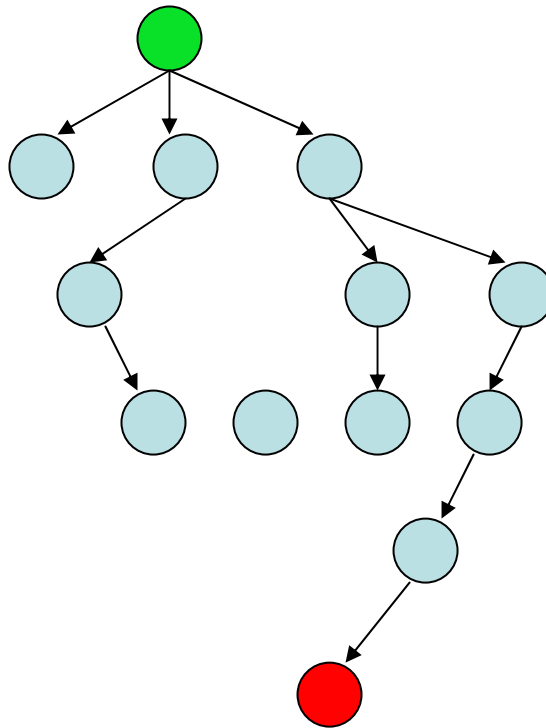
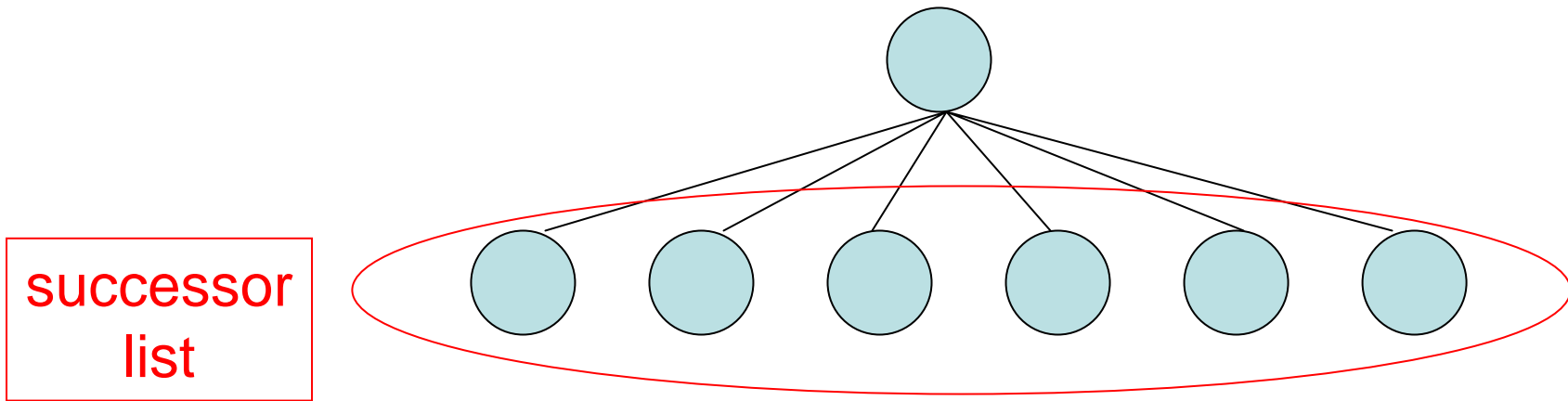


# Informed Search

Idea: be **smart**  
about what paths  
to try.



# Expanding a Node



How should we implement this?

# Blind Search vs. Informed Search

- What's the difference?
- How do we formally specify this?

# General Tree Search Paradigm

(adapted from Chapter 3)

```
function tree-search(root-node)
  fringe ← successors(root-node)
  while ( notempty(fringe) )
    {node ← remove-first(fringe)
     state ← state(node)
     if goal-test(state) return solution(node)
     fringe ← insert-all(successors(node),fringe) }
  return failure
end tree-search
```

Does this look familiar?

