CSE 473

Chapters 8-9

More First-Order Logic

Recall: FOL Definitions

- ***Constants***: George, Monkey2, etc.
  Name a specific object.
- ***Variables***: X, Y.
  Refer to an object without naming it.
- ***Functions***: banana-of, grade-of, etc.
  Mapping from objects to objects.
- ***Terms***: George, grade-of(stdnt1)
  Refer to objects
- ***Relations***: Curious, PokesInTheEyes, etc.
  State relationships between objects.
- ***Atomic Sentences***: PokesInTheEyes(Moe, Curly)
  Can be true or false
  Correspond to propositional symbols P, Q

More Definitions

- **Logical connectives**: and, or, not, $\Rightarrow$, $\Leftrightarrow$
- **Quantifiers**:

  $\forall$ For all (Universal quantifier)

  $\exists$ There exists (Existential quantifier)

- **Examples**

  Monkeys are curious

  $\forall$m: **Monkey(m)** $\Rightarrow$ **Curious(m)**

  There is a curious monkey

  $\exists$m: **Monkey(m)** $\wedge$ **Curious(m)**

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Nested Quantifiers: Order matters!

$$\forall x \exists y \; P(x,y) \neq \exists y \forall x \; P(x,y)$$

- Examples

  Every monkey has a tail

  $\forall m \exists t \; \text{has}(m,t)$

  ?

  Every monkey ***shares*** a tail!

  $\exists t \forall m \; \text{has}(m,t)$

Everybody loves somebody *vs.* Someone is loved by everyone

$\forall x \exists y \; \text{loves}(x, y)$ $\exists y \forall x \; \text{loves}(x, y)$

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Semantics

- ***Semantics*** = what the arrangement of symbols *means* in the world
- Propositional logic

  Basic elements are *variables*

  (refer to facts about the world)

  Possible worlds: mappings from variables to T/F
- First-order logic

  Basic elements are *terms*

  (refer to objects)

  Interpretations: mappings from terms to real-world elements.

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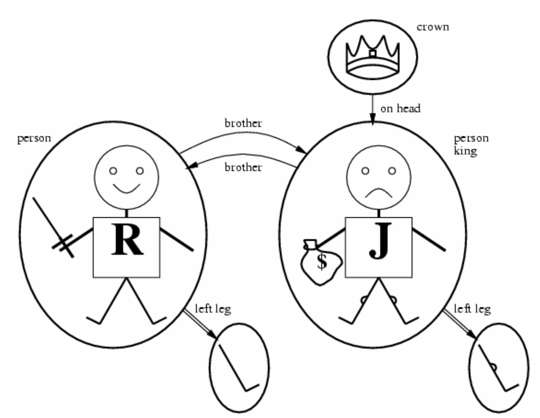
Example: A World of Kings and Legs



- Syntactic elements:

Constants:	Functions:	Relations:
Richard John	**LeftLeg(p)**	**On(x,y) King(p)**

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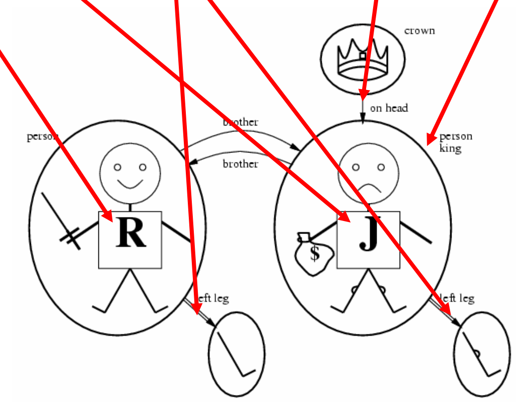
Interpretation I

- Interpretations map syntactic tokens to model elements

Constants: Richard John

Functions: LeftLeg(p)

Relations: On(x,y) King(p)

