First-order logic

Chapter 7



## Outline

◊ Syntax and semantics of FOL

◊ Fun with sentences

◊ Wumpus world in FOL



## Syntax of FOL: Basic elements

| | |
|---|---|
| Constants | $KingJohn,\ 2,\ UCB, \ldots$ |
| Predicates | $Brother,\ >, \ldots$ |
| Functions | $Sqrt,\ LeftLegOf, \ldots$ |
| Variables | $x,\ y,\ a,\ b, \ldots$ |
| Connectives | $\wedge\ \vee\ \neg\ \Rightarrow\ \Leftrightarrow$ |
| Equality | $=$ |
| Quantifiers | $\forall\ \exists$ |



## Atomic sentences

Atomic sentence = $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term = $function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$



## Complex sentences

Complex sentences are made from atomic sentences using connectives

$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg>(1,2)$



## Truth in first-order logic

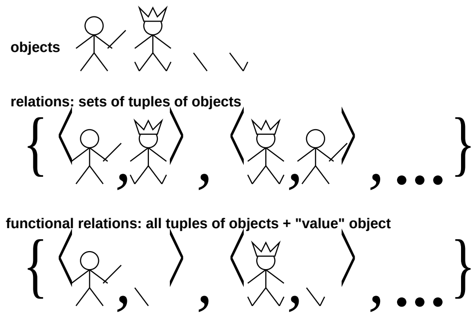
Sentences are true with respect to a model and an interpretation

Model contains objects and relations among them

Interpretation specifies referents for
*constant symbols* → objects
*predicate symbols* → relations
*function symbols* → functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

## Models for FOL: Example






## Universal quantification

$\forall \langle variables \rangle \;\; \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \;\; At(x, Berkeley) \;\Rightarrow\; Smart(x)$

$\forall x \;\; P$ is equivalent to the conjunction of instantiations of $P$

$$\begin{aligned} & At(KingJohn, Berkeley) \;\Rightarrow\; Smart(KingJohn) \\ \wedge \;& At(Richard, Berkeley) \;\Rightarrow\; Smart(Richard) \\ \wedge \;& At(Berkeley, Berkeley) \;\Rightarrow\; Smart(Berkeley) \\ \wedge \;& \ldots \end{aligned}$$

Typically, $\Rightarrow$ is the main connective with $\forall$.
Common mistake: using $\wedge$ as the main connective with $\forall$:

$\forall x \;\; At(x, Berkeley) \wedge Smart(x)$

means "Everyone is at Berkeley and everyone is smart"



## Existential quantification

$\exists \langle variables \rangle \;\; \langle sentence \rangle$

Someone at Stanford is smart:
$\exists x \;\; At(x, Stanford) \wedge Smart(x)$

$\exists x \;\; P$ is equivalent to the disjunction of instantiations of $P$

$$\begin{aligned} & At(KingJohn, Stanford) \wedge Smart(KingJohn) \\ \vee \;& At(Richard, Stanford) \wedge Smart(Richard) \\ \vee \;& At(Stanford, Stanford) \wedge Smart(Stanford) \\ \vee \;& \ldots \end{aligned}$$

Typically, $\wedge$ is the main connective with $\exists$.
Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$\exists x \;\; At(x, Stanford) \;\Rightarrow\; Smart(x)$

is true if there is anyone who is not at Stanford!



## Properties of quantifiers

$\forall x \;\; \forall y$ is the same as $\forall y \;\; \forall x$ (why??)

$\exists x \;\; \exists y$ is the same as $\exists y \;\; \exists x$ (why??)

$\exists x \;\; \forall y$ is not the same as $\forall y \;\; \exists x$

$\exists x \;\; \forall y \;\; Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y \;\; \exists x \;\; Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x \;\; Likes(x, IceCream) \qquad \neg\exists x \;\; \neg Likes(x, IceCream)$

$\exists x \;\; Likes(x, Broccoli) \qquad \neg\forall x \;\; \neg Likes(x, Broccoli)$



## Fun with sentences

Brothers are siblings

.

"Sibling" is reflexive

.

One's mother is one's female parent

.

A first cousin is a child of a parent's sibling

.



.

$\forall x, y \;\; Brother(x, y) \;\Leftrightarrow\; Sibling(x, y).$

.

$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$

.

$\forall x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) and Parent(x, y))$

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$\forall x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists p, ps \;\; Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$

## Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., $1 = 2$ and $\forall x\ \times(Sqrt(x), Sqrt(x)) = x$ are satisfiable
$2 = 2$ is valid

E.g., definition of (full) $Sibling$ in terms of $Parent$:
$\forall x, y\ \ Sibling(x, y) \Leftrightarrow [\neg(x = y) \land \exists m, f\ \ \neg(m = f) \land Parent(m, x) \land Parent(f, x) \land Parent(m, y) \land Parent(f, y)]$



## Interacting with FOL KBs

Suppose a wumpus-world agent is using an FOL KB
and perceives a smell and a breeze (but no glitter) at $t = 5$:

$\textsc{Tell}(KB, Percept([Smell, Breeze, None], 5))$
$\textsc{Ask}(KB, \exists a\ \ Action(a, 5))$

I.e., does the KB entail any particular actions at $t = 5$?

