Knowledge Representation IV
Inference in First-Order Logic

CSE 473
Logistics

• PS2
  
  Snapshot due today
  
  Final version due in 1 week

• Turn lights down
473 Topics

Perception  NLP  Robotics  Multi-agent

Inference  Supervised Learning  Reinforcement Learning

Logic  Knowledge Representation  Planning  Probability

Search  Problem Spaces  Agency
Logic-Based KR

✓ Propositional logic
  Syntax (CNF, Horn clauses, ...)
  Semantics (Truth Tables)
  Inference (FC, Resolution, DPLL, WalkSAT)
  Restricted Subsets

✓ First-order logic
  Syntax (quantifiers, skolem functions, ...)
  Semantics (Interpretations)
  Inference (FC, Resolution, Compilation)
  Restricted Subsets (e.g. Frame Systems)

Representing events, action & change
# Propositional Logic vs. First Order Logic

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<th><strong>Ontology</strong></th>
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<th><strong>Semantics</strong></th>
<th><strong>Inference Algorithm</strong></th>
<th><strong>Complexity</strong></th>
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<td>Facts (P, Q)</td>
<td>Atomic sentences Connectives</td>
<td>Truth Tables</td>
<td>DPLL, GSAT Fast in practice</td>
<td>NP-Complete</td>
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<tr>
<td>Objects, Properties, Relations</td>
<td>Variables &amp; quantification Sentences have structure: terms father-of(mother-of(X)))</td>
<td>Interpretations (Much more complicated)</td>
<td>Unification Forward, Backward chaining Prolog, theorem proving</td>
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1) One of the women is a biology major
2) Lisa is not next to Dave in the ranking
3) Dave is immediately ahead of Jim
4) Jim is immediately ahead of a bio major
5) Mary or Lisa is ranked first

1. Choose Vocabulary
   Universe: Lisa, Dave, Jim, Mary
   LD = “Lisa is directly ahead of Dave”
   D = “Dave is a Bio Major”

2. Choose initial sentences (wffs)

   1) \( L \lor M \)
   2) \( \neg LD \land \neg DL \)
   3) DJ
   4) \( (JD \land D) \lor (JL \land L) \lor (JM \land M) \)

   Error!
Knowledge Engineering in FOPC

1) One of the women is a biology major
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3) Dave is immediately ahead of Jim
4) Jim is immediately ahead of a bio major
5) Mary or Lisa is ranked first

1. Choose Vocabulary
   Objects, Relations

2. Choose initial sentences
FOL Reasoning

√ FO Forward & Backward Chaining
FO Resolution
Many other types of theorem proving
Restricted representations
Description logics
Compilation to SAT
Unification

- **Match expressions w/ variables**
  - Denoted \( ?x \)

- **Unify**\((x, y)\) returns “mgu”
  - \( \text{Unify(city(?a), city(kent)) returns } {?a/kent} \)

- **Substitute**\((\text{expr, mapping})\) returns new expr
  - \( \text{Substitute(connected(?a, ?b), {?a/kent}) returns connected(kent, ?b)} \)
Unification Example I

- Unify(f(g(?x, dog), ?y)),
  f(g(cat, ?y), dog)

  Unification ok
  Returns {?x / cat,  ?y / dog}

  When substitute in both expressions, they match
  Each is  f(g(cat, dog))
Unification Example II

- **Unify**($f(g(?x)), f(?x))$
  - They don’t unify
  - $\not\exists$ binding, such that substitution makes both expressions the same.
  - E.g. consider: {?x / g(?x) }
  - We get $f(g(g(?x)))$ and $f(g(?x))$ ... not equal!
- A variable value may not **contain** itself
  - Directly or indirectly

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Reminder Resolution

• When trying to prove
  \[ \Sigma \models \Phi \]

• Use proof by contradiction
  Show \( \Sigma \land \neg \Phi \) unsatisfiable

• Resolution rule
Propositional Resolution

[Robinson 1965]

\[ \{ (p \lor \alpha), (\neg p \lor \beta \lor \gamma) \} \vDash_R (\alpha \lor \beta \lor \gamma) \]

Recall Propositional Case:
- Literal in one clause
- Its negation in the other
- Result is disjunction of other literals
First-Order Resolution
[Robinson 1965]

