CSE 473: Artificial Intelligence
Assignment #3 Solutions
May 4, 2005

1. Chapter 7, exercise 7.4.

   (a) α is valid if and only if $True \models \alpha$.

   Forward direction: If $True \models \alpha$, then α is valid.

   By definition, $True \models \alpha$ means that α is true in all worlds where $True$ is True; this is all worlds. Thus α is true in all worlds, which is exactly the defintion of validity. So α is in this case valid.

   Backward direction: If α is valid, then $True \models \alpha$.

   By definition, if α is valid then it is true in all worlds. In this case *anything* entails α, so clearly $True$ entails α.

   (b) For any $\alpha, False \models \alpha$.

   Recall the definition of entailment: $p \models q$ means that in all worlds in which p is true, q is true as well. So, $False \models \alpha$ means that in all worlds in which $False$ is true, α is true. But there are no worlds in which $False$ is true! Clearly if there are no worlds in a set, then that set satisfies the condition that α be true in all worlds in that set.

   (c) $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

   Forward direction: If $\alpha \models \beta$, then the sentence $(\alpha \Rightarrow \beta)$ is valid.

   By definition, $\alpha \models \beta$ means that β is true in all worlds in which α is true. Thus, in all worlds in which α is true $\alpha \Rightarrow \beta$ holds because both α and β will be true. We must also consider worlds in which α is false; in these worlds, $\alpha \Rightarrow \beta$ also holds by definition of the falsehood of α.

   Backward direction: If the sentence $(\alpha \Rightarrow \beta)$ is valid, then $\alpha \models \beta$.

   If the sentence $(\alpha \Rightarrow \beta)$ is valid, then it is true in all worlds. Thus, for every world, it must be the case that either both α and β are true, *or* α is false. This is enough to tell us that in every world in which α is true, β is also true, which is the definition of entailment.

2. Write axioms describing the following predicates: ***GrandChild, GreatGrandparent, Brother, Aunt, SisterInLaw, FirstCousin***. You may assume that the predicates ***ParentOf(x,y), ChildOf(x,y), Male(x), Female(x), Married(x, y)*** are already defined.

   (a) **GrandChild:** $GrandChild(z, x) \Leftrightarrow \exists y(ChildOf(z, y) \wedge ChildOf(y, x)))$

   Here, y is the person whose parent (x) is the grandparent of their child (z).

   (b) **GreatGrandparent:** $GreatGrandparent(w, z) \Leftrightarrow \exists x, y(ParentOf(w, x) \wedge ParentOf(x, y) \wedge ParentOf(y, z))$

Here, x and y are the two generations between the great-grandparent and the great-grandchild.

(c) **Brother:** $Brother(x,y) \Leftrightarrow \exists a, b(ParentOf(a,x) \wedge ParentOf(b,x) \wedge ParentOf(a,y) \wedge ParentOf(b,y) \wedge Male(x) \wedge x \neq y)$

This answer assumes that brother means full brother: answers in which only one parent is shared (e.g. half-brother definitions) are also correct. We disallow $x = y$ because one cannot be the brother of oneself.

(d) **Aunt:** $Aunt(x,y) \Leftrightarrow (\exists a(ParentOf(a,y) \wedge Sister(x,a)) \vee (\exists a, b(ParentOf(a,y) \wedge Brother(b,a) \wedge Married(b,y))$

In this case $Sister(x,y)$ is defined analogously with $Brother(x,y)$ in the previous line. An aunt is either the sister of a, who is y's parent; or, an aunt is the wife of a parent's (a's) brother b.

(e) **SisterInLaw:** $SisterInLaw(x,y) \Leftrightarrow \exists a(Female(x) \wedge Married(x,a) \wedge (Sister(a,y) \vee Brother(a,y))))$

Here, a is a sibling of y whose wife, x, is y's sister-in-law.

(f) **FirstCousin:** $FirstCousin(x,y) \Leftrightarrow \exists a, b, p, q(a \neq b \wedge x \neq y \wedge p \neq q \wedge ParentOf(a,x) \wedge ParentOf(p,a) \wedge ParentOf(q,a) \wedge ParentOf(b,y) \wedge ParentOf(p,b) \wedge ParentOf(q,b))$

Here, p and q are the pair of grandparents shared by first-cousins x and y. a and b are siblings, and the parents of x and y, respectively. We disallow $x = y$ because this would collapse to "I am my own first cousin". We disallow $a = b$ because this would collapse to "I am the first cousin of my sibling". Finally, we disallow $p = q$ to enforce that the first cousins share two grandparents.