# General Graph Search Paradigm

(adapted from Chapter 3)

```
function graph-search(root-node)
  closed ← { }
  fringe ← successors(root-node)
  while ( notempty(fringe) )
    {node ← remove-first(fringe)
     state ← state(node)
     if goal-test(state) return solution(node)
     if notin(state,closed)
       {add(state,closed)
        fringe ← insert-all(successors(node),fringe) }}
  return failure
end graph-search
```

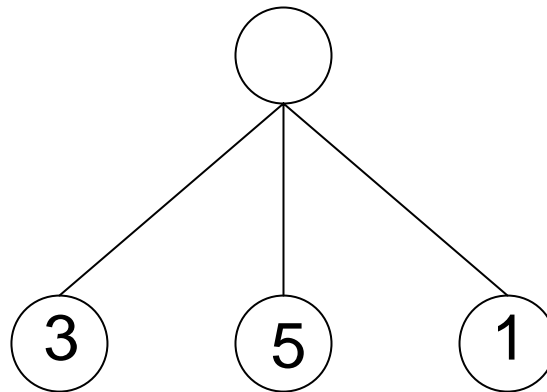
What's the difference between this and tree-search?

# Tree Search or Graph Search

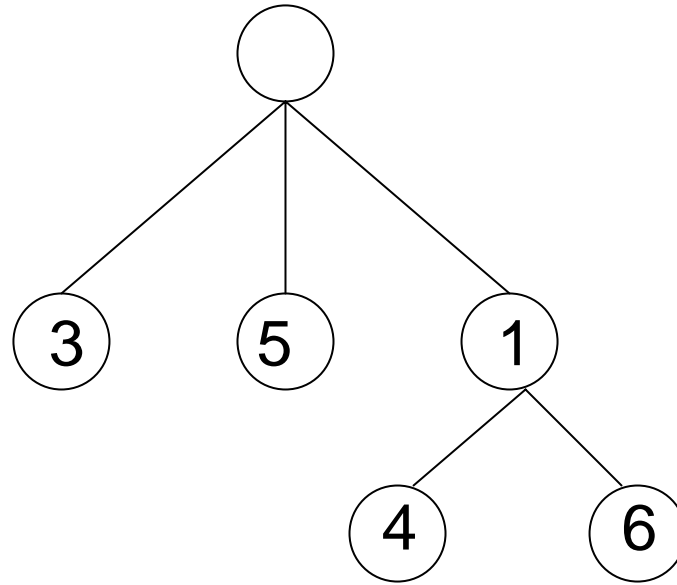
- What's the key to the order of the search?

# Best-First Search

- Use an **evaluation function  $f(n)$** .
- Always choose the node from fringe that has the **lowest**  $f$  value.



# Best-First Search Example





# Old Friends

- Breadth first = best first
  - with  $f(n) = \text{depth}(n)$
- Dijkstra's Algorithm = best first
  - with  $f(n) = g(n)$
  - where  $g(n) = \text{sum of edge costs from start to } n$
  - space bound (stores all generated nodes)

# Heuristics

- What is a heuristic?
- What are some examples of heuristics we use?
- We'll call the heuristic function  $h(n)$ .

# Greedy Best-First Search

- $f(n) = h(n)$
- What does that mean?
- Is greedy search optimal?
- Is it complete?
- What is its worst-case complexity for a tree with branching factor  $b$  and maximum depth  $m$ ?