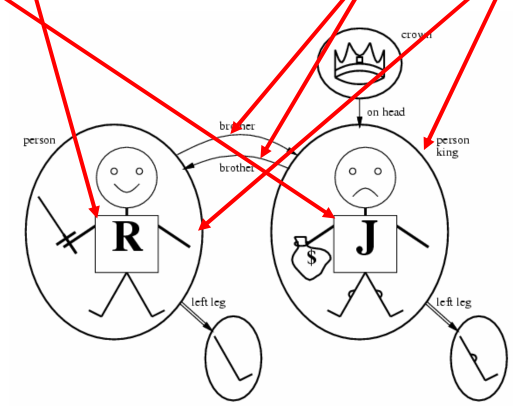


Interpretation II

Constants: Richard John

Functions: LeftLeg(p)

Relations: On(x,y) King(p)



How Many Interpretations?

- Two constants (and 5 objects in world)

  Richard, John (R, J, crown, RL, JL)

  5^2 = 25 object mappings

- One unary relation

  King(x)

  Infinite **number of values for x   infinite mappings**
  Even if we restricted x to: R, J, crown, RL, JL:

  2^5 = 32 unary truth mappings

- Two binary relations

  Leg(x, y); On(x, y)

  Infinite. But even restricting x, y to five objects still yields 2^{25} mappings ***for each*** **binary relation**

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Satisfiability, Validity, & Entailment

- S is **valid** if it is true in all interpretations

- S is **satisfiable** if it is true in some interp

- S is **unsatisfiable** if it is false all interps

- S1 **entails** S2 if ($\models$)

  For all interps where S1 is true,
  S2 is also true

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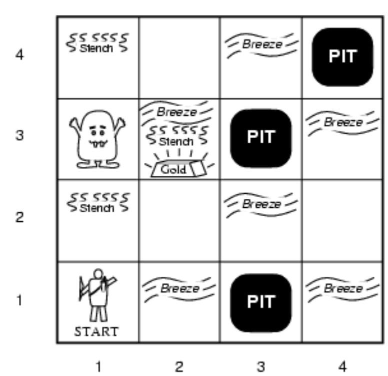
Propositional. Logic *vs.* **First Order**

Ontology	Facts (P, Q,…)	Objects, Properties, Relations
Syntax	Atomic sentences Connectives	Variables & quantification Sentences have structure: terms father-of(mother-of(X)))
Semantics	Truth Tables	Interpretations (Much more complicated)
Inference Algorithm	DPLL, WalkSAT Fast in practice	Unification Forward, Backward chaining Prolog, theorem proving
Complexity	NP-Complete	Semi-decidable

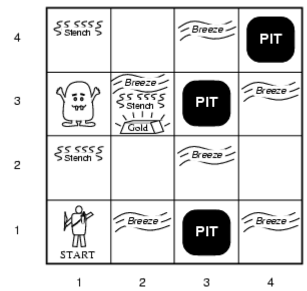
First-Order Wumpus World

- **Objects**
  Squares, wumpuses, agents, gold, pits, stinkiness, breezes
- **Relations**
  Square topology (adjacency), Pits/breezes, Wumpus/stinkiness



Wumpus World: Squares

- Each square as an object:
  $Square_{1,1}$, $Square_{1,2}$, …,
  $Square_{3,4}$, $Square_{4,4}$
- Square topology relations?
  $Adjacent(Square_{1,1}, Square_{2,1})$
  …
  $Adjacent(Square_{3,4}, Square_{4,4})$



- **Better**: Squares as lists:
  [1, 1], [1,2], …, [4, 4]
- Square topology relations:
  $\forall x, y, a, b$: Adjacent([x, y], [a, b])
  [a, b] ∈ {[x+1, y], [x-1, y], [x, y+1], [x, y-1]}

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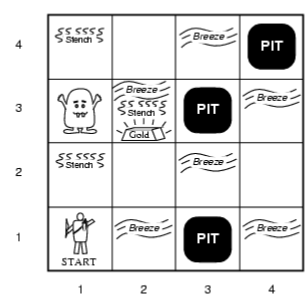
Wumpus World: Pits

