

Inference from joint distributions

Typically, we are interested in
the posterior joint distribution of the **query variables** \mathbf{Y}
given specific values e for the **evidence variables** \mathbf{E}

Let the **hidden variables** be $\mathbf{H} = \mathbf{X} - \mathbf{Y} - \mathbf{E}$

Then the required summation of joint entries is done by summing out the hidden variables:

$$P(\mathbf{Y}|\mathbf{E}=e) = \alpha P(\mathbf{Y}, \mathbf{E}=e) = \alpha \sum_{\mathbf{h}} P(\mathbf{Y}, \mathbf{E}=e, \mathbf{H}=\mathbf{h})$$

The terms in the summation are joint entries because \mathbf{Y} , \mathbf{E} , and \mathbf{H} together exhaust the set of random variables

Obvious problems:

- 1) Worst-case time complexity $O(d^n)$ where d is the largest arity
- 2) Space complexity $O(d^n)$ to store the joint distribution
- 3) How to find the numbers for $O(d^n)$ entries???

Independence

Two random variables A B are (absolutely) **independent** iff

$$P(A|B) = P(A)$$

$$\text{or } P(A, B) = P(A|B)P(B) = P(A)P(B)$$

e.g., A and B are two coin tosses

If n Boolean variables are independent, the full joint is

$$P(X_1, \dots, X_n) = \prod_i P(X_i)$$

hence can be specified by just n numbers

Absolute independence is a very strong requirement, seldom met

Conditional independence

Consider the dentist problem with three random variables:

$Toothache$, $Cavity$, $Catch$ (steel probe catches in my tooth)

The full joint distribution has $2^3 - 1 = 7$ independent entries

If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:

(1) $P(Catch|Toothache, Cavity) = P(Catch|Cavity)$
i.e., $Catch$ is **conditionally independent** of $Toothache$ given $Cavity$

The same independence holds if I haven't got a cavity:

(2) $P(Catch|Toothache, \neg Cavity) = P(Catch|\neg Cavity)$

Conditional independence contd.

Equivalent statements to (1)

(1a) $P(Toothache|Catch, Cavity) = P(Toothache|Cavity)$ **Why??**

(1b) $P(Toothache, Catch|Cavity) = P(Toothache|Cavity)P(Catch|Cavity)$ **Why'?**

Full joint distribution can now be written as

$$\begin{aligned} \mathbf{P}(Toothache, Catch, Cavity) &= \mathbf{P}(Toothache, Catch|Cavity)\mathbf{P}(Cavity) \\ &= \mathbf{P}(Toothache|Cavity)\mathbf{P}(Catch|Cavity)\mathbf{P}(Cavity) \end{aligned}$$

i.e., $2 + 2 + 1 = 5$ independent numbers