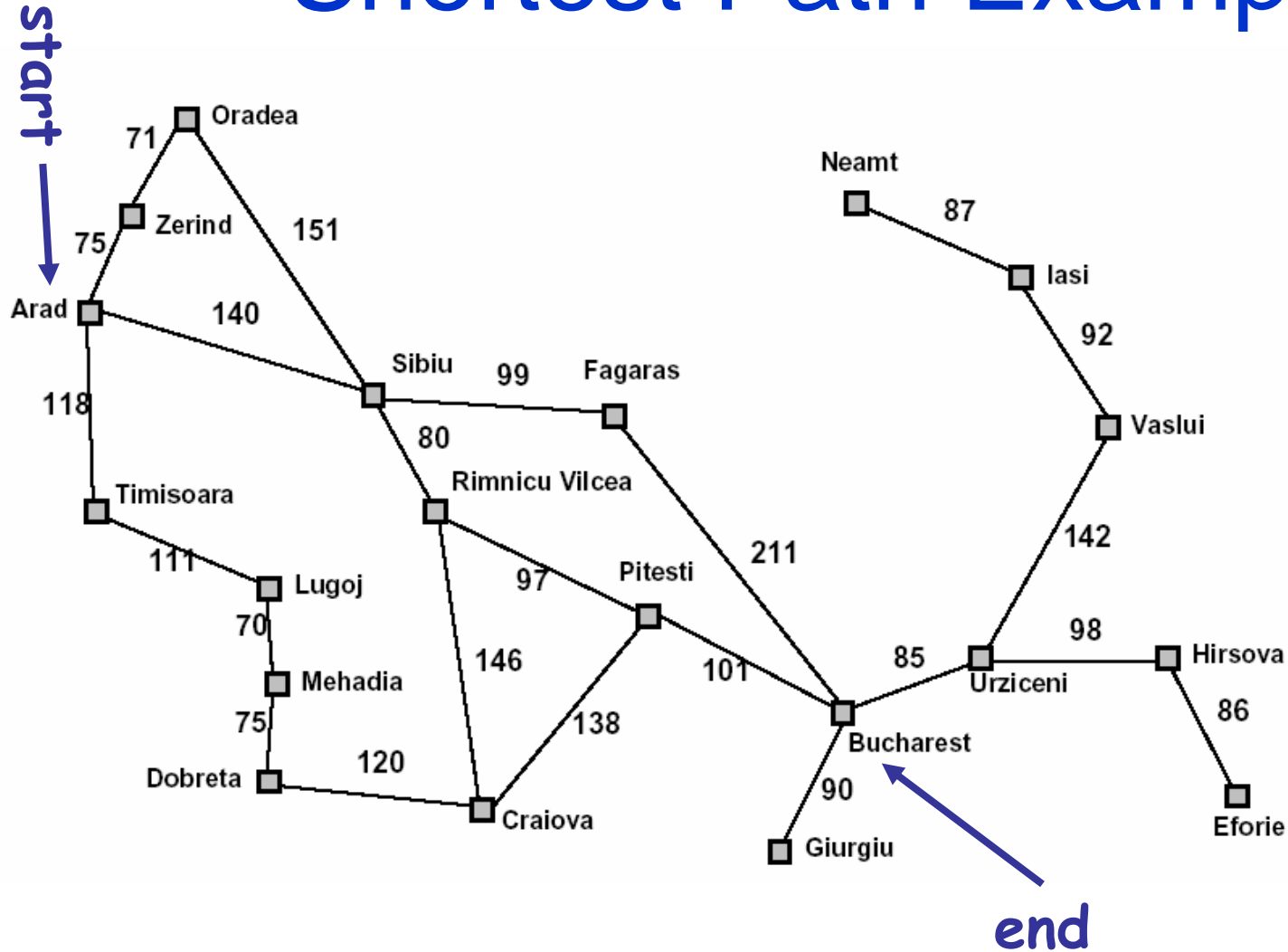
# A\* Search

- Hart, Nilsson & Rafael 1968
  - Best first search with  $f(n) = g(n) + h(n)$   
where  $g(n)$  = sum of edge costs from start to  $n$   
and  $h(n)$  = estimate of lowest cost path  $n \rightarrow$  goal
  - If  $h(n)$  is **admissible** then search will find optimal solution.

↑ { Never overestimates the true cost of any solution which can be reached from a node.

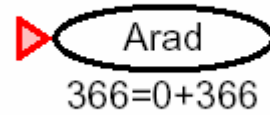
Space bound since the queue must be maintained.

