Constraint Satisfaction

CSE 473
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Question from last time.

- What's the relationship between Uniform Cost search and Dijkstra's algorithm?
- Answer: essentially the same.
- Paradox: Dijkstra's algorithm is "fast" O(e*logn) and search is "slow" O(b^d), but they are the same…how can that be?

Today:
Constraint Satisfaction Problems

Definition

Factoring state spaces

- Backtracking policies
- Variable-ordering heuristics
- Preprocessing algorithms

Constraint Satisfaction

- Kind of ***search*** in which
  - States are ***factored*** into sets of variables
  - Search = assigning values to these variables
  - Structure of space is encoded with constraints
- Backtracking-style algorithms work

- But other techniques add speed
  - Propagation
  - Variable ordering
  - Preprocessing

Chinese Food as Search?

- **States?**
  - Partially specified meals
- **Operators?**
  - Add, remove, change dishes
- **Start state?**
  - Null meal
- **Goal states?**
  - Meal meeting certain conditions (rating?)

Factoring States

- Rather than state = meal
- Model state's (independent) parts, e.g.

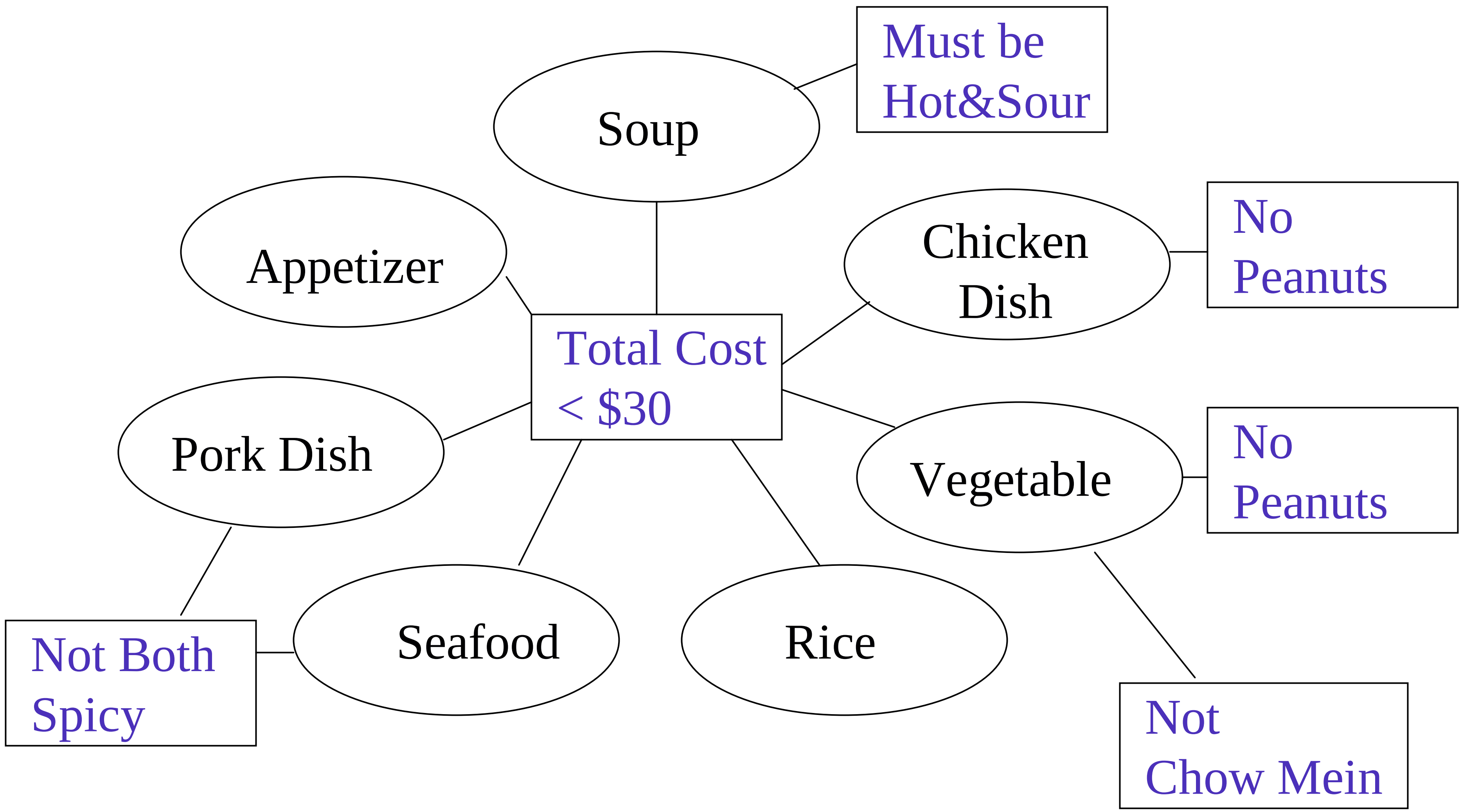
    Suppose every meal for n people
    Has n dishes plus soup

