Simulated Annealing

10/7/2005

Local Search algorithms

- Search algorithms like breadth-first, depth-first or A* explore all the search space systematically by keeping one or more paths in memory and by recording which alternatives have been explored.
- When a goal is found, the path to that goal constitutes a solution.
- Local search algorithms can be very helpful if we are interested in the solution state but not in the path to that goal. They operate only on the current state and move into neighboring states .

Local Search algorithms

- Local search algorithms have 2 key advantages:
  - They use very little memory
  - They can find reasonable solutions in large or infinite (continuous) state spaces.
- Some examples of local search algorithms are:
  - Hill-climbing
  - Random walk
  - Simulated annealing

Annealing

- Annealing is a thermal process for obtaining low energy states of a solid in a heat bath.
- The process contains two steps:
  - Increase the temperature of the heat bath to a maximum value at which the solid melts.
  - Decrease carefully the temperature of the heat bath until the particles arrange themselves in the ground state of the solid. Ground state is a minimum energy state of the solid.
- The ground state of the solid is obtained only if the maximum temperature is high enough and the cooling is done slowly.

Simulated Annealing

- The process of annealing can be simulated with the Metropolis algorithm, which is based on Monte Carlo techniques.
- We can apply this algorithm to generate a solution to combinatorial optimization problems assuming an analogy between them and physical many-particle systems with the following equivalences:
  - Solutions in the problem are equivalent to states in a physical system.
  - The cost of a solution is equivalent to the “energy” of a state.

Simulated Annealing

- To apply simulated annealing with optimization purposes we require the following:
  - A successor function that returns a “close” neighboring solution given the actual one. This will work as the “disturbance” for the particles of the system.
  - A target function to optimize that depends on the current state of the system. This function will work as the energy of the system.
- The search is started with a randomized state. In a polling loop we will move to neighboring states always accepting the moves that decrease the energy while only accepting bad moves accordingly to a probability distribution dependent on the “temperature” of the system.

Simulated Annealing

- Decrease the temperature slowly, accepting less bad moves at each temperature level until at very low temperatures the algorithm becomes a greedy hill-climbing algorithm.
  - The distribution used to decide if we accept a bad movement is know as Boltzman distribution.

$$P(\gamma) = \frac{e^{-E_\gamma / T}}{Z(T)},$$

$$Z(T) = \sum_{\gamma'} e^{-E_{\gamma'} / T},$$

  - This distribution is very well known is in solid physics and plays a central role in simulated annealing. Where γ is the current configuration of the system, E_γ is the energy related with it, and Z is a normalization constant.

Simulated Annealing: the code

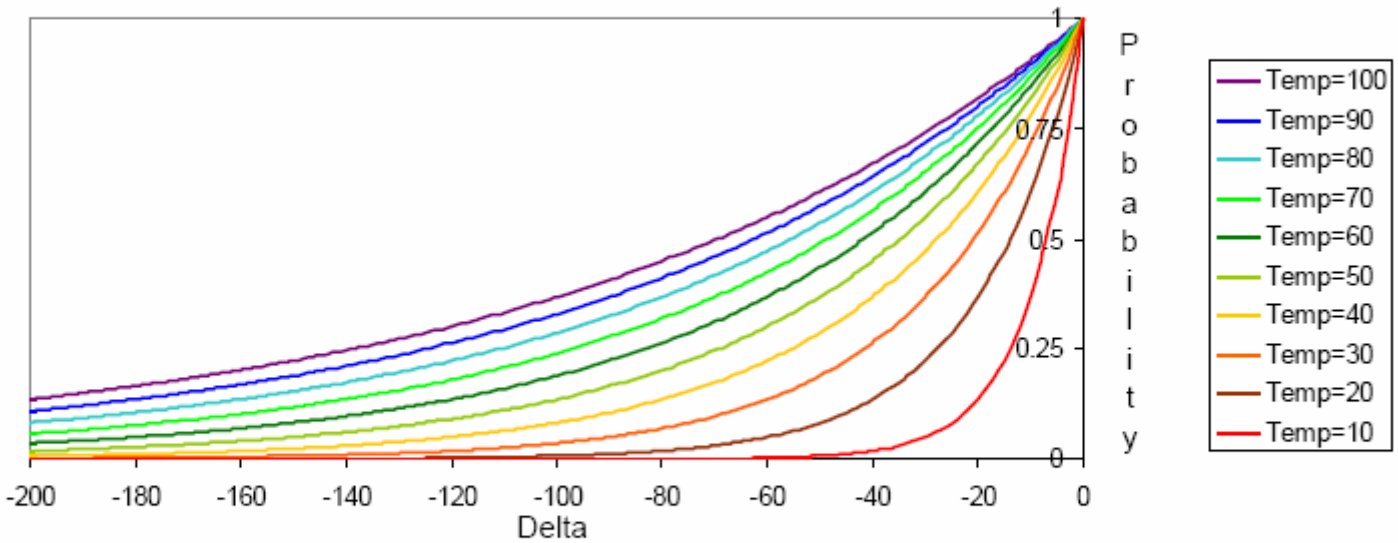
```
1. Create random initial solution γ
2. E_old=cost(γ);
3. for(temp=temp_max; temp>=temp_min;temp=next_temp(temp) ) {
4.     for(i=0;i<i_max; i++ ) {
5.             succesor_func(γ); //this is a randomized function
6.             E_new=cost(γ);
7.             delta=E_new-E_old;
8.             if(delta>0)
9.                     if(random() >= exp(-delta/K*temp);
10.                            undo_func(γ); //rejected bad move
11.                    else
12.                            E_old=E_new  //accepted bad move
13.            else
14.                    E_old=E_new; //always accept good moves
        }
    }
```