# Shortest Path Example

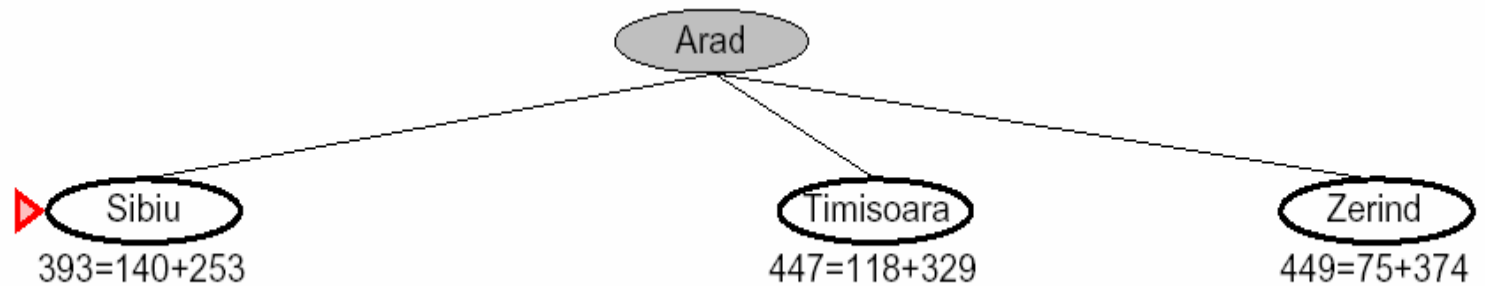


	Straight-line distance to Bucharest
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