    Soup =
    Meal 1 =
    Meal 2 =
    …
    Meal n =

- Or… physical state =

    X coordinate =
    Y coordinate =

Chinese Constraint Network



CSPs in the Real World

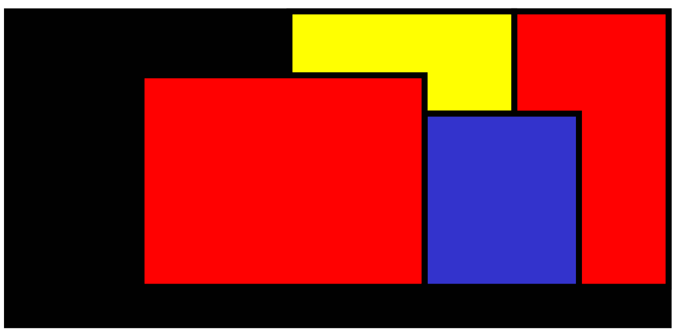
- Scheduling space shuttle repair
- Airport gate assignments
- Transportation Planning
- Supply-chain management
- Computer configuration
- Diagnosis
- UI optimization
- Etc...

Binary Constraint Network

- Any FD constraints can be reduced to 'binary'
- Set of n ***variables***: $x_1 \dots x_n$
- Value ***domains*** for each variable: $D_1 \dots D_n$
- Set of binary ***constraints*** (also "relations")

  $R_{ij} \subseteq D_i \times D_j$

  Specifies which values pair $(x_i\ x_j)$ are consistent



- V for each country
- Each domain = 4 colors
- R_{ij} enforces $\neq$

Binary Constraint Network

Partial ***assignment*** of values = tuple of pairs

- {...(x, a)...} means variable x gets value a...

Tuple= ***consistent*** if all constraints satisfied

Tuple= ***full solution*** if consistent + has all vars

Tuple $\{(x_i, a_i) \ldots (x_j, a_j)\}$ = ***consistent w/ a set of vars*** $\{x_m \ldots x_n\}$

iff $\exists\ a_m \ldots a_n$ such that

$\{(x_i, a_i)\ldots(x_j, a_j), (x_m, a_m)\ldots(x_n, a_n)\}$ = consistent

Difficulty of CSP

- How hard is it to solve a Boolean CSP?
  (so we why do we care about algorithms?)
- CSPs can have infinite domains (e.g., integers)
- Linear programming = integer domain + linear constraints.

Cryptarithmetic

- ~~State Space~~
  - ~~Set of states~~
  - ~~Operators [and costs]~~
  - ~~Start state~~
  - ~~Goal states~~

```
   SEND
+  MORE
------
  MONEY
```

- Variables?
- Domains (variable values)?
- Constraints? (alldiff(..) can be represented as binary inequalities!

