Knowledge Representation III
First-Order Logic

CSE 473
473 Topics

Perception  NLP  Robotics  Multi-agent

Inference  Supervised Learning  Reinforcement Learning

Logic  Knowledge Representation  Planning

Search  Problem Spaces  Probability  Agency

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Logic-Based KR

✓ Propositional logic
   Syntax (CNF, Horn clauses, ...)
   Semantics (Truth Tables)
   Inference (FC, Resolution, DPLL, WalkSAT)
   Restricted Subsets

First-order logic
   Syntax (quantifiers, skolem functions, ...)
   Semantics (Interpretations)
   Inference (FC, Resolution, Compilation)
   Restricted Subsets (e.g. Frame Systems)

Representing events, action & change

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# Propositional Logic vs. First Order Logic

<table>
<thead>
<tr>
<th><strong>Ontology</strong></th>
<th>Facts (P, Q)</th>
<th>Objects, Properties, Relations</th>
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<tr>
<td><strong>Syntax</strong></td>
<td>Atomic sentences, Connectives</td>
<td>Variables &amp; quantification, Sentences have structure: terms father-of(mother-of(X)))</td>
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<tr>
<td><strong>Semantics</strong></td>
<td>Truth Tables</td>
<td>Interpretations (Much more complicated)</td>
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<tr>
<td><strong>Inference Algorithm</strong></td>
<td>DPLL, GSAT, Fast in practice</td>
<td>Unification, Forward, Backward chaining, Prolog, theorem proving</td>
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<td><strong>Complexity</strong></td>
<td>NP-Complete</td>
<td>Semi-decidable</td>
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FOL Definitions

- **Constants**: a, b, dog33.
  Name a specific object.
- **Variables**: X, Y.
  Refer to an object without naming it.
- **Functions**: dad-of
  Mapping from objects to objects.
- **Terms**: dad-of(dog33)
  Refer to objects
- **Atomic Sentences**: in(dad-of(dog33), food6)
  Can be true or false
  Correspond to propositional symbols P, Q
More Definitions

• **Logical connectives:** and, or, not, =>

• **Quantifiers:**
  - ∀ Forall
  - ∃ There exists

• **Examples**
  - Dumbo is grey
  
  Elephants are grey
  
  There is a grey elephant
Quantifier / Connective Interaction

1. $\forall x \; E(x) \land G(x)$

2. $\forall x \; E(x) \Rightarrow G(x)$

3. $\exists x \; E(x) \land G(x)$

4. $\exists x \; E(x) \Rightarrow G(x)$

E(x) == “x is an elephant”
G(x) == “x has the color grey”
Nested Quantifiers: Order matters!

\[ \forall x \exists y \ P(x,y) \neq \exists y \ \forall x \ P(x,y) \]

• Examples

Every dog has a tail
\[ \forall d \exists t \ has(d,t) \]

Every dog shares a tail!
\[ \exists t \ \forall d \ has(d,t) \]

Someone is loved by everyone
\[ \exists x \ \forall y \ loving(y, x) \]
Semantics

- **Syntax**: a description of the legal arrangements of symbols
  (Def “sentences”)
- **Semantics**: what the arrangement of symbols *means* in the world
Propositional Logic: SEMANTICS

- “Interpretation” (or “possible world”)
- Specifically, TRUTH TABLES

Assignment to each variable either T or F
Assignment of T or F to each connective via defns

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
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</tbody>
</table>

\[ P \land Q \]

Mapping from vars \(\rightarrow\) T/F
Models

• Depiction of one possible model
Interpretations=Mappings
syntactic tokens → model elements

Depiction of one possible interpretation, assuming
Constants: Richard John
Functions: Leg(p,l)
Relations: On(x,y) King(p)
Interpretations = Mappings
syntactic tokens $\rightarrow$ model elements

Another interpretation, same assumptions

Constants: Richard, John
Functions: Leg(p, l), On(x, y)
Relations: King(p)
Satisfiability, Validity, & Entailment

- **S is **valid **if it is true in all interpretations**

- **S is satisfiable if it is true in some interp**

- **S is unsatisfiable if it is false all interps**

- **S1 entails S2 if**
  
  - for all interps where S1 is true, S2 is also true
Skolemization

• Existential quantifiers aren’t necessary!
  Existential variables can be replaced by
  • Skolem functions (or constants)
  • Args to function are all surrounding \( \forall \) vars

• \( \forall d \exists \, \text{has}(d, t) \)
  \( \forall d \, \text{has}(d, f(d)) \)

• \( \exists x \forall y \, \text{loves}(y, x) \)
  \( \forall y \, \text{loves}(y, f()) \)
  \( \forall y \, \text{loves}(y, f_{97}) \)
FOL Reasoning

- FO Forward & Backward Chaining
- FO Resolution
- Many other types of theorem proving
- Restricted representations
  - Description logics
- Compilation to SAT
Forward Chaining

• Given
  \[ \forall x \ lifeform(x) \Rightarrow mortal(x) \]
  \[ \forall x \ mammal(x) \Rightarrow lifeform(x) \]
  \[ \forall x \ dog(x) \Rightarrow mammal(x) \]
  \[ \text{dog(fido)} \]