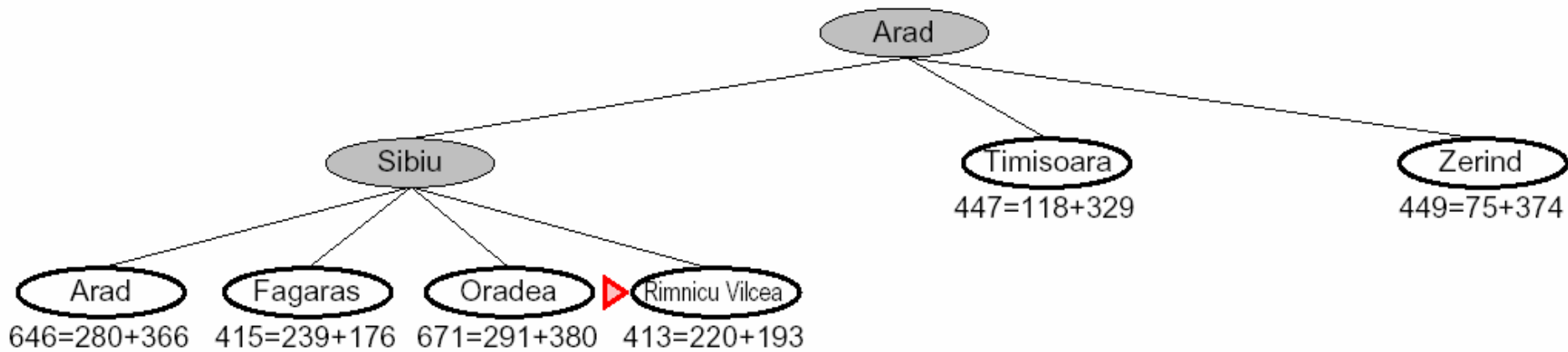
# A\* Shortest Path Example



# A\* Shortest Path Example

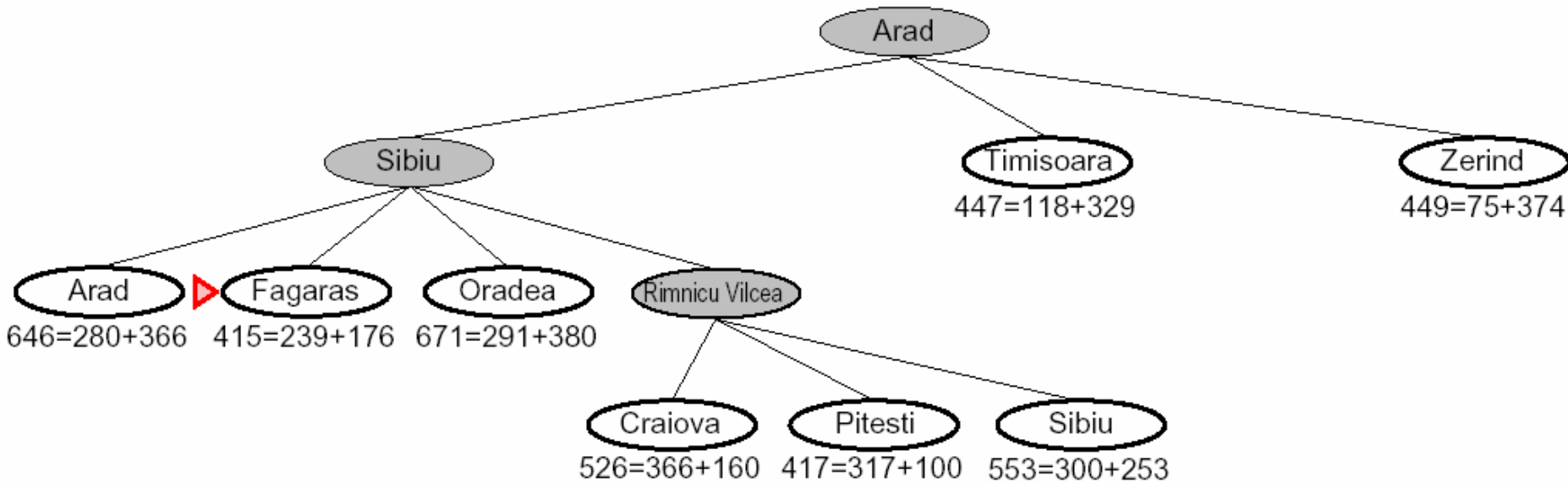


# A\* Shortest Path Example

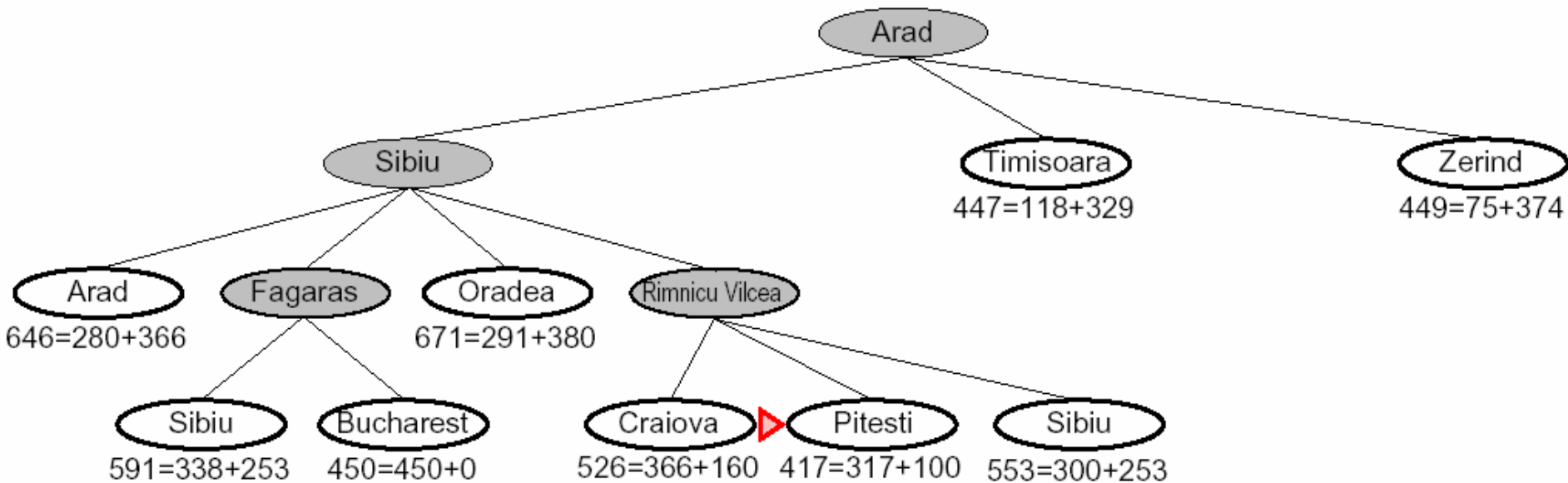




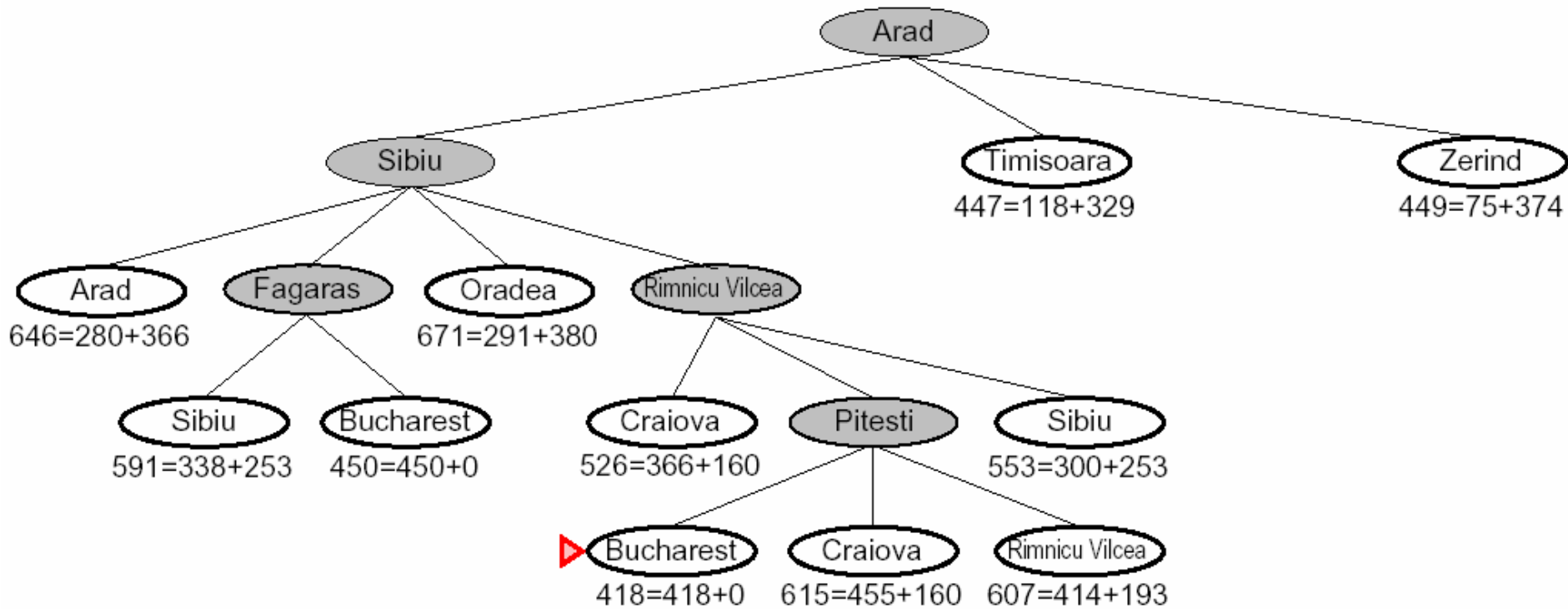
# A\* Shortest Path Example



# A\* Shortest Path Example



# A\* Shortest Path Example



# 8 Puzzle Example

- $f(n) = g(n) + h(n)$
- What is the usual  $g(n)$ ?
- two well-known  $h(n)$ 's
  - $h_1$  = the number of misplaced tiles
  - $h_2$  = the sum of the distances of the tiles from their goal positions, using city block distance, which is the sum of the horizontal and vertical distances

