Informed Search

CSE 473
University of Washington

Last Time

- Agents
- Problem Spaces
- Blind Search
  - DFS
  - BFS
  - Iterative Deepening

Best-first Search

Generalization of breadth first search
Priority queue of nodes to be explored
Cost function f(n) applied to each node

```
Add initial state to priority queue
While queue not empty
    Node = head(queue)
    If goal?(node) then return node
    Add children of node to queue
```


Old Friends

- Breadth first = best first
  With f(n) = depth(n)
- Dijkstra's Algorithm = best first
  With f(n) = g(n)
  Where g(n) = sum of edge costs from start to n

A* Search

- Hart, Nilsson & Rafael 1968

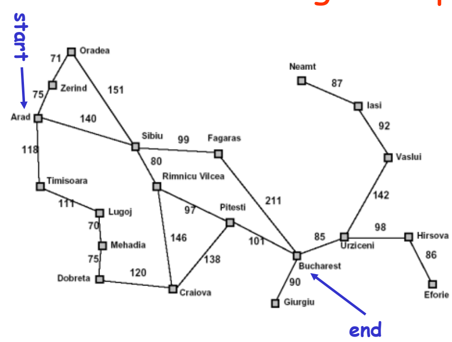
  Best first search with f(n) = g(n) + h(n)

  Where g(n) = sum of edge costs from start to n
  And heuristic function h(n) = estimate of lowest cost path n-->goal

  If h(n) is **admissible** then search will find optimal

  Underestimates cost of any solution which can be reached from node

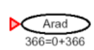
Route Finding Example



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* Example



A* Example

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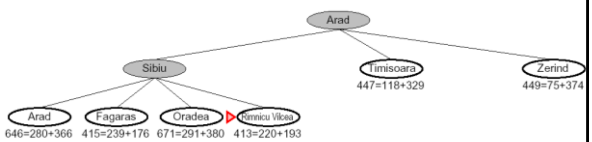
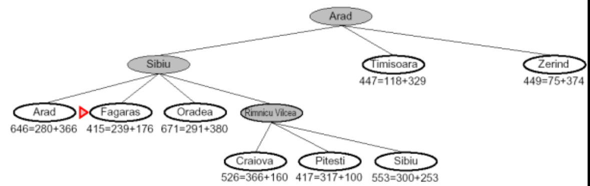
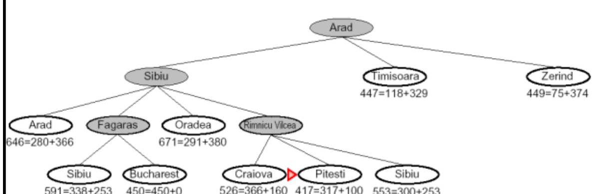


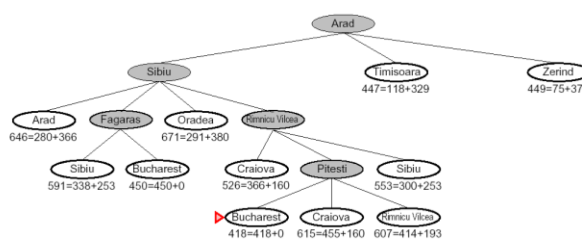
A* Example



A* Example



A* Example



Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \le h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
-
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
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- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.
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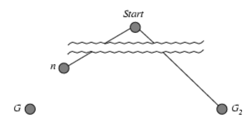

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $> g(G)$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

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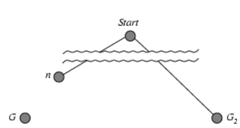

- $f(G_2) > f(G)$ from above
- $h(n) \le h^*(n)$ since h is admissible
- $g(n) + h(n) \le g(n) + h^*(n)$
- $f(n) \le f(G)$
-

Hence $f(n) < f(G_2)$, and A* will never select G_2 for expansion

Consistent heuristics

- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,
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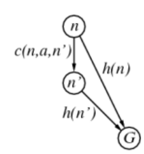
$h(n) \le c(n,a,n') + h(n')$



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- If h is consistent, then $f(n)$ is non-decreasing along any path.
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- Theorem: If $h(n)$ is consistent, A* using `GRAPH-SEARCH` is optimal

Optimality of A*

- A* expands nodes in order of increasing *f* value
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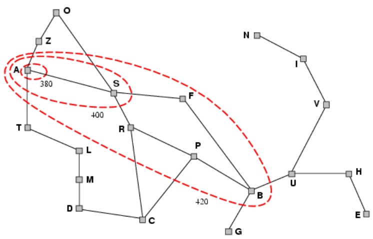
- Gradually adds "*f*-contours" of nodes



Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
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- Space? Keeps all nodes in memory
- Optimal? Yes
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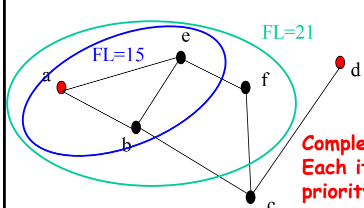
Iterative-Deepening A*

- Like iterative-deepening depth-first, but...
- Depth bound modified to be an **f-limit**
  - Start with limit = h(start)
  - Prune any node if f(node) > f-limit
  - Next f-limit=min-cost of any node pruned



Complete & Optimal (ala A*)
Each iteration is DFS - no priority queue!