3. Recall the nursery rhyme:

Fuzzy Wuzzy was a bear,
Fuzzy Wuzzy had no hair.
Was he fuzzy?

In this exercise you will prove that Fuzzy Wuzzy is indeed fuzzy. Fuzzy Wuzzy's universe is governed by the following six rules:

$\forall b\, \exists c\; Bear(b) \Rightarrow Coat(c) \wedge Has(b,c)$

Every bear owns a coat.

$\neg \exists c\; Raincoat(c) \wedge Furcoat(c)$

No coat is both a raincoat and a furcoat.

$\forall c\; Coat(c) \Rightarrow (Raincoat(c) \vee Furcoat(c))$

Every coat is either a raincoat or a furcoat, or both.

$\forall x, c\ Has(x, c) \wedge Furcoat(c) \Rightarrow Fuzzy(x)$

Anything that owns a fur coat is fuzzy.

$\forall x\ HasHair(x) \Rightarrow Fuzzy(x)$

Anything that has hair is fuzzy.

$\neg \exists c\ Has(FuzzyWuzzy, c) \wedge Raincoat(c)$

Fuzzy Wuzzy doesn't own a raincoat.

In addition, our knowledge base contains the following, thanks to the rhyme:

$Bear(FuzzyWuzzy)$

$\neg HasHair(FuzzyWuzzy)$

(a) Write down the English-sentence equivalents of the six rules governing Fuzzy Wuzzy's universe.

See above.

(b) Convert each of the six rules into Clausal Normal Form (CNF). *Show your intermediate steps for each rule.*

i. **Rule 1**: $\boldsymbol{\{\neg Bear(b), Coat(F(b))\}\ \{\neg Bear(b), Has(b, F(b))\}}$
$\forall b\ \exists c\ Bear(b) \Rightarrow Coat(c) \wedge Has(b, c)$ (Original)
$\forall b\ \exists c\ \neg Bear(b) \vee (Coat(c) \wedge Has(b, c))$ (Eliminate implications)
No need to move $\neg$ inwards or standardize variables
$\forall b\ \neg Bear(b) \vee (Coat(F(b)) \wedge Has(b, F(b)))$ (Skolemize out c)
$\neg Bear(b) \vee (Coat(F(b)) \wedge Has(b, F(b)))$ (Drop universal quantifiers)
$(\neg Bear(b) \vee Coat(F(b)) \wedge (\neg Bear(b) \vee Has(b, F(b)))$ (Distribute $\vee$ over $\wedge$).

ii. **Rule 2**: $\boldsymbol{\{\neg Raincoat(c), \neg Furcoat(c)\}}$
$\neg \exists c\ Raincoat(c) \wedge Furcoat(c)$ (Original)
No implications to eliminate.
$\forall c\ \neg(Raincoat(c) \wedge Furcoat(c))$ (Move $\neg$ inwards)
$\forall c\ \neg Raincoat(c) \vee \neg Furcoat(c)$ (Move $\neg$ inwards)
No need to standardize variables or skolemize.
$\neg Raincoat(c) \vee \neg Furcoat(c)$ (Drop universal quantifiers)
No need to distribute $\vee$ over $\wedge$.

iii. **Rule 3**: $\boldsymbol{\{\neg Coat(c), Raincoat(c), Furcoat(c)\}}$
$\forall c\ Coat(c) \Rightarrow (Raincoat(c) \vee Furcoat(c))$ (Original)
$\forall c\ \neg Coat(c) \vee (Raincoat(c) \vee Furcoat(c))$ (Eliminate implications)
No need to move $\neg$ inwards, standardize variables, or skolemize.
$\neg Coat(c) \vee (Raincoat(c) \vee Furcoat(c))$ (Drop universal quantifiers)
$\neg Coat(c) \vee Raincoat(c) \vee Furcoat(c)$ (Drop unnecessary parentheses)
No need to distribute $\vee$ over $\wedge$.