# 8 Puzzle Using Number of Misplaced Tiles

1	2	3
8		4
7	6	5

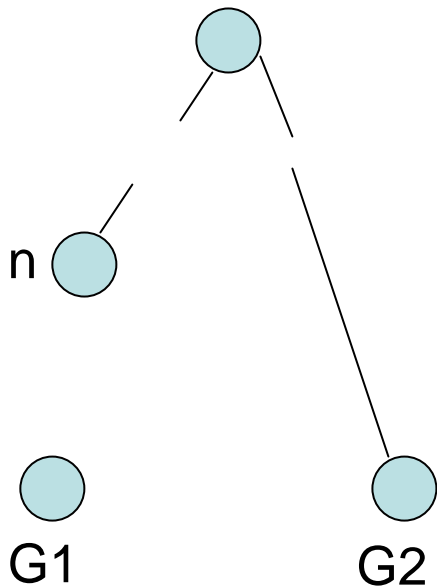
goal

2	8	3
1	6	4
7		5

# Continued

# Optimality of A\*

Suppose a suboptimal goal G2 has been generated and is in the queue. Let n be an unexpanded node on the shortest path to an optimal goal G1.



$$\begin{aligned} f(n) &= g(n) + h(n) \\ &\leq g(G1) \\ &< g(G2) \\ &= f(G2) \end{aligned}$$

Why?

G2 is suboptimal  
 $f(G2) = g(G2)$

So  $f(n) < f(G2)$  and A\* will never select G2 for expansion.

# Algorithms for A\*

- Since Nilsson defined A\* search, many different authors have suggested algorithms.
- Using Tree-Search, the optimality argument holds, but you search too many states.
- Using Graph-Search, it can break down, because an optimal path to a **repeated state** can be discarded if it is not the first one found.
- One way to solve the problem is that whenever you come to a repeated node, discard the **longer** path to it.



# The Rich/Knight Implementation

- a **node** consists of
  - state
  - g, h, f values
  - list of successors
  - pointer to parent
- **OPEN** is the list of nodes that have been generated and had h applied, but not expanded and can be implemented as a priority queue.
- **CLOSED** is the list of nodes that have already been expanded.

# Rich/Knight

1) /\* Initialization \*/

OPEN <- start node

Initialize the start node

g:

h:

f:

CLOSED <- empty list

# Rich/Knight

2) repeat until goal (or time limit or space limit)

- if OPEN is empty, fail
- BESTNODE  $\leftarrow$  node on OPEN with lowest  $f$
- if BESTNODE is a goal, exit and succeed
- remove BESTNODE from OPEN and add it to CLOSED
- generate successors of BESTNODE

# Rich/Knight

for each successor  $s$  do

1. set its parent field

2. compute  $g(s)$

3. if there is a node **OLD** on OPEN with the same state info as  $s$

{ add **OLD** to successors(BESTNODE)

if  $g(s) < g(\text{OLD})$ , update **OLD** and

throw out  $s$  }

# Rich/Knight

4. if ( $s$  is not on OPEN and there is a node  $OLD$  on CLOSED with the same state info as  $s$ )
  - { add  $OLD$  to successors(BESTNODE)
  - if  $g(s) < g(OLD)$ , update  $OLD$ ,
  - throw out  $s$ ,
  - \*\*\*propagate the lower costs to successors( $OLD$ ) }

That sounds like a LOT of work. What could we do instead?

# Rich/Knight

```
5. If s was not on OPEN or CLOSED
   { add s to OPEN
     add s to successors(BESTNODE)
     calculate g(s), h(s), f(s) }
```

end of repeat loop

# The Heuristic Function $h$

- If  $h$  is a **perfect estimator** of the true cost then  $A^*$  will always pick the correct successor with no search.
- If  $h$  is **admissible**,  $A^*$  with TREE-SEARCH is guaranteed to give the optimal solution.
- If  $h$  is **consistent**, too, then GRAPH-SEARCH without extra stuff is optimal.  
 $h(n) \leq c(n,a,n') + h(n')$  for every node  $n$  and each of its successors  $n'$  arrived at through action  $a$ .
- If  $h$  is not admissible, no guarantees, but it can work well if  $h$  is not often greater than the true cost.