Knowledge Representation IV
Inference in First-Order Logic

CSE 473

Logistics

• PS2
  Snapshot due today
  Final version due in 1 week

• Turn lights down

473 Topics

Agency
Problem Spaces
Search
Knowledge Representation
Perception
NLP
Multi-agent

Logic-Based KR

Propositional logic
  Syntax (CNF, Horn clauses, …)
  Semantics (Truth Tables)
  Inference (FC, Resolution, DPLL, WalkSAT)
  Restricted Subsets
First-order logic
  Syntax (quantifiers, skolem functions, …)
  Semantics (Interpretations)
  Inference (FC, Resolution, Compilation)
  Restricted Subsets (e.g. Frame Systems)

Representing events, action & change

Propositional Logic vs. First Order

<table>
<thead>
<tr>
<th>Ontology</th>
<th>Facts (P, Q)</th>
<th>Objects, Properties, Relations</th>
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</thead>
<tbody>
<tr>
<td>Syntax</td>
<td>Atomic sentences, Connectives</td>
<td>Variables &amp; quantification, Sentences have structure: terms father-of(mother-of(X))</td>
</tr>
<tr>
<td>Semantics</td>
<td>Truth Tables</td>
<td>Interpretations, (Much more complicated)</td>
</tr>
<tr>
<td>Inference Algorithm</td>
<td>DPLL, GSAT</td>
<td>Fast in practice, Unification, Forward, Backward chaining, Prolog, theorem proving</td>
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<tr>
<td>Complexity</td>
<td>NP-Complete</td>
<td>Semi-decidable</td>
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Prop. Logic: Knowledge Engr

1) One of the women is a biology major
2) Lisa is not next to Dave in the ranking
3) Dave is immediately ahead of Jim
4) Jim is immediately ahead of a bio major
5) Mary or Lisa is ranked first

1. Choose Vocabulary
   Universe: Lisa, Dave, Jim, Mary
   LD = “Lisa is directly ahead of Dave”
   D = “Dave is a Bio Major”

2. Choose initial sentences (wffs)

   1) L ∨ M
   2) ¬LD ∧ ¬DL
   3) DJ
   4) (JD ∧ D) ∨ (JL ∧ L) ∨ (JM ∧ M)

Error!
Knowledge Engineering in FOPC

1) One of the women is a biology major
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1. Choose Vocabulary
   - Objects, Relations
2. Choose initial sentences

FOL Reasoning

- FO Forward & Backward Chaining
- FO Resolution
- Many other types of theorem proving
- Restricted representations
- Description logics
- Compilation to SAT

Unification

- Match expressions w/ variables
  - Denoted ?x
- Unify(x, y) returns "mgu"
  - Unify(city(?a), city(kent)) returns (?a/kent)
- Substitute(expr, mapping) returns new expr
  - Substitute(connected(?a, ?b), (?a/kent))
    returns connected(kent, ?b)

Unification Example I

- Unify(f(g(?x, dog), ?y)), f(g(cat, ?y), dog)
  - Unification ok
  - Returns (?x / cat, ?y / dog)
  - When substitute in both expressions, they match
    Each is f(g(cat, dog))

Unification Example II

- Unify(f(g(?x)), f(?x))
  - They don't unify
  - ¬∃ binding, such that substitution makes both expressions the same.
  - E.g. consider: (?x / g(?x) )
    We get f(g(?x))) and f(g(?x)) ... not equal!
- A variable value may not contain itself
  - Directly or indirectly

Reminder Resolution

- When trying to prove
  - Σ ⊨ Φ
- Use proof by contradiction
  - Show Σ ∧ ¬Φ unsatisfiable
- Resolution rule
Propositional Resolution
[Robinson 1965]
\[
\{ (p \lor \alpha), (\neg p \lor \beta \lor \gamma) \} \vdash_R (\alpha \lor \beta \lor \gamma)
\]
Recall Propositional Case:
- Literal in one clause
- Its negation in the other
- Result is disjunction of other literals

