

## First-order logic

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### Chapter 7

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- ◇ Syntax and semantics of FOL
- ◇ Fun with sentences
- ◇ Wumpus world in FOL

## Syntax of FOL: Basic elements

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Constants     *KingJohn, 2, UCB, ...*  
Predicates    *Brother, >, ...*  
Functions     *Sqrt, LeftLegOf, ...*  
Variables     *x, y, a, b, ...*  
Connectives    $\wedge \vee \neg \Rightarrow \Leftrightarrow$   
Equality       =  
Quantifiers    $\forall \exists$

Atomic sentence = *predicate(term<sub>1</sub>, ..., term<sub>n</sub>)*  
                  OR *term<sub>1</sub> = term<sub>2</sub>*

Term = *function(term<sub>1</sub>, ..., term<sub>n</sub>)*  
          OR *constant OR variable*

E.g., *Brother(KingJohn, RichardTheLionheart)*  
       $>$  (*Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn))*)

## Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

- E.g.  $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$   
 $>(1, 2) \vee \leq(1, 2)$   
 $>(1, 2) \wedge \neg >(1, 2)$

## Truth in first-order logic

Sentences are true with respect to a **model** and an **interpretation**

Model contains objects and relations among them

Interpretation specifies referents for

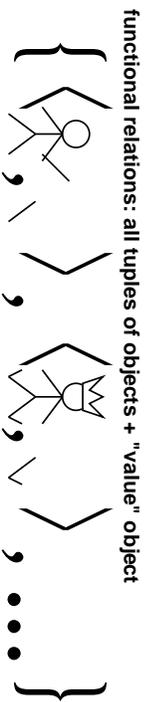
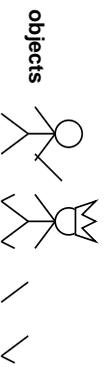
**constant symbols**  $\rightarrow$  **objects**

**predicate symbols**  $\rightarrow$  **relations**

**function symbols**  $\rightarrow$  **functional relations**

An atomic sentence  $predicate(term_1, \dots, term_n)$  is true iff the **objects** referred to by  $term_1, \dots, term_n$  are in the **relation** referred to by  $predicate$

## Models for FOL: Example



## Universal quantification

$\forall$  (*variables*) (*sentence*)

Everyone at UW is smart:

$$\forall x \text{ At}(x, UW) \Rightarrow \text{Smart}(x)$$

$\forall x \text{ } P$  is equivalent to the **conjunction of instantiations** of  $P$

$$\begin{aligned} & \text{At}(KingJohn, UW) \Rightarrow \text{Smart}(KingJohn) \\ & \wedge \text{At}(Richard, UW) \Rightarrow \text{Smart}(Richard) \\ & \wedge \text{At}(UW, UW) \Rightarrow \text{Smart}(UW) \\ & \wedge \dots \end{aligned}$$

Typically,  $\Rightarrow$  is the main connective with  $\forall$ .

Common mistake: using  $\wedge$  as the main connective with  $\forall$ :

$$\forall x \text{ At}(x, UW) \wedge \text{Smart}(x)$$

## Universal quantification

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$\forall$  (*variables*) (*sentence*)

Everyone at UW is smart:

$\forall x \text{ At}(x, UW) \Rightarrow \text{Smart}(x)$

$\forall x P$  is equivalent to the **conjunction of instantiations of  $P$**

$\text{At}(\text{KingJohn}, UW) \Rightarrow \text{Smart}(\text{KingJohn})$   
 $\wedge \text{At}(\text{Richard}, UW) \Rightarrow \text{Smart}(\text{Richard})$   
 $\wedge \text{At}(UW, UW) \Rightarrow \text{Smart}(UW)$   
 $\wedge \dots$

Typically,  $\Rightarrow$  is the main connective with  $\forall$ .

