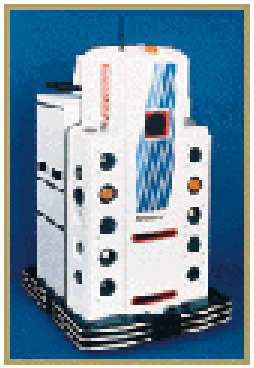
Example: service robot



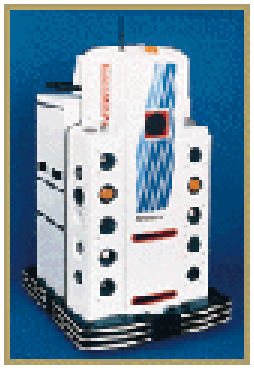
states??:

operators??:

goal test??:

path cost??:

Example: service robot



states??: real-valued coordinates of manipulator joint angles (retracted?)
real-valued position of robot (which room, floor?)
doors (where,open,closed) , people (where)
coffee machine (where, full)

operators??: continuous motions of robot (abstract navigation actions)

goal test??: coffee delivered to Henry

path cost??: time to execute

Search algorithms

Basic idea:

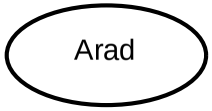
offline, simulated exploration of state space

by generating successors of already-explored states

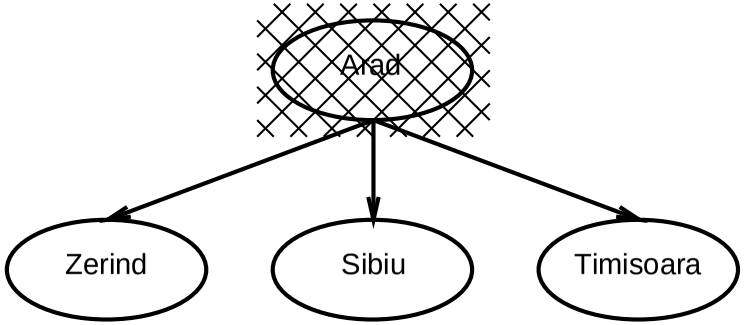
(a.k.a. *expanding* states)

```
function GENERAL-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
       if there are no candidates for expansion then return failure
       choose a leaf node for expansion according to strategy
       if the node contains a goal state then return the corresponding solution
       else expand the node and add the resulting nodes to the search tree
   end
```

