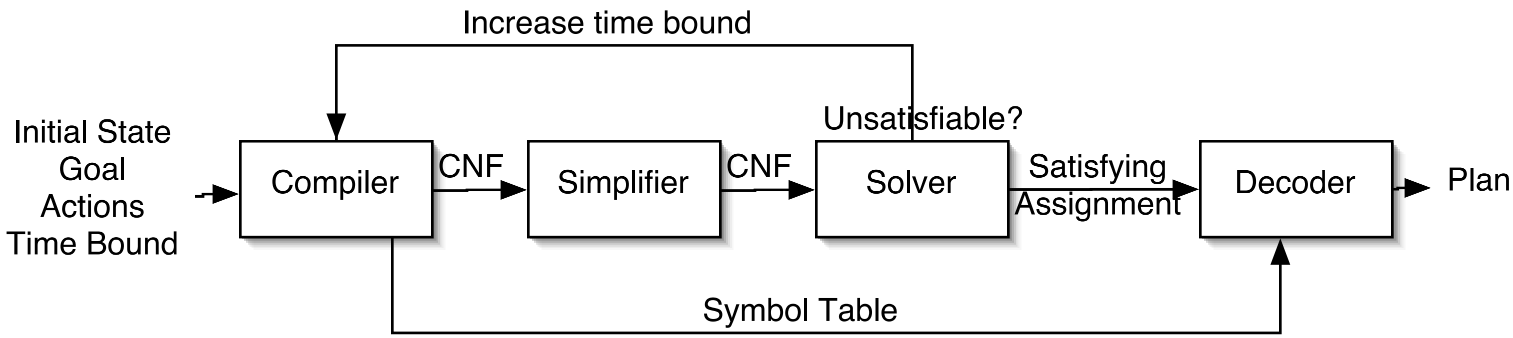
Planning and Satisfiability

Planning and Satisfiability: Kautz & Selman ECAI-92, KR-96, AAAI-96

- Planning is traditionally seen as deduction:
  - Find a proof of the statement "Initial conditions $\wedge$ Actions $\supset$ Goals"
- Recall that satisfiability is the problem of finding a model of a set of formulas (axioms)
  - First-order satisfiability is not decidable (testing entailment is semi-decidable)
  - Propositional satisfiability is decidable (although NP-complete)
- Key insight: Encode a planning problem as a set of axioms with the following property:
  - Any model of the axioms corresponds to a valid plan
- Then a procedure that computes models (satisfying assignments) will effectively compute plans!
- Bonus: Fast SAT solvers will yield fast planners

SAT Planning Architecture



– Weld, AI Magazine 20, 1999

- Two issues:
  - Encoding the planning problem
  - Solving the SAT problem

Elements of an Encoding: Initial State

- The initial state is asserted as a conjunction of all literals at time 0 (including any closed world assumptions):

$$garbage_0 \wedge cleanHands_0 \wedge quiet_0 \wedge \neg dinner_0 \wedge \neg present_0$$

Elements of an Encoding: Goals

- To test for a plan of length n, assert all desired goal properties at time $2n$: For $n = 1$:

$$\neg garbage_2 \wedge dinner_2 \wedge present_2$$

Elements of an Encoding: Action Occurrences

- The result (the plan) is a set of actions to be executed at specific times
- For each odd time t between 1 and $2n-1$, we have a disjunction of all possible actions at that time:

$$(carry_1 \vee dolly_1 \vee cook_1 \vee wrap_1) \wedge$$

$$(carry_3 \vee dolly_3 \vee cook_3 \vee wrap_3) \wedge$$

$$\ldots$$

- Note: This can lead to many axioms if actions are parameterized.

Elements of an Encoding: Preconditions and Effects

- Actions imply their preconditions and effects: For any odd time t between 1 and $2n-1$, and for each action, an axiom states that execution of the action implies its preconditions hold at $t-1$ and its effects hold at $t+1$. For example:

$$(\neg cook_1 \vee \neg dinner_2) \wedge (\neg cook_1 \vee cleanHands_0)$$

- Note: This can lead to many axioms!

Elements of an Encoding: Frame Axioms

- Frame axioms state what isn't changed as a result of an action; or
- Explanation axioms state what must have happened given that a change occurs (Schubert, J. Logic and Computation 4(5), 1994; Gerevini & Schubert, AAAI-98)
- For every odd time t between 1 and $2n-1$, for example:

$$\neg present_0 \wedge \neg wrap_1 \supset \neg present_2$$

$$garbage_0 \wedge \neg(carry_1 \vee dolly_1) \supset garbage_2$$

- Note: This can lead to many, many axioms!

Elements of an Encoding: Summary

- Initial Conditions
- Goals
- Action Occurrences
- Preconditions and Effects
- Frame Axioms
- Other axioms for technical reasons (see papers)

SAT Encoding Considerations

- Complexity of SAT is exponential in the size of the formula being tested
- Parameters of an encoding:
  - Number of propositional variables
  - Number of clauses
  - Number of literals (in all clauses)

SAT Encoding Complexity

- Let $\mathcal{A}$ be the number of actions
- Let $\mathcal{F}$ be the number of fluents
- Linear encoding (Kautz & Selman ECAI-92):
  - Number of propositional variables: $O(n|\mathcal{A}| + n|\mathcal{F}|)$
  - Number of literals is dominated by frame axioms: $O(n|\mathcal{A}|^2 + n|\mathcal{A}||\mathcal{F}|)$
- Operator splitting (lifting) reduces both numbers
- Explanatory frame axioms: $O(n|\mathcal{F}|)$ (but each clause may be longer; worst-case same total size)
- Many more optimizations: see Kautz, McAllester, & Selman, KR-96

Solving SAT Problems

- Systematic (deterministic) solvers
- Stochastic solvers

Recent Advances in AI Planning: Summary

- GraphPlan precomputes constraints to improve the search for plans
- SAT Planners use the power and scalability of SAT solvers to solve large problems
- Both approaches (and they can be combined) exploit the fact that planning problems are instances of more general reasoning problems (constraint satisfaction and satisfiability, respectively)
- Although planning is still a difficult computational problem, these techniques are allowing fully automated systems to solve real-world problems with practical importance (i.e., $$$)