Common mistake: using  $\wedge$  as the main connective with  $\forall$ :

$\forall x \text{ At}(x, UW) \wedge \text{Smart}(x)$

means “Everyone is at UW and everyone is smart”

## Existential quantification

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$\exists$  (*variables*) (*sentence*)

Someone at Berkeley is smart:

$\exists x \text{ At}(x, Berkeley) \wedge \text{Smart}(x)$

$\exists x P$  is equivalent to the **disjunction of instantiations of  $P$**

$\text{At}(\text{KingJohn}, Berkeley) \wedge \text{Smart}(\text{KingJohn})$   
 $\vee \text{At}(\text{Richard}, Berkeley) \wedge \text{Smart}(\text{Richard})$   
 $\vee \text{At}(Berkeley, Berkeley) \wedge \text{Smart}(Berkeley)$   
 $\vee \dots$

Typically,  $\wedge$  is the main connective with  $\exists$ .

Common mistake: using  $\Rightarrow$  as the main connective with  $\exists$ :

$\exists x \text{ At}(x, Berkeley) \Rightarrow \text{Smart}(x)$

## Existential quantification

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$\exists$  (*variables*) (*sentence*)

Someone at Berkeley is smart:

$\exists x \text{ At}(x, Berkeley) \wedge \text{Smart}(x)$

$\exists x P$  is equivalent to the **disjunction of instantiations of  $P$**

$\text{At}(\text{KingJohn}, Berkeley) \wedge \text{Smart}(\text{KingJohn})$   
 $\vee \text{At}(\text{Richard}, Berkeley) \wedge \text{Smart}(\text{Richard})$   
 $\vee \text{At}(Berkeley, Berkeley) \wedge \text{Smart}(Berkeley)$   
 $\vee \dots$

Typically,  $\wedge$  is the main connective with  $\exists$ .

Common mistake: using  $\Rightarrow$  as the main connective with  $\exists$ :

$\exists x \text{ At}(x, Berkeley) \Rightarrow \text{Smart}(x)$

is true if there is anyone who is not at Berkeley!

## Properties of quantifiers

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$\forall x \forall y$  is the same as  $\forall y \forall x$  (**why??**)

$\exists x \exists y$  is the same as  $\exists y \exists x$  (**why??**)

$\exists x \forall y$  is **not** the same as  $\forall y \exists x$

$\exists x \forall y \text{ Loves}(x, y)$

“There is a person who loves everyone in the world”

$\forall y \exists x \text{ Loves}(x, y)$

“Everyone in the world is loved by at least one person”

**Quantifier duality:** each can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream})??$

$\exists x \text{ Likes}(x, \text{Broccoli})??$

## Properties of quantifiers

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“There is a person who loves everyone in the world”

$\forall y \exists x \text{ Loves}(x, y)$

“Everyone in the world is loved by at least one person”

**Quantifier duality:** each can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream})$ :  $\neg \exists x \neg \text{Likes}(x, \text{IceCream})$

$\exists x \text{ Likes}(x, \text{Broccoli})$ :  $\neg \forall x \neg \text{Likes}(x, \text{Broccoli})$

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## Equality

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$term_1 = term_2$  is true under a given interpretation if and only if  $term_1$  and  $term_2$  refer to the same object

E.g.,  $1 = 2$  and  $\forall x \times(\text{Sqrt}(x), \text{Sqrt}(x)) = x$  are satisfiable  
 $2 = 2$  is valid

E.g., definition of (full) *Sibling* in terms of *Parent*:

$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow [\neg(x=y) \wedge \exists m, f \neg(m=f) \wedge \text{Parent}(m, x) \wedge \text{Parent}(f, x) \wedge \text{Parent}(m, y) \wedge \text{Parent}(f, y)]$

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## Fun with sentences

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Brothers are siblings

$\forall x, y \text{ Brother}(x, y) \Rightarrow \text{Sibling}(x, y)$ .

“Sibling” is reflexive

$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$

One’s mother is one’s female parent

$\forall x, y \text{ Mother}(x, y) \Leftrightarrow (\text{Female}(x) \wedge \text{Parent}(x, y))$

A first cousin is a child of a parent’s sibling

$\forall x, y \text{ FirstCousin}(x, y) \Leftrightarrow \exists p, ps \text{ Parent}(p, x) \wedge \text{Sibling}(ps, p) \wedge \text{Parent}(ps, y)$

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## Interacting with FOL KBs

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Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at  $t = 5$ :

$Tell(KB, \text{Percept}([\text{Smell}, \text{Breeze}, \text{None}], 5))$

$Ask(KB, \exists a \text{ Action}(a, 5))$

I.e., does the KB entail any particular actions at  $t = 5$ ?