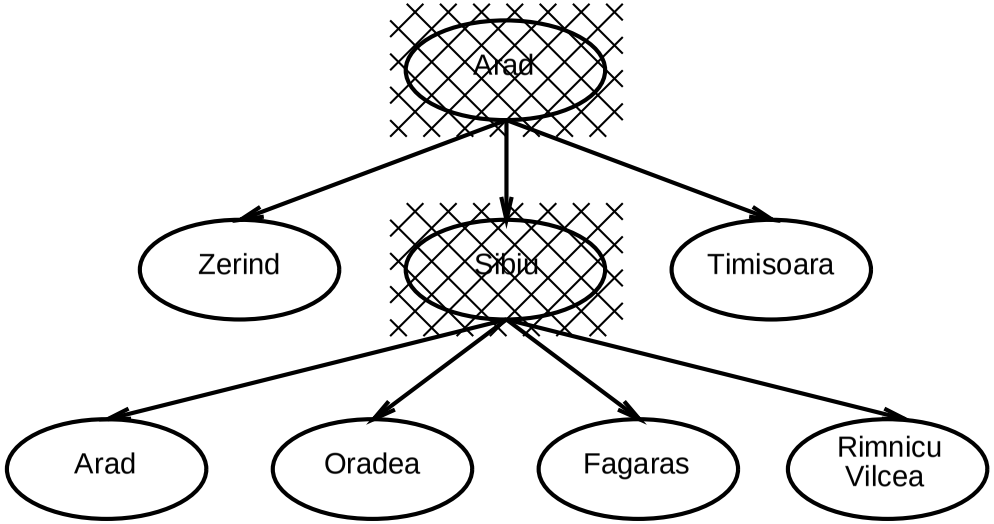
General search example



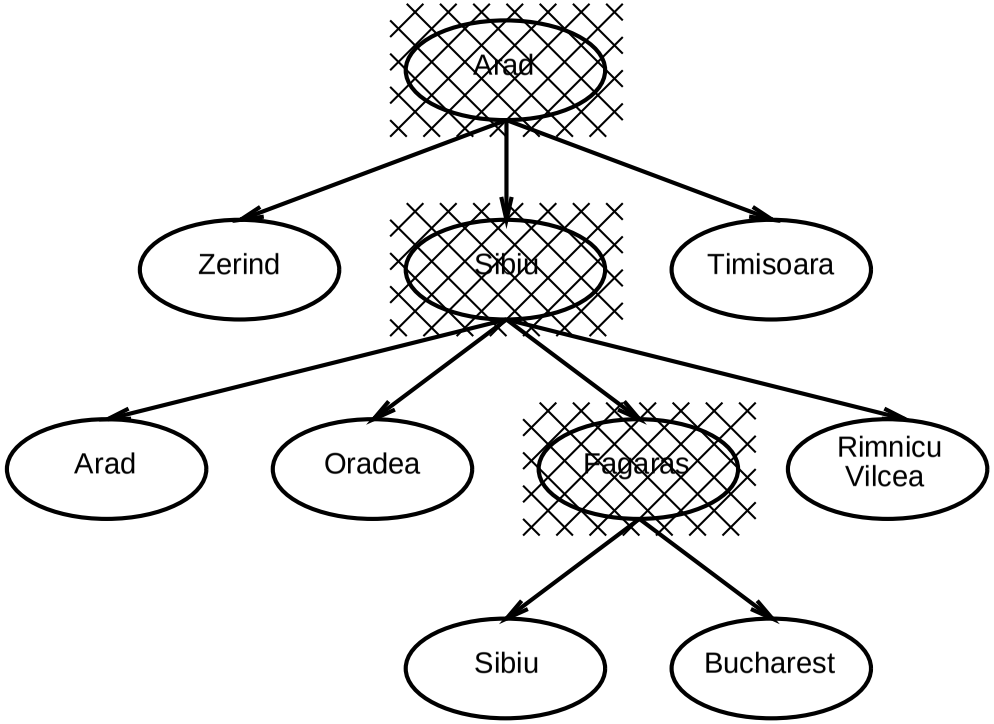
General search example



General search example



General search example



Implementation of search algorithms

```
function GENERAL-SEARCH( problem, QUEUING-FN) returns a solution, or failure
    nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
    loop do
        if nodes is empty then return failure
        node ← REMOVE-FRONT(nodes)
        if GOAL-TEST[problem] applied to STATE(node) succeeds then return node
        nodes ← QUEUING-FN(nodes, EXPAND(node, OPERATORS[problem]))
    end
```

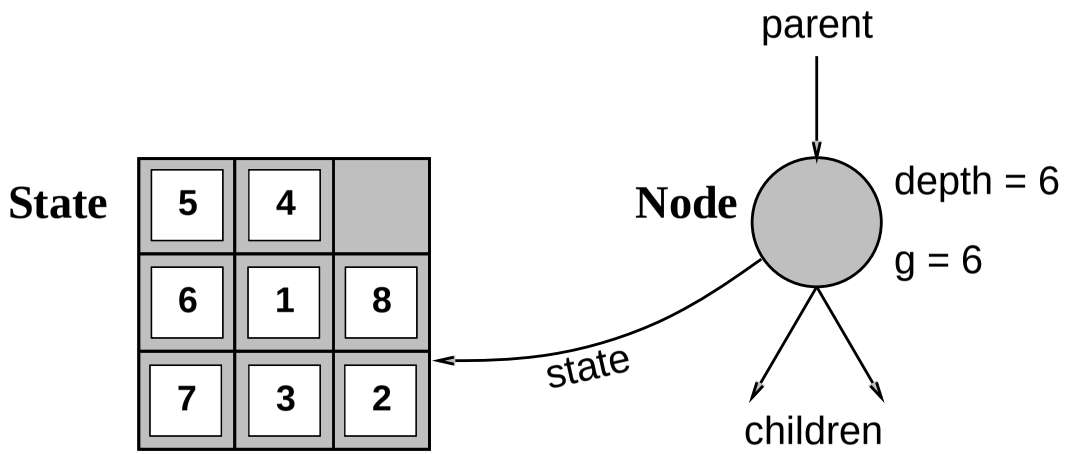
Implementation contd: states vs. nodes

A *state* is a (representation of) a physical configuration

A *node* is a data structure constituting part of a search tree

includes *parent*, *children*, *depth*, *path cost* $g(x)$

States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in the various fields and using the OPERATORS (or SUCCESSORFN) of the problem to create the corresponding states.

Search strategies

A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

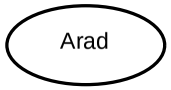
Iterative deepening search

Breadth-first search

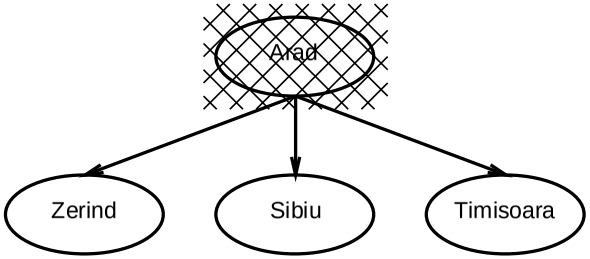
Expand shallowest unexpanded node

Implementation:

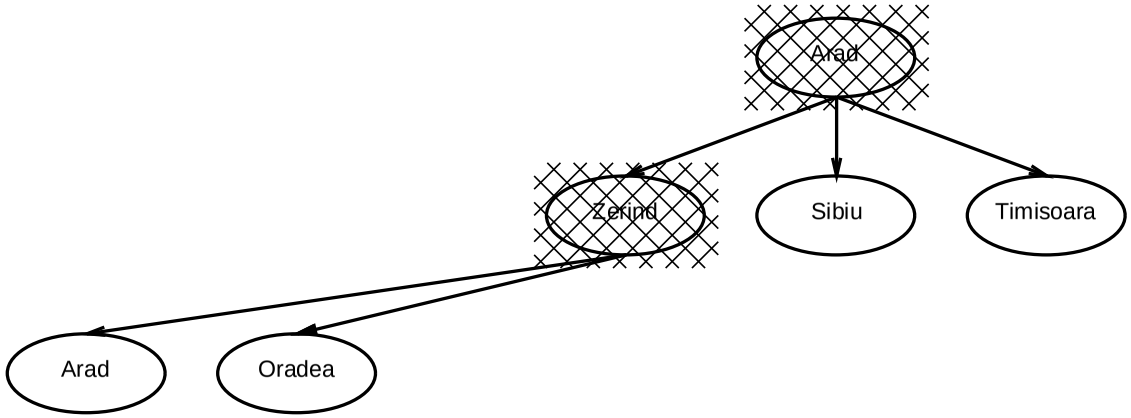
$\textsc{QueueingFn}$ = put successors at end of queue