Answer: $Yes$, $\{a/Shoot\}$ $\leftarrow$ substitution (binding list)

Given a sentence $S$ and a substitution $\sigma$,
$S\sigma$ denotes the result of plugging $\sigma$ into $S$; e.g.,
$S = Smarter(x, y)$
$\sigma = \{x/Hillary, y/Bill\}$
$S\sigma = Smarter(Hillary, Bill)$

$\textsc{Ask}(\text{KB}, \text{S})$ returns some/all $\sigma$ such that $KB \models S\sigma$



## Knowledge base for the wumpus world

"Perception"
$\forall b, g, t\ \ Percept([Smell, b, g], t) \Rightarrow Smelt(t)$
$\forall s, b, t\ \ Percept([s, b, Glitter], t) \Rightarrow AtGold(t)$

Reflex: $\forall t\ \ AtGold(t) \Rightarrow Action(Grab, t)$

Reflex with internal state: do we have the gold already?
$\forall t\ \ AtGold(t) \land \neg Holding(Gold, t) \Rightarrow Action(Grab, t)$

$Holding(Gold, t)$ cannot be observed
$\Rightarrow$ keeping track of change is essential



## Deducing hidden properties

Properties of locations:
$\forall l, t\ \ At(Agent, l, t) \land Smelt(t) \Rightarrow Smelly(l)$
$\forall l, t\ \ At(Agent, l, t) \land Breeze(t) \Rightarrow Breezy(l)$

Squares are breezy near a pit:

Diagnostic rule—infer cause from effect
$\forall y\ \ Breezy(y) \Rightarrow \exists x\ \ Pit(x) \land Adjacent(x, y)$

Causal rule—infer effect from cause
$\forall x, y\ \ Pit(x) \land Adjacent(x, y) \Rightarrow Breezy(y)$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

Definition for the $Breezy$ predicate:
$\forall y\ \ Breezy(y) \Leftrightarrow [\exists x\ \ Pit(x) \land Adjacent(x, y)]$

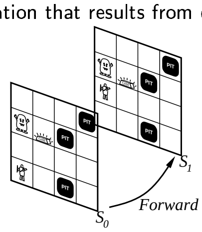


## Keeping track of change

Facts hold in situations, rather than eternally
E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

Situation calculus is one way to represent change in FOL:
Adds a situation argument to each non-eternal predicate
E.g., $Now$ in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing $a$ is $s$






## Describing actions I

"Effect" axiom—describe changes due to action
$\forall s\ \ AtGold(s) \Rightarrow Holding(Gold, Result(Grab, s))$

"Frame" axiom—describe non-changes due to action
$\forall s\ \ HaveArrow(s) \Rightarrow HaveArrow(Result(Grab, s))$

Frame problem: find an elegant way to handle non-change
(a) representation—avoid frame axioms
(b) inference—avoid repeated "copy-overs" to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .

## Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is "about" a predicate (not an action per se):

P true afterwards $\Leftrightarrow$ [an action made P true

$\vee$ P true already and no action made P false]

For holding the gold:

$\forall a, s \;\; Holding(Gold, Result(a, s)) \;\Leftrightarrow$
$[(a = Grab \wedge AtGold(s))$
$\vee\, (Holding(Gold, s) \wedge a \neq Release)]$



## Making plans

Initial condition in KB:
$At(Agent, [1,1], S_0)$
$At(Gold, [1,2], S_0)$

Query: $\textsc{Ask}(KB, \exists\, s \;\; Holding(Gold, s))$
i.e., in what situation will I be holding the gold?

Answer: $\{s/Result(Grab, Result(Forward, S_0))\}$
i.e., go forward and then grab the gold

This assumes that the agent is interested in plans starting at $S_0$ and that $S_0$ is the only situation described in the KB



## Making plans: A better way

Represent plans as action sequences $[a_1, a_2, \ldots, a_n]$

$PlanResult(p, s)$ is the result of executing $p$ in $s$

Then the query $\textsc{Ask}(KB, \exists\, p \;\; Holding(Gold, PlanResult(p, S_0)))$
has the solution $\{p/[Forward, Grab]\}$

Definition of $PlanResult$ in terms of $Result$:
$\forall s \;\; PlanResult([], s) = s$
$\forall a, p, s \;\; PlanResult([a|p], s) = PlanResult(p, Result(a, s))$

Planning systems are special-purpose reasoners designed to do this type of inference more efficiently than a general-purpose reasoner



## Summary

First-order logic:
– objects and relations are semantic primitives
– syntax: constants, functions, predicates, equality, quantifiers

Increased expressive power: sufficient to define wumpus world

Situation calculus:
– conventions for describing actions and change in FOL
– can formulate planning as inference on a situation calculus KB