iv. **Rule 4**: $\boldsymbol{\{\neg Has(x, c), \neg Furcoat(c), Fuzzy(x)\}}$
$\forall x, c\ Has(x, c) \wedge Furcoat(c) \Rightarrow Fuzzy(x)$ (Original)

$\forall x, c\ \neg(Has(x, c) \wedge Furcoat(c)) \vee Fuzzy(x)$ (Eliminate implications)
$\forall x, c\ (\neg Has(x, c) \vee \neg Furcoat(c)) \vee Fuzzy(x)$ (Move $\neg$ inwards)
$\forall x, c\ \neg Has(x, c) \vee \neg Furcoat(c) \vee Fuzzy(x)$ (Drop unnecessary parentheses)
No need to standardize variables or skolemize.
$\neg Has(x, c) \vee \neg Furcoat(c) \vee Fuzzy(x)$ (Drop universal quantifiers)
No need to distribute $\vee$ over $\wedge$.

v. **Rule 5**: $\boldsymbol{\{\neg HasHair(x), Fuzzy(x)\}}$
$\forall x\ HasHair(x) \Rightarrow Fuzzy(x)$ (Original)
$\forall x\ \neg HasHair(x) \vee Fuzzy(x)$ (Eliminate implications)
No need to move $\neg$ inwards, standardize variables, or skolemize.
$\neg HasHair(x) \vee Fuzzy(x)$ (Drop universal quantifiers)
No need to distribute $\vee$ over $\wedge$.

vi. **Rule 6**: $\boldsymbol{\{\neg Has(FuzzyWuzzy, c), \neg Raincoat(c)\}}$
$\neg \exists c\ Has(FuzzyWuzzy, c) \wedge Raincoat(c)$ (Original)
No need to eliminate implications.
$\forall c\ \neg(Has(FuzzyWuzzy, c) \wedge Raincoat(c))$ (Move $\neg$ inwards)
$\forall c\ \neg Has(FuzzyWuzzy, c) \vee \neg Raincoat(c)$ (Move $\neg$ inwards)
No need to standardize variables or skolemize.
$\neg Has(FuzzyWuzzy, c) \vee \neg Raincoat(c)$ (Drop universal quantifiers)
No need to distribute $\vee$ over $\wedge$.

c) Using resolution on your clauses from part b), prove that Fuzzy Wuzzy is fuzzy. Number the steps in your resolution so that steps $1 - n$ are the n CNF clauses from part b) (plus the two clauses from the KB and the negated goal), and each subsequent clause is labeled with the numbers of the two clauses that you resolved to produce it. Also, if any unification was required for a particular step, write the substitution to the right of the resulting clause. For example:

1) $\neg Bear(b), Coat(F(b))$	Clause 1 from Rule 1
2) $\neg Bear(b), Has(b, F(b))$	Clause 2 from Rule 1
3) $\neg Raincoat(c), \neg Furcoat(c)$	From Rule 2
4) $\neg Coat(c), Raincoat(c), Furcoat(c)$	From Rule 3
5) $\neg Has(x, c), \neg Furcoat(c), Fuzzy(x)$	From Rule 4
6) $\neg HasHair(x), Fuzzy(x)$	From Rule 5
7) $\neg Has(FuzzyWuzzy, c), \neg Raincoat(c)$	From Rule 6
8) $Bear(FuzzyWuzzy)$	From KB
9) $\neg HasHair(FuzzyWuzzy)$	From KB
10) $\neg Fuzzy(FuzzyWuzzy)$	Negated Goal
11) $\neg Has(FuzzyWuzzy, c), \neg Furcoat(c)$	(10, 5) {x/FuzzyWuzzy}
12) $\neg Coat(c), Raincoat(c), \neg(Has(FuzzyWuzzy, c))$	(11, 4)
13) $\neg Bear(b), Raincoat(F(b)), \neg Has(FuzzyWuzzy, F(b))$	(12, 1) {c/F(b)}
14) $Raincoat(F(FuzzyWuzzy)), \neg Has(FuzzyWuzzy, F(FuzzyWuzzy))$	(13, 8) {b/FuzzyWuzzy}
15) $\neg Has(FuzzyWuzzy, F(FuzzyWuzzy))$	(14, 7) {c/F(FuzzyWuzzy)}
16) $\neg Bear(FuzzyWuzzy)$	(15, 2) {b/FuzzyWuzzy}
17) $\{\}$	(16, 8)