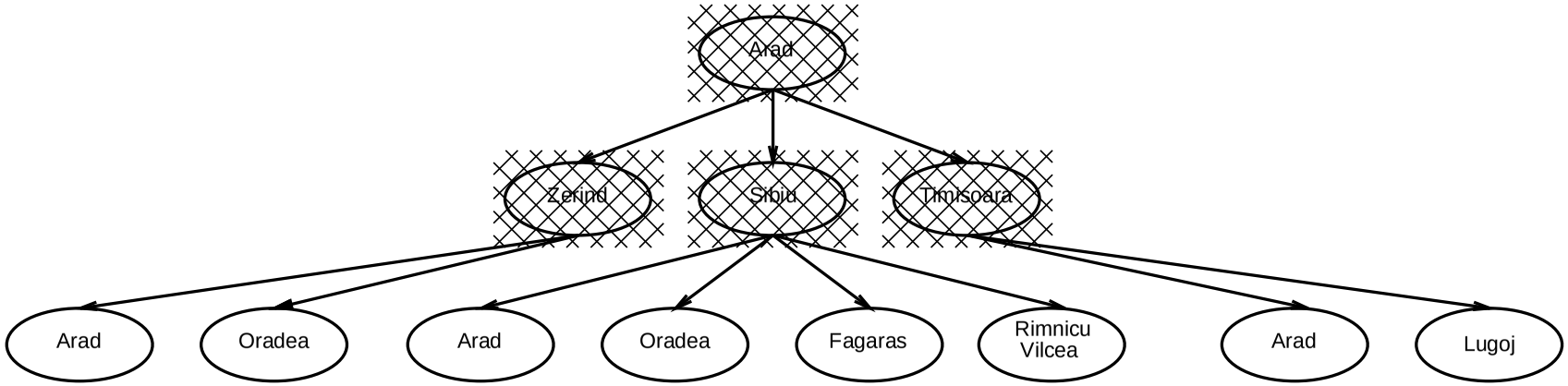
Breadth-first search



Breadth-first search



Breadth-first search



Properties of breadth-first search

Complete??

Time??

Space??

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

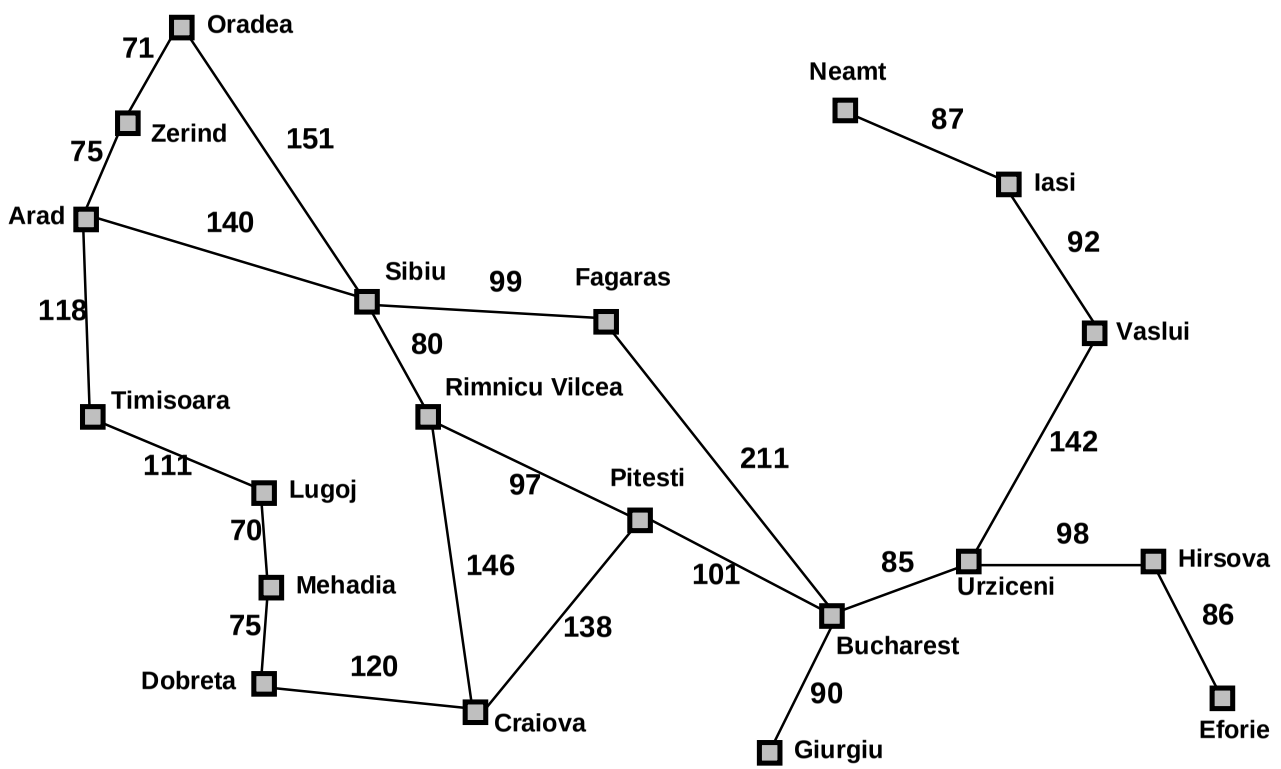
Time?? $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d

Space?? $O(b^d)$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Depth	Nodes	Time	Memory
0	1	1 millisecond	100 bytes
2	111	.1 seconds	11 kilobytes
4	11,111	11 seconds	1 megabyte
6	10^6	18 minutes	111 megabytes
8	10^8	31 hours	11 gigabytes
10	10^{10}	128 days	1 terabyte
12	10^{12}	35 years	111 terabytes
14	10^{14}	3500 years	11,111 terabytes