• Prove
  \[ \text{mortal(fido)} \]

  \[ \forall x \ dog(x) \Rightarrow mammal(x) \]
  \[ \text{dog(fido)} \]
  \[ \frac{}{\text{mammal(fido)}} \]
Unification

- Emphasize variables with ?
- Useful for FO inference (modus ponens, ...)
  Also for compilation of FOPC \( \rightarrow \) propositional

- \texttt{Unify}(x, y) \text{ returns "mgu"}
  \texttt{Unify(city(?a), city(kent)) \text{ returns } ?a/kent}

- \texttt{Substitute}(expr, mapping) \text{ returns new expr}
  \texttt{Substitute(connected(?a, ?b), {?a/kent}) \text{ returns connected(kent, ?b)}}
Unification Examples

• Unify(road(?a, kent), road(seattle, ?b))

• Unify(road(?a, ?a), road(seattle, kent))

• Unify(f(g(?x, dog), ?y)), f(g(cat, ?y), dog)

• Unify(f(g(?x)), f(?x))
Recall Propositional Case:
- Literal in one clause
- Its negation in the other
- Result is disjunction of other literals

\[ \{ (p \lor \alpha), (\neg p \lor \beta \lor \gamma) \} \vdash_R (\alpha \lor \beta \lor \gamma) \]
First-Order Resolution
[Robinson 1965]

\{ (p(?x) \lor a(a), \neg p(q) \lor b(?x) \lor c(?y)) \} \models_R (a(a) \lor b(q) \lor c(?y))

• Literal in one clause
• The negation of something which unifies in the other
• Result is disjunction of other literals / mgu
First-Order Resolution

• Is it the case that $\Sigma \models \Phi$?
• Method

  Let $\emptyset = \Sigma \land \neg \Phi$
  
  Convert $\emptyset$ to clausal form
  • Standardize variables
  • Move quantifiers to front, skolemize to remove $\exists$
  • Replace $\Rightarrow$ with $\lor$ and $\neg$
  • Demorgan’s laws...
  
  Resolve until get empty clause
Example

• **Given**

  \[\forall ?x \text{ man}(?x) \implies \text{mortal}(?x)\]
  \[\forall ?x \text{ woman}(?x) \implies \text{mortal}(?x)\]
  \[\forall ?x \text{ person}(?x) \implies \text{man}(?x) \lor \text{woman}(?x)\]
  \[\text{person}(\text{kelly})\]

• **Prove**

  \[\text{mortal}(\text{kelly})\]

\[\neg \text{m}(?x), \text{d}(?x) \]
\[\neg \text{w}(?y), \text{d}(?y) \]
\[\neg \text{p}(?z), \text{m}(?z), \text{w}(?z) \]
\[\text{p}(\text{k})[\neg \text{d}(\text{k})]\]
Example Continued

\[ \neg m(?x), d(?x) \] \[ \neg w(?y), d(?y) \] \[ \neg p(?z), m(?z), w(?z) \] \[ p (k) \] \[ \neg d(k) \]

\[ m(k), w(k) \]

\[ w(k), d(k) \]

\[ w(k) \]

\[ d(k) \]

\[ \]
KR with Description Logics

**Tbox**
- person
  - father
  - mother
  - grandmother

**Abox**
- mother(jane)
- child-of(jane, bob)
  ...

**Assertions**

**TermDefs**
Tbox

- Term definitions
- FO Language + inference organized into a taxonomy, e.g:
  \[ \text{father}(x) = \text{person}(x) \land \text{male}(x) \land \exists y \text{ childof}(y, x) \]
  \[ \text{parent}(x) = \text{person}(x) \land \exists y \text{ childof}(y, x) \]
- Complexity of classifying new terms
  subsumption

Subsumption hierarchy
Abox

- **Assertions**
- **Abox** – separate language + inference for “propositional” assertions using Tbox terms
  e.g. `person(kelley)"`
Debate

- **Restricted language thesis**
  Disjunction, negation, particularization, order...
  Natural kinds

- **Restricted classification thesis**
  Concepts using contingent information:
  Treatable disease, democratic country, illegal act

- **Counterargument**

- **Constructs: Omit vs limit**
  Completeness
  Efficiency
Compilation to Prop. Logic I

- Typed Logic
  \[ \forall_{\text{city}} a,b \text{ connected}(a,b) \]
- Universe
  Cities: seattle, tacoma, enumclaw
- Equivalent propositional formula:
Compilation to Prop. Logic II

- **Universe**
  - Cities: seattle, tacoma, enumclaw
  - Firms: IBM, Microsoft, Boeing

- **First-Order formula**
  \[ \forall_{\text{city}} c \ \exists_{\text{firm}} f \ \text{hasHQ}(c, f) \]

- **Equivalent propositional formula**
Hey!

• You said FO Inference is semi-decidable
• But you compiled it to SAT
  Which is NP Complete
• So now we can always do the inference?!?
  Tho it might take exponential time...

• Something seems wrong here....?????
Restricted Forms of FO Logic

• Known, Finite Universes
  Compile to SAT
• Frame Systems
  Ban certain types of expressions
• Horn Clauses
  Aka Prolog
• Function-Free Horn Clauses
  Aka Datalog