- Each pit as an object:
  $Pit_{1,1}$, $Pit_{1,2}$, …,
  $Pit_{3,4}$, $Pit_{4,4}$
- Problem?
  Not all squares have pits
- List only the pits we have?
  $Pit_{3,1}$, $Pit_{3,3}$, $Pit_{4,4}$



- Problem?
  No reason to distinguish pits (same properties)
- **Better:** pit as unary predicate
  Pit(x)
  Pit([3,1]); Pit([3,3]); Pit([4,4]) will be true

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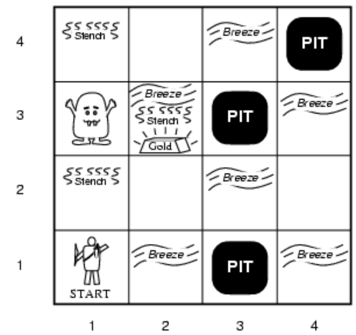
Wumpus World: Breezes

- Represent breezes like pits, as unary predicates:
  Breezy(x)



- "Squares next to pits are breezy":
  $\forall$x, y, a, b:
  Pit([x, y]) $\wedge$ Adjacent([x, y], [a, b]) $\Rightarrow$ Breezy([a, b])

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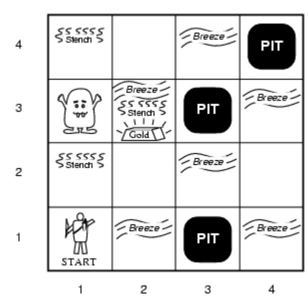
Wumpus World: Wumpuses

- Wumpus as object:
  Wumpus

- Wumpus home as unary predicate:
  WumpusIn(x)



- **Better:** Wumpus's home as a function:
  Home(Wumpus) references the wumpus's home square.

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FOL Reasoning: Outline

- Basics of FOL reasoning
- Classes of FOL reasoning methods
  - Forward & Backward Chaining
  - Resolution
  - Compilation to SAT

Basics: Universal Instantiation

- Universally quantified sentence:
  - $\forall x$: Monkey(x) $\Rightarrow$ Curious(x)
- Intutively, x can be anything:
  - Monkey(George) $\Rightarrow$ Curious(George)
  - Monkey(473Student1) $\Rightarrow$ Curious(473Student1)
  - Monkey(DadOf(George)) $\Rightarrow$ Curious(DadOf(George))
- Formally: (example)

$$\frac{\forall x \;\; S}{\text{Subst}(\{x/p\}, S)} \qquad \frac{\forall x \;\; \text{Monkey}(x) \quad \text{Curious}(x)}{\text{Monkey}(\text{George}) \quad \text{Curious}(\text{George})}$$

x is replaced with p in S, and the quantifier removed

x is replaced with George in S, and the quantifier removed

Basics: Existential Instantiation

- Existentially quantified sentence:

  $\exists x$: Monkey(x) $\wedge$ ¬Curious(x)

- **Intutively, x must name something. But what?**

  Monkey(George) $\wedge$ ¬Curious(George) ???

  No! S might not be true for George!

- Use a *Skolem Constant* :

  Monkey(K) $\wedge$ ¬Curious(K)

  …where K is a **completely new** symbol (stands for the monkey for which the statement is true)

- Formally:

$$\frac{\exists x \ \ S}{\mathrm{Subst}(\{x/K\},\ S)}$$

K is called a Skolem constant

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Basics: Generalized Skolemization

- What if our existential variable is nested?

  $\forall x\ \exists y$: Monkey(x) $\Rightarrow$ HasTail(x, y)

  $\forall x$: Monkey(x) $\Rightarrow$ HasTail(x, K_Tail) ???

- Existential variables can be replaced by **Skolem functions**

  Args to function are all surrounding $\forall$ vars

$\forall x$: Monkey(x) $\Rightarrow$ HasTail(x, f(x))

"tail-of" function

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Next Time

- Reasoning with FOL
  - Unification
  - Chaining
  - Resolution