Romania with step costs in km



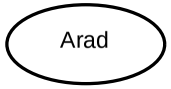
Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

QUEUEINGFN = insert in order of increasing path cost

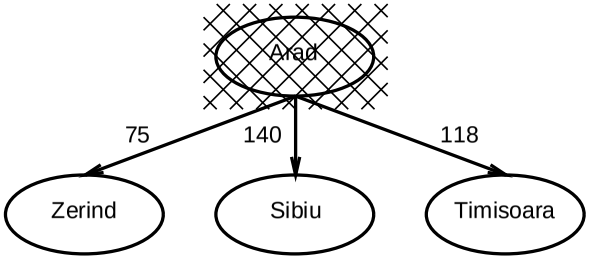


Uniform-cost search

-

-

-



Uniform-cost search

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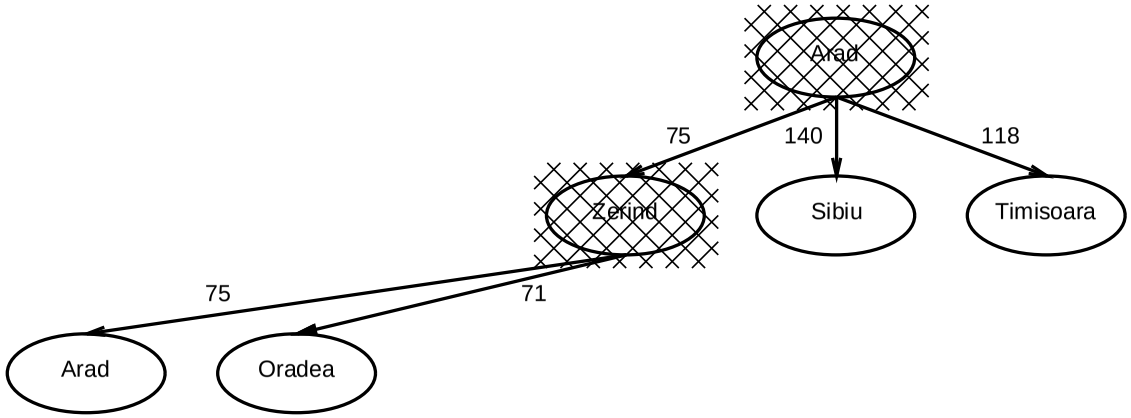

Uniform-cost search

-

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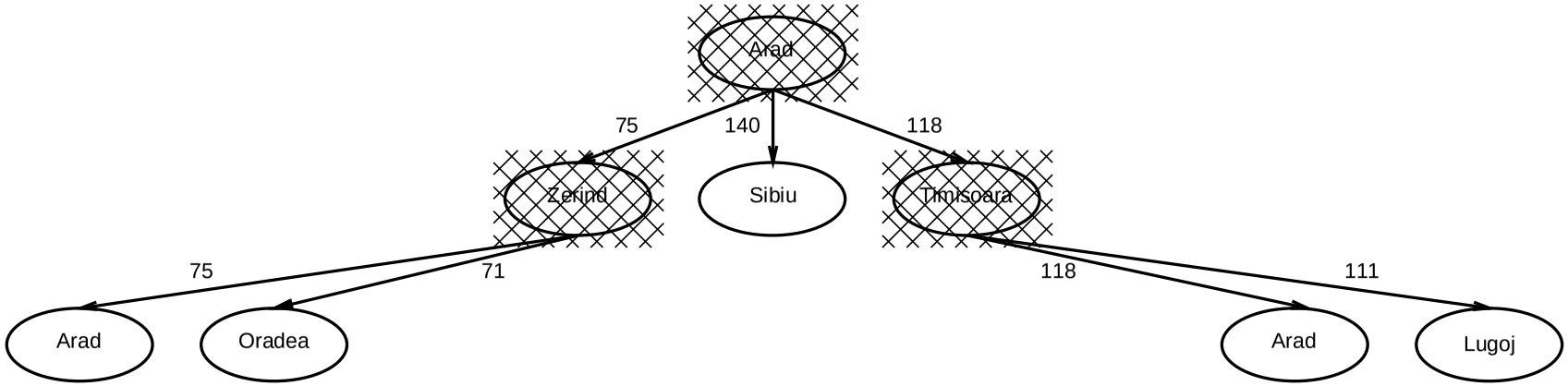


Properties of uniform-cost search

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution

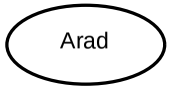
Optimal?? Yes

Depth-first search

Expand deepest unexpanded node

Implementation:

QUEUEINGFN = insert successors at front of queue



Depth-first search

-

-

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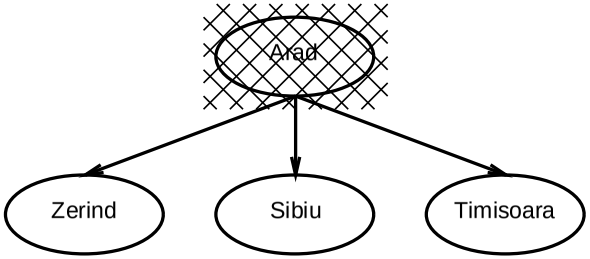

