Knowledge Representation I
(Propositional Logic)

CSE 473
Some KR Languages

- Propositional Logic
- Predicate Calculus
- Frame Systems
- Rules with Certainty Factors
- Bayesian Belief Networks
- Influence Diagrams
- Semantic Networks
- Concept Description Languages
- Nonmonotonic Logic
In Fact...

• All popular knowledge representation systems are equivalent to (or a subset of) Logic
  • Either Propositional Logic
  • Or Predicate Calculus
Probability Theory
What is Propositional Logic?

• And why have you studied it?

• And why are we torturing you again?
473 Topics

Perception  NLP  Robotics  Multi-agent

Inference  Supervised Learning  Reinforcement Learning

Logic  Knowledge Representation  Planning

Search  Problem Spaces  Probability

Agency

© Daniel S. Weld
AI=Knowledge Representation & Reasoning

- Syntax
- Semantics
- Inference Procedure
  - Algorithm
  - Sound?
  - Complete?
  - Complexity

Knowledge Engineering
Basic Idea of Logic

• By starting with true assumptions, you can deduce true conclusions.
Truth

• Francis Bacon (1561-1626)
  • No pleasure is comparable to the standing upon the vantage-ground of truth.

• Thomas Henry Huxley (1825-1895)
  • Irrationally held truths may be more harmful than reasoned errors.

• John Keats (1795-1821)
  • Beauty is truth, truth beauty; that is all
  • Ye know on earth, and all ye need to know.

• Blaise Pascal (1623-1662)
  • We know the truth, not only by the reason, but also by the heart.

• François Rabelais (c. 1490-1553)
  • Speak the truth and shame the Devil.

• Daniel Webster (1782-1852)
  • There is nothing so powerful as truth, and often nothing so strange.
Propositional Logic

• Syntax
  Atomic sentences: P, Q, ...
  Connectives: ∧, ∨, ¬, ⇒

• Semantics
  Truth Tables

• Inference
  Modus Ponens
  Resolution
  DPLL
  GSAT

• Complexity
Propositional Logic: Syntax

• **Atoms**
  
  \( P, Q, R, \ldots \)

• **Literals**

  \( P, \neg P \)

• **Sentences**

  Any literal is a sentence
  
  If \( S \) is a sentence
  
  • Then \((S \land S)\) is a sentence
  
  • Then \((S \lor S)\) is a sentence

• **Conveniences**

  \( P \supset Q \) same as \( \neg P \lor Q \)
Special Syntactic Forms

- **General Form:**
  \[
  ((q \land \neg r) \supset s) \land \neg (s \land t)
  \]

- **Conjunction Normal Form (CNF)**
  \[
  (\neg q \lor r \lor s) \land (\neg s \lor \neg t)
  \]
  
  Set notation: \{ (\neg q, r, s), (\neg s, \neg t) \}
  
  empty clause () = false

- **Binary clauses:** 1 or 2 literals per clause
  \[
  (\neg q \lor r) \quad (\neg s \lor \neg t)
  \]

- **Horn clauses:** 0 or 1 positive literal per clause
  \[
  (\neg q \lor \neg r \lor s) \quad (\neg s \lor \neg t)
  \]
  \[
  (q \land r) \supset s \quad (s \land t) \supset false
  \]
Semantics

- **Syntax**: a description of the *legal* arrangements of symbols
  (Def “sentences”)
- **Semantics**: what the arrangement of symbols *means* in the world
Propositional Logic: SEMANTICS

- "Interpretation" (or "possible world")
  Assignment to each variable either T or F
  Assignment of T or F to each connective via defns

\[
\begin{array}{c|c|c}
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

\[
\begin{array}{c|c|c}
P & Q & P \lor Q \\
\hline
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

\[
\begin{array}{c|c}
P & \neg P \\
\hline
T & F \\
F & T \\
\end{array}
\]
Satisfiability, Validity, & Entailment

• S is **satisfiable** if it is true in *some* world

• S is **unsatisfiable** if it is false *all* worlds

• S is **valid** if it is true in *all* worlds

• S1 **entails** S2 if *wherever* S1 is true S2 is also true
Examples

\[ P \Rightarrow Q \]

\[ R \Rightarrow \neg R \]

\[ S \land (W \land \neg S) \]

\[ T \lor \neg T \]

\[ X \Rightarrow X \]
Notation

\[ \Rightarrow \quad \cap \quad \cup \quad \) \quad \textbf{Implication} \ (\text{syntactic symbol})

\begin{align*}
\text{Proves:} & \quad S_1 \dashv\vdash S_2 \text{ if `ie' algorithm says `S2' from } S_1 \\
\text{Entails:} & \quad S_1 \models S_2 \text{ if wherever } S_1 \text{ is true } S_2 \text{ is also true}
\end{align*}

- \textbf{Sound} \quad \dashv \Rightarrow \models
- \textbf{Complete} \quad \models \Rightarrow \dashv
Prop. Logic: Knowledge Engr

1) One of the women is a biology major
2) Lisa is not next to Dave in the ranking
3) Dave is immediately ahead of Jim
4) Jim is immediately ahead of a bio major
5) Mary or Lisa is ranked first

1. Choose Vocabulary
   Universe: Lisa, Dave, Jim, Mary
   LD = “Lisa is immediately ahead of Dave"
   D = “Dave is a Bio Major”

2. Choose initial sentences (wffs)
Reasoning Tasks

• **Model finding**
  
  KB = background knowledge
  
  S = description of problem
  
  Show \((KB \land S)\) is satisfiable
  
  A kind of *constraint satisfaction*

• **Deduction**

  S = question

  Prove that \(KB \models S\)

  Two approaches:
  
  1. Rules to derive new formulas from old
     (inference)
  
  2. Show \((KB \land \neg S)\) is unsatisfiable
Propositional Logic: Inference

A mechanical process for computing new sentences

1. Backward & Forward Chaining
   Based on rule of *modus ponens*

   If know $P_1, ..., P_n$ & know $(P_1 \land ... \land P_n) \Rightarrow Q$
   Then can conclude $Q$

2. Resolution (Proof by Contradiction)
3. GSAT
4. Davis Putnam