First-Order Resolution
[Robinson 1965]
\[
\{ (p(?x) \lor a(a), (\neg p(?q) \lor b(?x) \lor c(?y)) \} \\
\vdash_R (a(a) \lor b(?q) \lor c(?y))
\]
- Literal in one clause
- The negation of something which unifies in the other
- Result is disjunction of other literals / mgu

First-Order Resolution
• Is it the case that \( \Sigma \models \Phi \)?
• Method
  \[
  \vartheta = \Sigma \land \neg \Phi
  \]
  Convert \( \vartheta \) to clausal form
  • Standardize variables
  • Move quantifiers to front, skolemize to remove \( \exists \)
  • Replace \( \Rightarrow \) with \( \lor \) and \( \neg \)
  • Demorgan’s laws...
  Resolve until get empty clause

Skolemization
• Existential quantifiers aren’t necessary!
  Existential variables can be replaced by
  - Skolem functions (or constants)
  - Args to function are all surrounding \( \forall \) vars
  \[
  \forall d \exists t \text{ has}(d, t) \\
  \exists y \forall x \text{ loves}(y, x)
  \]
  \[
  \forall d \text{ has}(d, f(d)) \\
  \forall y \text{ loves}(y, f()) \\
  \forall y \text{ loves}(y, f_{97})
  \]

Example
• Given
  \[
  \forall x \text{ man}(?x) \Rightarrow \text{mortal}(?x) \\
  \forall x \text{ woman}(?x) \Rightarrow \text{mortal}(?x) \\
  \forall x \text{ person}(?x) \Rightarrow \text{man}(?x) \lor \text{woman}(?x) \\
  \text{person}(\text{kelly})
  \]
• Prove
  \text{mortal}(\text{kelly})

\[
[-\text{m}(?x), \text{d}(?x)] [-\text{w}(?y), \text{d}(?y)] [-\text{p}(?z), \text{m}(?z), \text{w}(?z)] [p \lor [\neg \text{d}(k)]]
\]

Example Continued

\[
[m(k), w(k)] \\
[w(k), d(k)] \\
d(k) \\
c[k] \\
[]
\]
May not Terminate

Given

\( \forall ?p \exists ?f \ P(?p) \Rightarrow P(?f) \)

\( P(\text{joe}) \)

Prove

\[ \neg P(?p), P(F(?p)) \quad P(?f) \quad \neg P(fred) \]

\[ P(F(\text{joe})) \]

\[ P(F(F(\text{joe}))) \]

\[ P(F(F(F(\text{joe})))) \]


Compilation to Prop. Logic I

- **Typed Logic**
  \( \forall \text{city } a, b \ connected(a, b) \)

- **Universe**
  Cities: seattle, tacoma, enumclaw

- Equivalent propositional formula:
  \[ \text{Cst} \land \text{Cse} \land \text{Cts} \land \text{Cte} \land \text{Ces} \land \text{Get} \]


Compilation to Prop. Logic II

- **Typed Logic**
  \( \exists \text{city } c \ biggest(c) \)

- **Universe**
  Cities: seattle, tacoma, enumclaw

- Equivalent propositional formula:
  \[ \text{Bs} \lor \text{Bt} \lor \text{Be} \]


Compilation to Prop. Logic III

- **Universe**
  Cities: seattle, tacoma, enumclaw
  Firms: IBM, Microsoft, Boeing

- **First-Order formula**
  \( \forall \text{firm } f \ \exists \text{city } c \ \text{HeadQuarters}(f, c) \)

- Equivalent propositional formula
  \[ (\text{HQis} \lor \text{HQit} \lor \text{HQie}) \land (\text{HQms} \lor \text{HQmt} \lor \text{HQme}) \land (\text{HQbs} \lor \text{HQbt} \lor \text{HQbe}) \]

Hey!

- You said FO Inference is semi-decidable
- But you compiled it to SAT
  Which is NP Complete
- So now we can always do the inference?!?
  Tho it might take exponential time...

- Something seems wrong here....????

Restricted Forms of FO Logic

- **Known, Finite Universes**
  Compile to SAT

- **Frame Systems**
  Ban certain types of expressions

- **Horn Clauses**
  Aka Prolog

- **Function-Free Horn Clauses**
  Aka Datalog