Simulated Annealing

- Acceptance criterion and cooling schedule

if (delta>=0) accept

else if ($random < e^{delta/Temp}$) accept, else reject /* 0<=random<=1 */



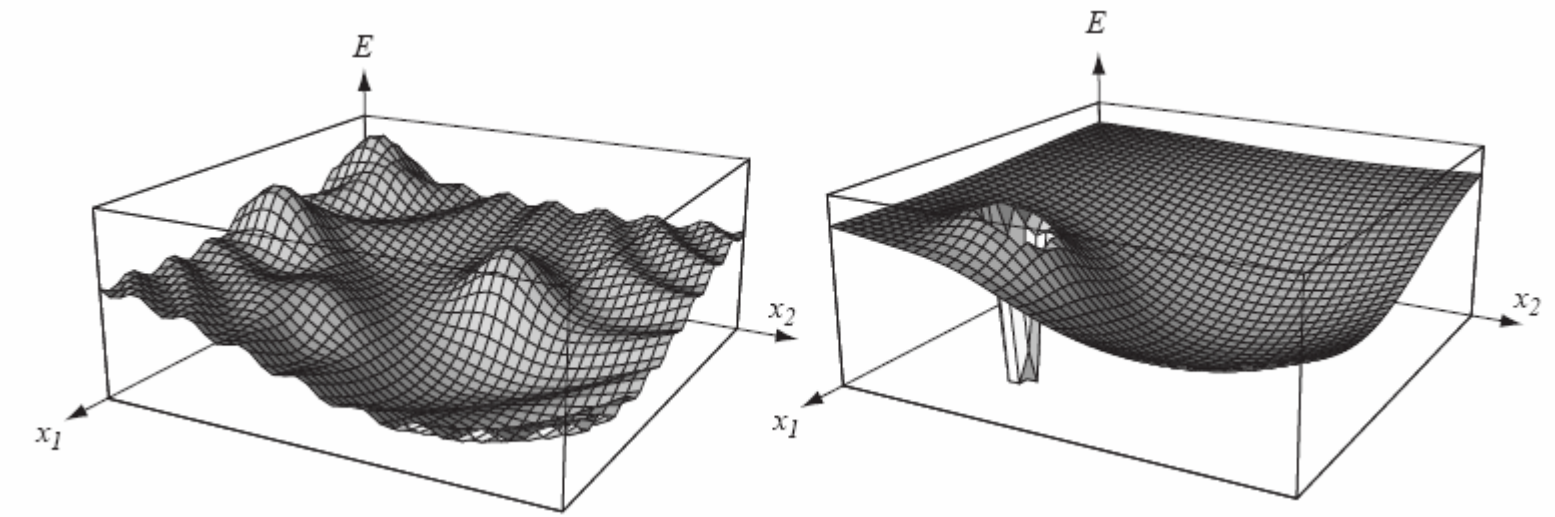
Initially temperature is very high (most bad moves accepted)

Temp slowly goes to 0, with multiple moves attempted at each temperature

Final runs with temp=0 (always reject bad moves) greedily “quench” the system

Practical Issues with simulated annealing

- Cost function must be carefully developed, it has to be “fractal and smooth”.
- The energy function of the left would work with SA while the one of the right would fail.



Practical Issues with simulated annealing

- The cost function should be fast it is going to be called “millions” of times.
- The best is if we just have to calculate the deltas produced by the modification instead of traversing through all the state.
- This is dependent on the application.

Practical Issues with simulated annealing

- In asymptotic convergence simulated annealing converges to globally optimal solutions.
- In practice, the convergence of the algorithm depends of the cooling schedule.
- There are some suggestion about the cooling schedule but it stills requires a lot of testing and it usually depends on the application.

Practical Issues with simulated annealing

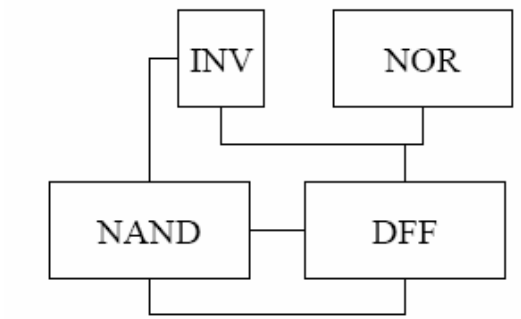
- Start at a temperature where 50% of bad moves are accepted.
- Each cooling step reduces the temperature by 10%
- The number of iterations at each temperature should attempt to move between 1-10 times each “element” of the state.
- The final temperature should not accept bad moves; this step is known as the quenching step.

Applications

- Basic Problems
  - Traveling salesman
  - Graph partitioning
  - Matching problems
  - Graph coloring
  - Scheduling
- Engineering
  - VLSI design
    - Placement
    - Routing
    - Array logic minimization
    - Layout
  - Facilities layout
  - Image processing
  - Code design in information theory

Applications: Placement

- Pick relative location for each gate.
- Seek to improve routeability, limit delay and reduce area.
- This is achieved through the reduction of the signals per routing channel, the balancing row width and the reduction of wirelength.
- The placement also has to follow some restrictions like:
  - I/Os at the periphery
  - Rectangular shape
  - Manhattan routing
- The cost function should balance this multiple objectives while the successor has to comply with the restrictions.



Applications: Placement

- In placement there are multiple complex objective.
- The cost function is difficult to balance and requires testing.
- An example of cost function that balances area efficiency vs. performance.
- Cost=$c_1$area+$c_2$delay+$c_3$power+$c_4$crosstalk
- Where the c_i weights heavily depend on the application and requirements of the project

References

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