \times D_j$



Binary Constraint Network

Partial ***assignment*** of values = tuple of pairs

{...(x, a)...} means variable x gets value a...

Tuple=***consistent*** if all constraints satisfied

Tuple=***full solution*** if consistent + has all vars

Tuple $\{(x_i, a_i) \dots (x_j, a_j)\}$ = ***consistent w/ a set of vars*** $\{x_m \dots x_n\}$

iff $\exists\ a_m \dots a_n$ such that

$\{(x_i, a_i)\dots(x_j, a_j), (x_m, a_m)\dots(x_n, a_n)\}$ = consistent

CSPs in the Real World

- Scheduling space shuttle repair
- Airport gate assignments
- Transportation Planning
- Supply-chain management
- Computer configuration
- Diagnosis
- UI optimization
- Etc...

CSP Example: Cryptarithmetic

- ~~State Space~~

  ~~Set of states~~

  ~~Operators [and costs]~~

  ~~Start state~~

  ~~Goal states~~

- Variables?
- Domains (variable values)?
- Constraints?

```
   T W O
 + T W O
 -------
 F O U R
```

CSP Example: Cryptarithmetic

- ~~State Space~~
  - ~~Set of states~~
  - ~~Operators [and costs]~~
  - ~~Start state~~
  - ~~Goal states~~
- Variables?
- Domains (variable values)?
- Constraints?

```
   T W O
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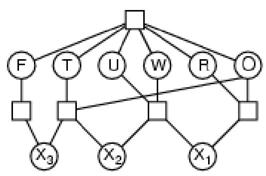


CSP Example: Classroom Scheduling

- Variables?
- Domains (possible values for variables)?
- Constraints?

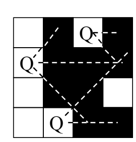
N Queens

- As a CSP?

N Queens

- Variables = board columns
- Domain values = rows
- $R_{ij} = \{(a_i, a_j) : (a_i \neq a_j) \wedge (|i-j| \neq |a_i-a_j|)$

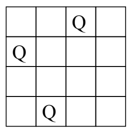
  e.g. $R_{12} = \{(1,3), (1,4), (2,4), (3,1), (4,1), (4,2)\}$



- $\{(x_1, 2), (x_2, 4), (x_3, 1)\}$ consistent with (x_4)
- Shorthand: "{2, 4, 1} consistent with x_4"

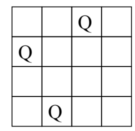
CSP as a search problem?

- What are states?
- What are the operators?
- Initial state?
- Goal test?



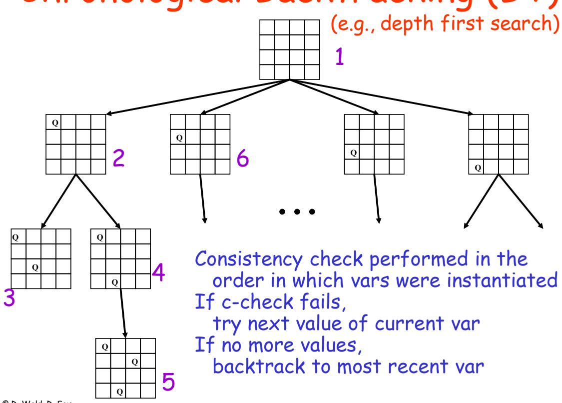
CSP as a search problem?

- What are states?
  (partial assignments)
- What are the operators?
  (add/change a single-variable assignment)
- Initial state?
  (empty assignment)
- Goal test?
  (full & consistent assignment)



Chronological Backtracking (BT)

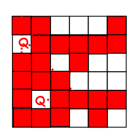
(e.g., depth first search)



Consistency check performed in the order in which vars were instantiated
If c-check fails, try next value of current var
If no more values, backtrack to most recent var

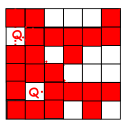
Variable and Value Ordering

- Which queen to place next, and where?



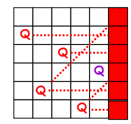
Variable and Value Ordering

- Which queen to place next, and where?
- Most constrained variable / minimum remaining values (MRV)
- Least constraining value
- Highest degree (as tiebreaker)



Backjumping (BJ)

- Similar to BT, but
  more efficient when no consistent instantiation can be found for the current var
- Instead of backtracking to most recent var…
  BJ reverts to deepest var which was c-checked against the current var

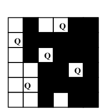


BJ Discovers
(2, 5, 3, 6) inconsistent with x_6
No sense trying other values of x_5
Backtrack to x_4 and try a new placement

Forward Checking (FC)

- Perform Consistency Check ***Forward***
- Whenever a var is assigned a value
  Prune inconsistent values from
  As-yet unvisited variables
  Backtrack if domain of any var ever collapses

FC can't detect that
(2, 5, 3) inconsistent with {x_5, x_6 }
→ Arc consistency can fix this



CSP Summary

- CSPs are a special kind of problem
  states defined by values of a fixed set of variables
  goal test defined by *constraints* on variable values
- Forward checking prevents assignments that guarantee later failure
- Variable ordering and value selection heuristics help significantly
- Complexity depends on constraint graph: linear time if tree structured