Answer:  $Yes, \{a/\text{Shoot}\} \leftarrow$  **substitution** (binding list)

Given a sentence  $S$  and a substitution  $\sigma$ ,

$S\sigma$  denotes the result of plugging  $\sigma$  into  $S$ ; e.g.,

$S = \text{Smarter}(x, y)$

$\sigma = \{x/\text{Hillar}, y/\text{Bill}\}$

$S\sigma = \text{Smarter}(\text{Hillar}, \text{Bill})$

$Ask(KB, S)$  returns some/all  $\sigma$  such that  $KB \models S\sigma$

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## Knowledge base for the wumpus world

### “Perception”

$\forall b, g, t \text{ Percept}([Smell, b, g], t) \Rightarrow Smelt(t)$   
 $\forall s, b, t \text{ Percept}([s, b, Glitter], t) \Rightarrow AtGold(t)$

**Reflex:**  $\forall t \text{ AtGold}(t) \Rightarrow \text{Action}(\text{Grab}, t)$

**Reflex with internal state:** do we have the gold already?

$\forall t \text{ AtGold}(t) \wedge \neg \text{Holding}(\text{Gold}, t) \Rightarrow \text{Action}(\text{Grab}, t)$

$\text{Holding}(\text{Gold}, t)$  cannot be observed

$\Rightarrow$  keeping track of change is essential

## Keeping track of change

Facts hold in **situations**, rather than eternally

E.g.,  $\text{Holding}(\text{Gold}, \text{Now})$  rather than just  $\text{Holding}(\text{Gold})$

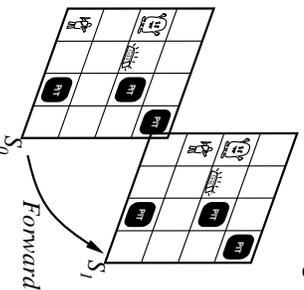
**Situation calculus** is one way to represent change in FOL:

Adds a situation argument to each non-eternal predicate

E.g.,  $\text{Now}$  in  $\text{Holding}(\text{Gold}, \text{Now})$  denotes a situation

Situations are connected by the  $\text{Result}$  function

$\text{Result}(a, s)$  is the situation that results from doing  $a$  in  $s$



## Deducing hidden properties

Properties of locations:

$\forall l, t \text{ At}(\text{Agent}, l, t) \wedge \text{Smelt}(t) \Rightarrow \text{Smelly}(l)$   
 $\forall l, t \text{ At}(\text{Agent}, l, t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(l)$

Squares are breezy near a pit:

**Diagnostic rule**—infer cause from effect

$\forall y \text{ Breezy}(y) \Rightarrow \exists x \text{ Pit}(x) \wedge \text{Adjacent}(x, y)$

**Causal rule**—infer effect from cause

$\forall x, y \text{ Pit}(x) \wedge \text{Adjacent}(x, y) \Rightarrow \text{Breezy}(y)$

Neither of these is complete—e.g., the causal rule doesn't say whether squares far away from pits can be breezy

**Definition** for the  $\text{Breezy}$  predicate:

$\forall y \text{ Breezy}(y) \Leftrightarrow [\exists x \text{ Pit}(x) \wedge \text{Adjacent}(x, y)]$

## Describing actions I

“Effect” axiom—describe changes due to action

$\forall s \text{ AtGold}(s) \Rightarrow \text{Holding}(\text{Gold}, \text{Result}(\text{Grab}, s))$

“Frame” axiom—describe **non-changes** due to action

$\forall s \text{ HaveArrow}(s) \Rightarrow \text{HaveArrow}(\text{Result}(\text{Grab}, s))$

**Frame problem**: find an elegant way to handle non-change

(a) representation—avoid frame axioms

(b) inference—avoid repeated “copy-overs” to keep track of state

**Qualification problem**: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or ...

**Ramification problem**: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, ...