Classroom Scheduling

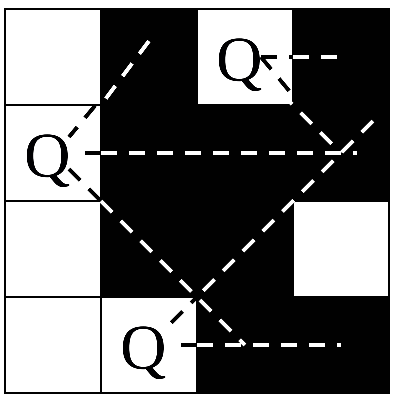
- Variables?
- Domains (possible values for variables)?
- Constraints?

N Queens

- As a CSP?

N Queens

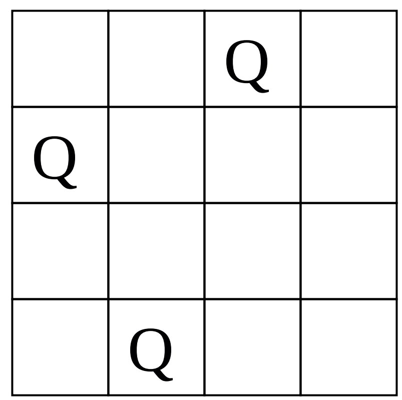
- Variables = board columns
- Domain values = rows



- {(x_1, 2), (x_2, 4), (x_3, 1)} consistent with (x_4)
- Shorthand: "{2, 4, 1} consistent with x_4"

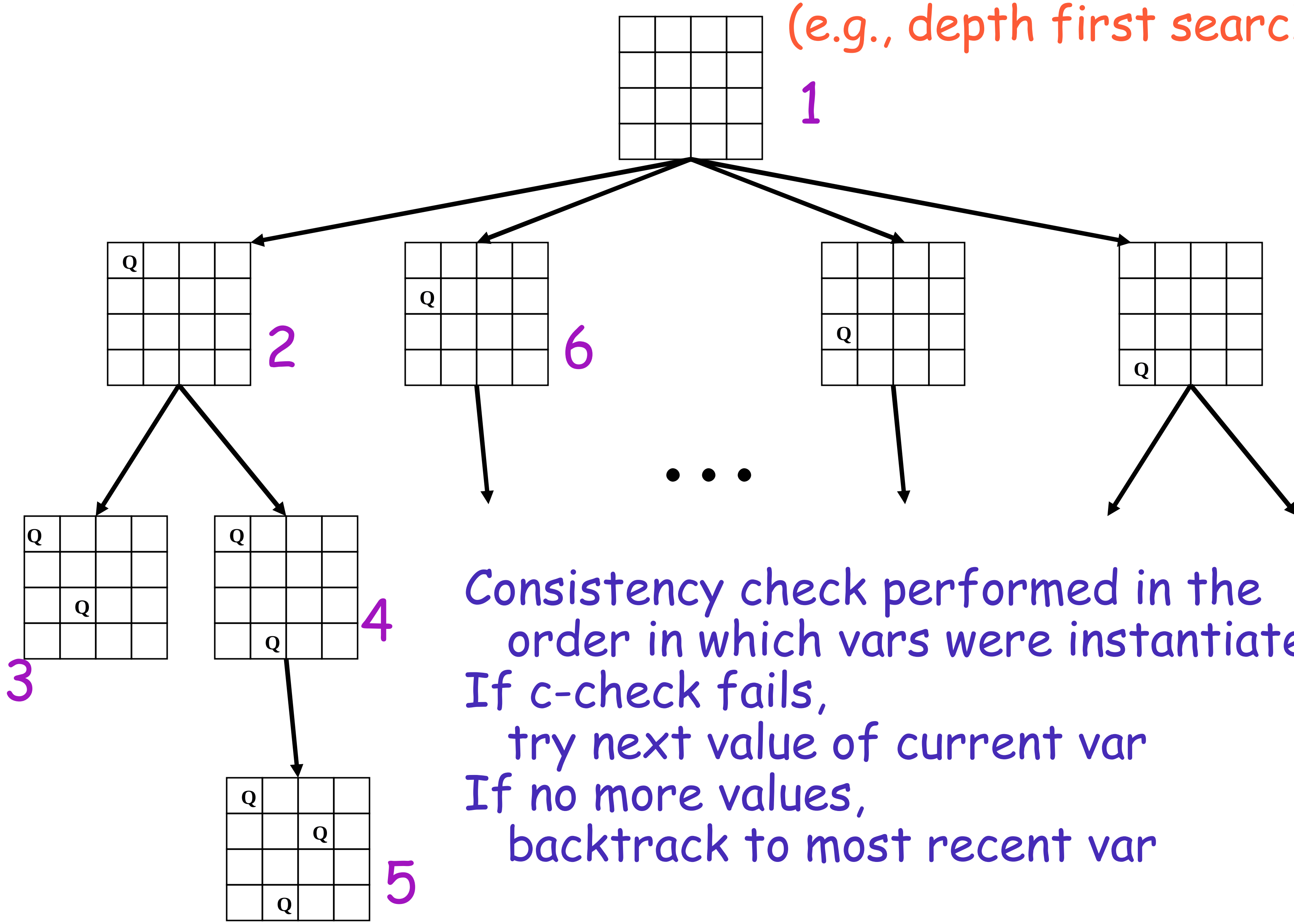
CSP as a search problem?