Depth-first search

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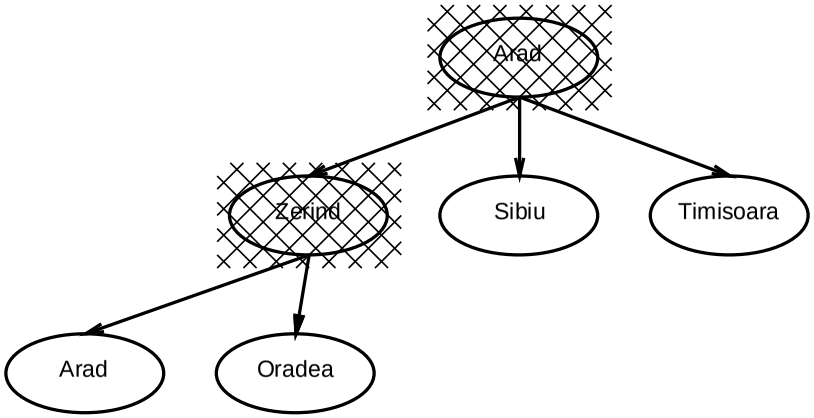

Depth-first search

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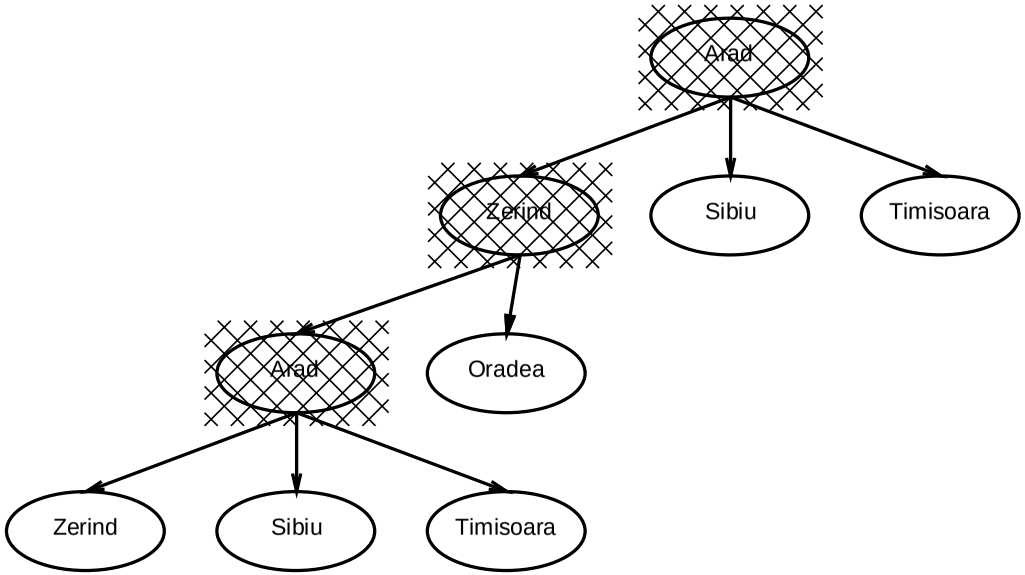
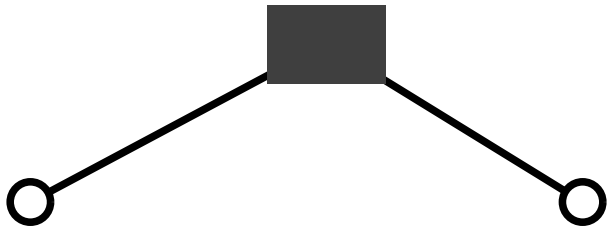


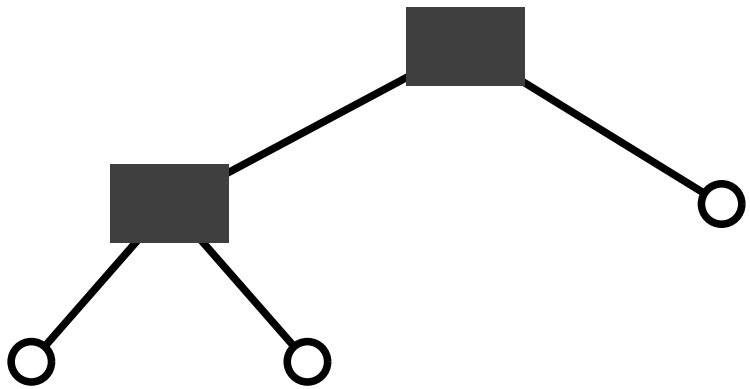
I.e., depth-first search can perform infinite cyclic excursions
Need a finite, non-cyclic search space (or repeated-state checking)