\{ (p(?x) \lor a(a), \ (\neg p(q) \lor b(?x) \lor c(?y))) \} \\

\text{\textasciitilde}_R \\
(a(a) \lor b(q) \lor c(?y))

- Literal in one clause
- The negation of something which unifies in the other
- Result is disjunction of other literals / mgu
First-Order Resolution

- Is it the case that $\Sigma \models \Phi$?
- **Method**
  
  Let $\emptyset = \Sigma \land \neg \Phi$
  
  Convert $\emptyset$ to clausal form
  
  - Standardize variables
  - Move quantifiers to front, skolemize to remove $\exists$
  - Replace $\Rightarrow$ with $\lor$ and $\neg$
  - Demorgan’s laws...

  Resolve until get empty clause
Skolemization

• Existential quantifiers aren’t necessary!
  Existential variables can be replaced by
  • Skolem functions (or constants)
  • Args to function are all surrounding \( \forall \) vars

• \( \forall d \ \exists t \ \text{has}(d, t) \)
  \( \forall d \ \text{has}(d, f(d)) \)

• \( \exists x \ \forall y \ \text{loves}(y, x) \)
  \( \forall y \ \text{loves}(y, f()) \)
  \( \forall y \ \text{loves}(y, f_{97}) \)
Example

• Given
  \( \forall x \, \text{man}(x) \Rightarrow \text{mortal}(x) \)
  \( \forall x \, \text{woman}(x) \Rightarrow \text{mortal}(x) \)
  \( \forall x \, \text{person}(x) \Rightarrow \text{man}(x) \lor \text{woman}(x) \)
  \( \text{person}(\text{kelly}) \)

• Prove
  \( \text{mortal}(\text{kelly}) \)

[\neg \text{m}(x), \text{d}(x)] \ [\neg \text{w}(y), \text{d}(y)] \ [\neg \text{p}(z), \text{m}(z), \text{w}(z)] \ [\text{p}(k)][\neg \text{d}(k)]
Example Continued

\[\neg m(?x), d(?x)\]  \[\neg w(?y), d(?y)\]  \[\neg p(?z), m(?z), w(?z)\]  \[p (k)\]  \[\neg d(k)\]

\[m(k), w(k)\]

\[w(k), d(k)\]

\[w(k)\]

\[d(k)\]

[]
May not Terminate

Given

∀p ∃f P(p) => P(f)
P(joe)

Prove

P(fred)

[ (¬P(p), P(F(p))) (P(joe)) (¬P(fred)) ]

(P(F(joe)))

(P(F(F(joe))))

(P(F(F(F(joe)))))

...
Compilation to Prop. Logic I

• Typed Logic
  \( \forall_{\text{city}} a,b \ \text{connected}(a,b) \)

• Universe
  Cities: seattle, tacoma, enumclaw

• Equivalent propositional formula:
  \( Cst \land Cse \land Cts \land Cte \land Ces \land Cet \)
Compilation to Prop. Logic II

• Typed Logic
  \( \exists_{\text{city}} \ c \ \text{biggest}(c) \)

• Universe
  Cities: seattle, tacoma, enumclaw

• Equivalent propositional formula:
  \( B_s \lor B_t \lor B_e \)
Compilation to Prop. Logic III

• Universe
  • Cities: seattle, tacoma, enumclaw
  • Firms: IBM, Microsoft, Boeing

• First-Order formula
  \[ \forall_{\text{firm } f} \exists_{\text{city } c} \text{ HeadQuarters}(f, c) \]

• Equivalent propositional formula
  \[
  \left[ (\text{HQis} \lor \text{HQit} \lor \text{HQie}) \land \\
  (\text{HQms} \lor \text{HQmt} \lor \text{HQme}) \land \\
  (\text{HQbs} \lor \text{HQbt} \lor \text{HQbe}) \right]
  \]
Hey!

• You said FO Inference is semi-decidable
• But you compiled it to SAT
  Which is NP Complete
• So now we can always do the inference?!?
  Tho it might take exponential time...

• Something seems wrong here....????
Restricted Forms of FO Logic

- **Known, Finite Universes**
  - Compile to SAT

- **Frame Systems**
  - Ban certain types of expressions

- **Horn Clauses**
  - Aka Prolog

- **Function-Free Horn Clauses**
  - Aka Datalog