- What are states?
  (N variables)
- What are the operators?
  (assign D possible values)
- What is the branching factor? O(N * D)
- Commutative→D
- Initial state?
- Goal test?



Chronological Backtracking (BT)

(e.g., depth first search)



Consistency check performed in the order in which vars were instantiated

If c-check fails, try next value of current var

If no more values, backtrack to most recent var

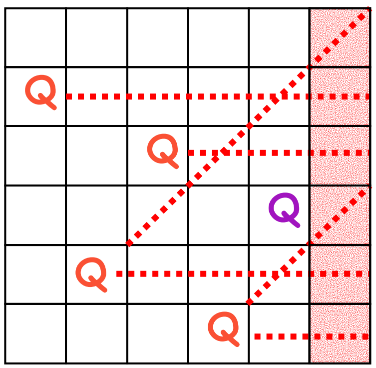
Backjumping (BJ)

- Similar to BT, but

  more efficient when no consistent instantiation can be found for the current var

- Instead of backtracking to most recent var…

  BJ reverts to deepest var which was c-checked against the current var



BJ Discovers

(2, 5, 3, 6) inconsistent with x_6

No sense trying other values of x_5

Conflict-Directed Backjumping (CBJ)

- More sophisticated backjumping behavior
- Each variable has ***conflict set CS***

    Set of vars that failed c-checks w/ current val

    Update this set on every failed c-check

- When no more values to try for x_i

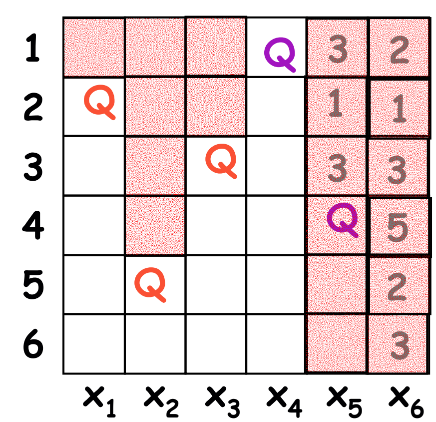
    Backtrack to deepest var, x_d, in $CS(x_i)$

    And update $CS(x_d) := CS(x_d) \cup CS(x_i) - \{x_d\}$

CBJ Discovers (2, 5, 3) inconsistent with $\{x_5, x_6\}$

$CS(x_5)$: 1,2,3

$CS(x_6)$: 1,2,3,5



BT vs. BJ vs. CBJ

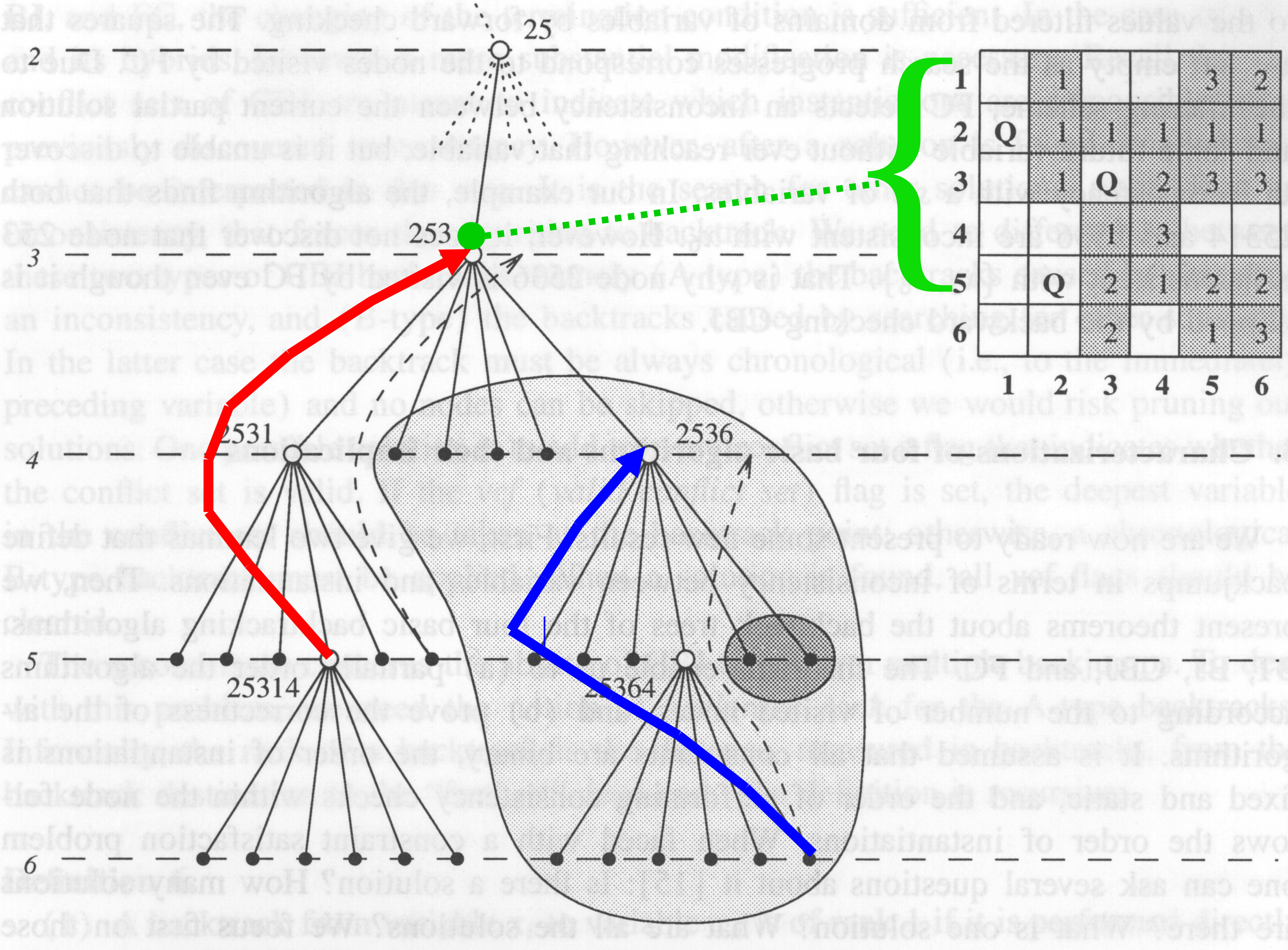


Fig. 2. A fragment of the BT backtrack tree for the 6-queens problem.

Forward Checking (FC)

- Perform Consistency Check ***Forward***
- Whenever a var is assigned a value

  Prune inconsistent values from

  As-yet unvisited variables

  Backtrack if domain of any var ever collapses

FC only visits consistent nodes

but not *all* such nodes

skips (2, 5, 3, 4) which CBJ visits

But FC can't detect that

(2, 5, 3) inconsistent with {x_5, x_6}