DFS on a depth-3 binary tree

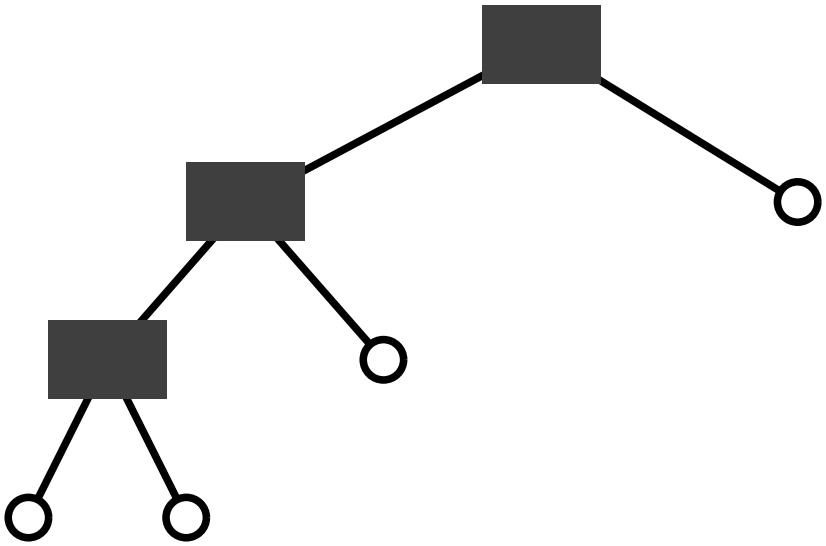
DFS on a depth-3 binary tree



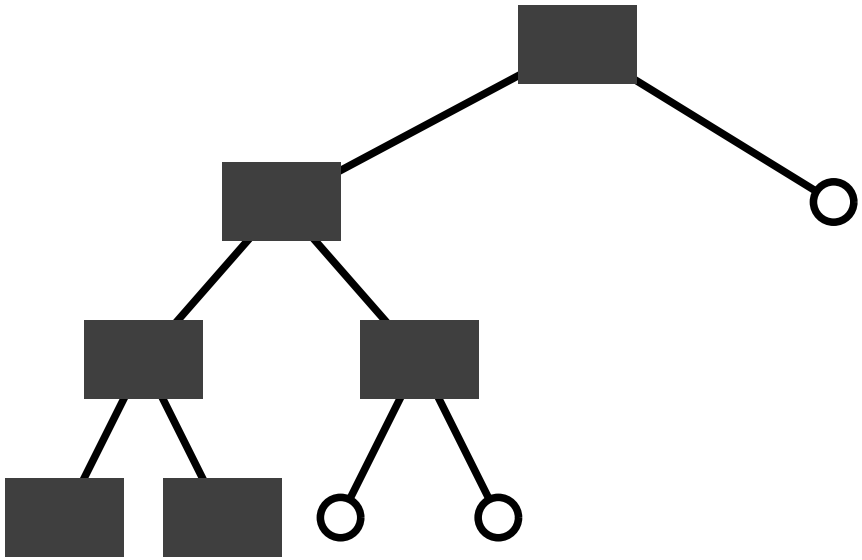
DFS on a depth-3 binary tree



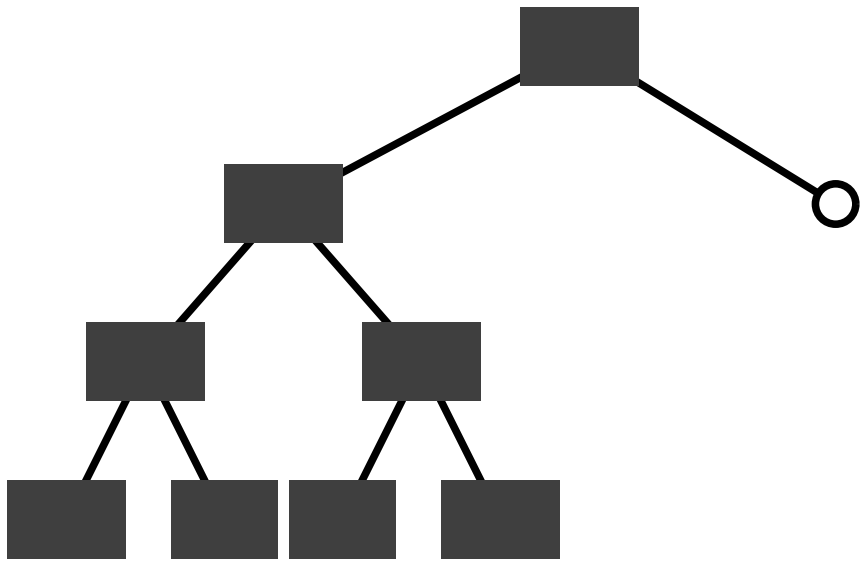
DFS on a depth-3 binary tree



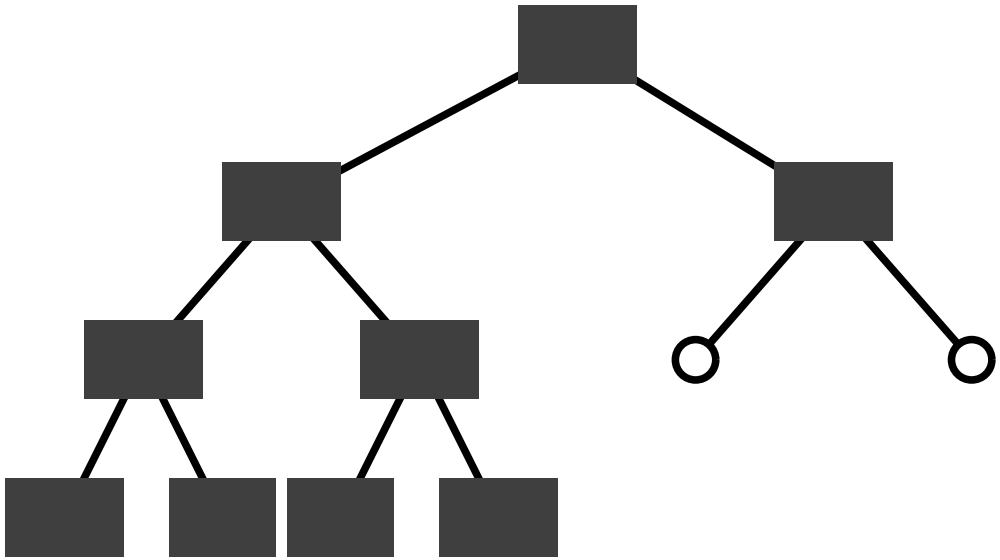
DFS on a depth-3 binary tree



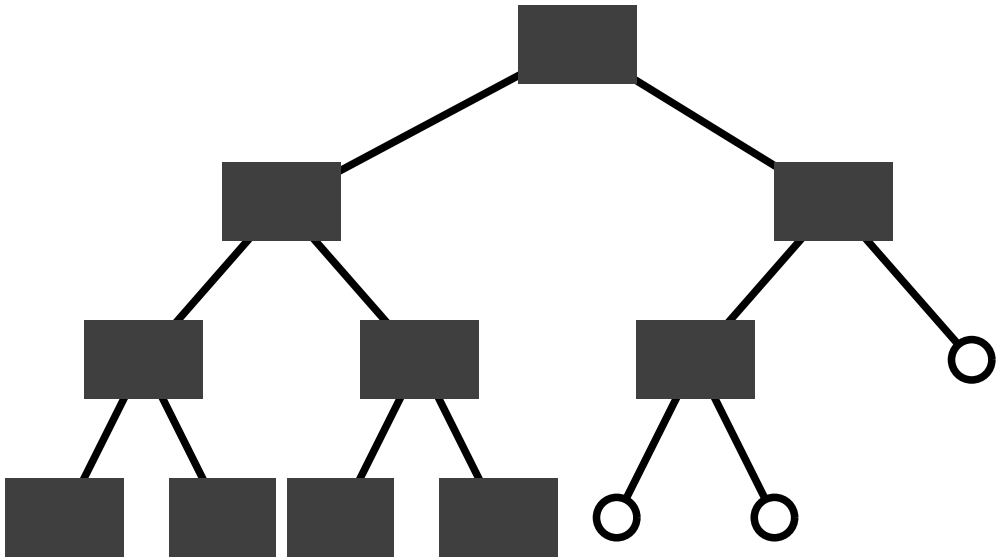
DFS on a depth-3 binary tree, contd.



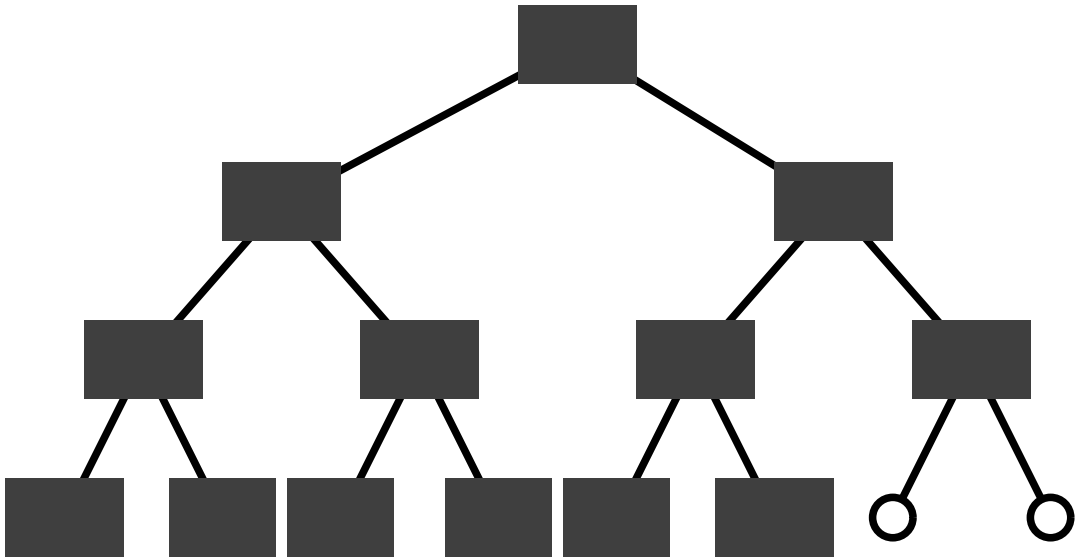
DFS on a depth-3 binary tree



DFS on a depth-3 binary tree



DFS on a depth-3 binary tree



Properties of depth-first search

Complete??

Time??

Space??

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Depth-limited search

= depth-first search with depth limit l

Implementation:

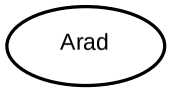
Nodes at depth l have no successors

Iterative deepening search

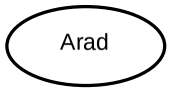
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution sequence
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

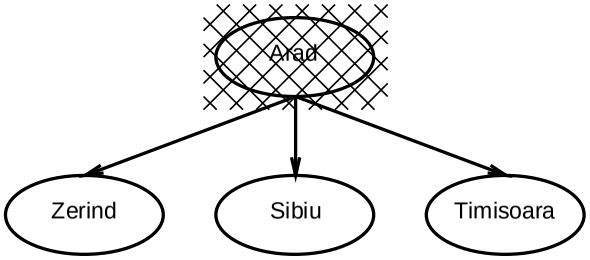
Iterative deepening search $l = 0$



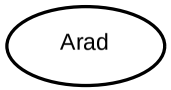
Iterative deepening search $l = 1$



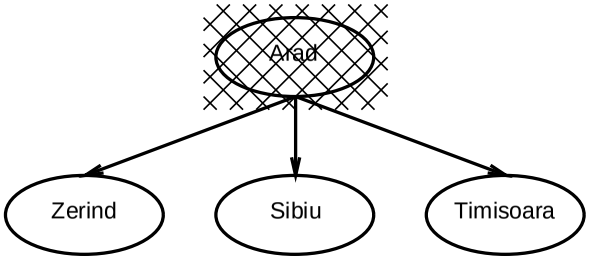
Iterative deepening search $l = 1$



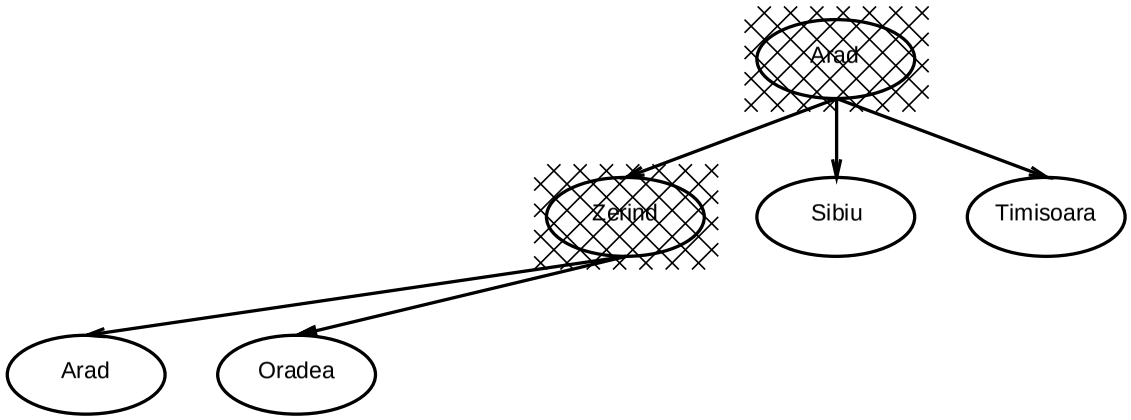
Iterative deepening search $l = 2$



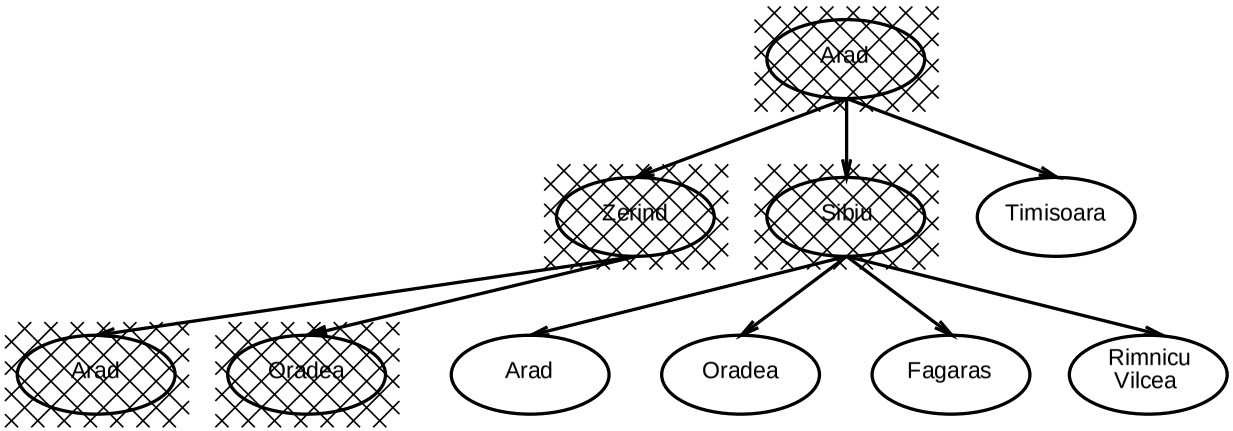
Iterative deepening search $l = 2$



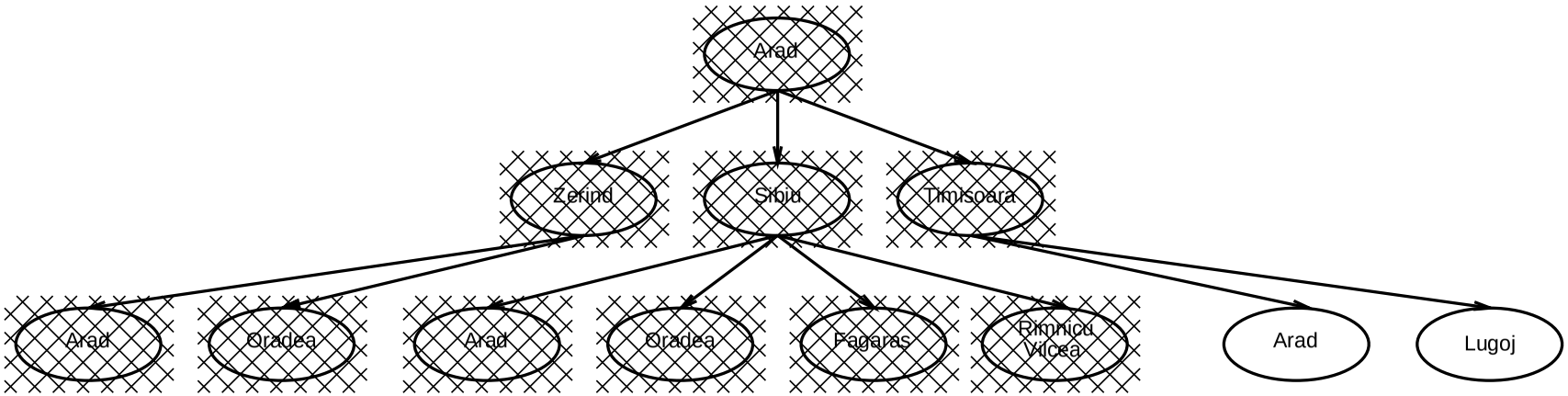
Iterative deepening search $l = 2$



Iterative deepening search $l = 2$



Iterative deepening search $l = 2$



Properties of iterative deepening search

Complete??

Time??

Space??

Optimal??

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$

Breadth-first:	1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
Iterative deepening:	6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456

For $b = 10$, iterative deepening search expands only 11% more nodes than breadth-first search!

Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms