

Resolution in First-Order Logic

Basic steps for proving a conclusion **S** given premises

Premise₁, ..., Premise_n
(all expressed in FOL):

1. Convert all sentences to CNF
2. Negate conclusion **S** & convert result to CNF
3. Add negated conclusion **S** to the premise clauses
4. Repeat until contradiction or no progress is made:
 - a. Select 2 clauses (call them parent clauses)
 - b. Resolve them together, performing all required unifications
 - c. If resolvent is the empty clause, a contradiction has been found (i.e., **S** follows from the premises)
 - d. If not, add resolvent to the premises

If we succeed in Step 4, we have proved the conclusion

Resolution Examples

Example 1:

- If something is intelligent, it has common sense
- Deep Blue does not have common sense
- Prove that Deep Blue is not intelligent

1. $\forall x. I(x) \Rightarrow H(x)$

2. $\neg H(D)$

Conclusion: $\neg I(D)$

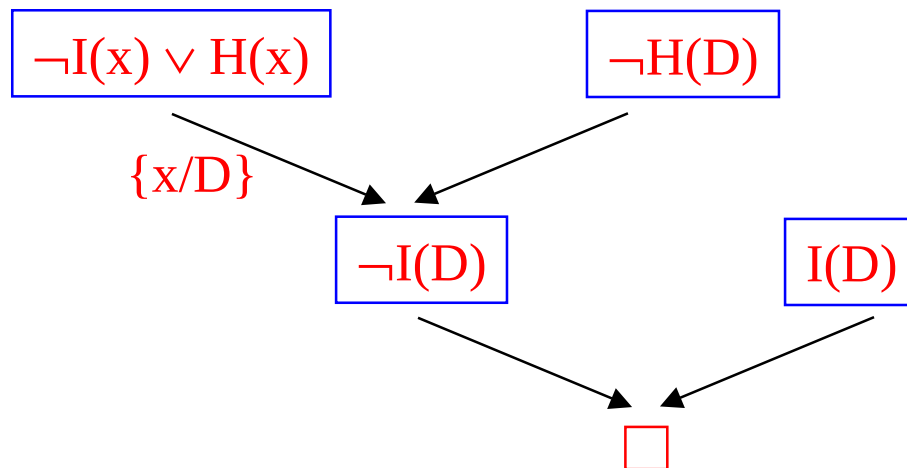
Denial: $C3: I(D)$

$C1: \neg I(x) \vee H(x)$

$C2: \neg H(D)$

CNF

A resolution proof of $\neg I(D)$:



Proof also written as:

$C4: \neg I(D)$

$C5: \square$

$r[C1b, C2]$

$r[C3, C4]$

2nd literal,
1st clause

Resolution Examples (cont.)

Example 2:

Premises:

$\text{Mother}(\text{Lulu}, \text{Fifi})$

$\text{Alive}(\text{Lulu})$

$\forall x \forall y. \text{Mother}(x, y) \Rightarrow \text{Parent}(x, y)$

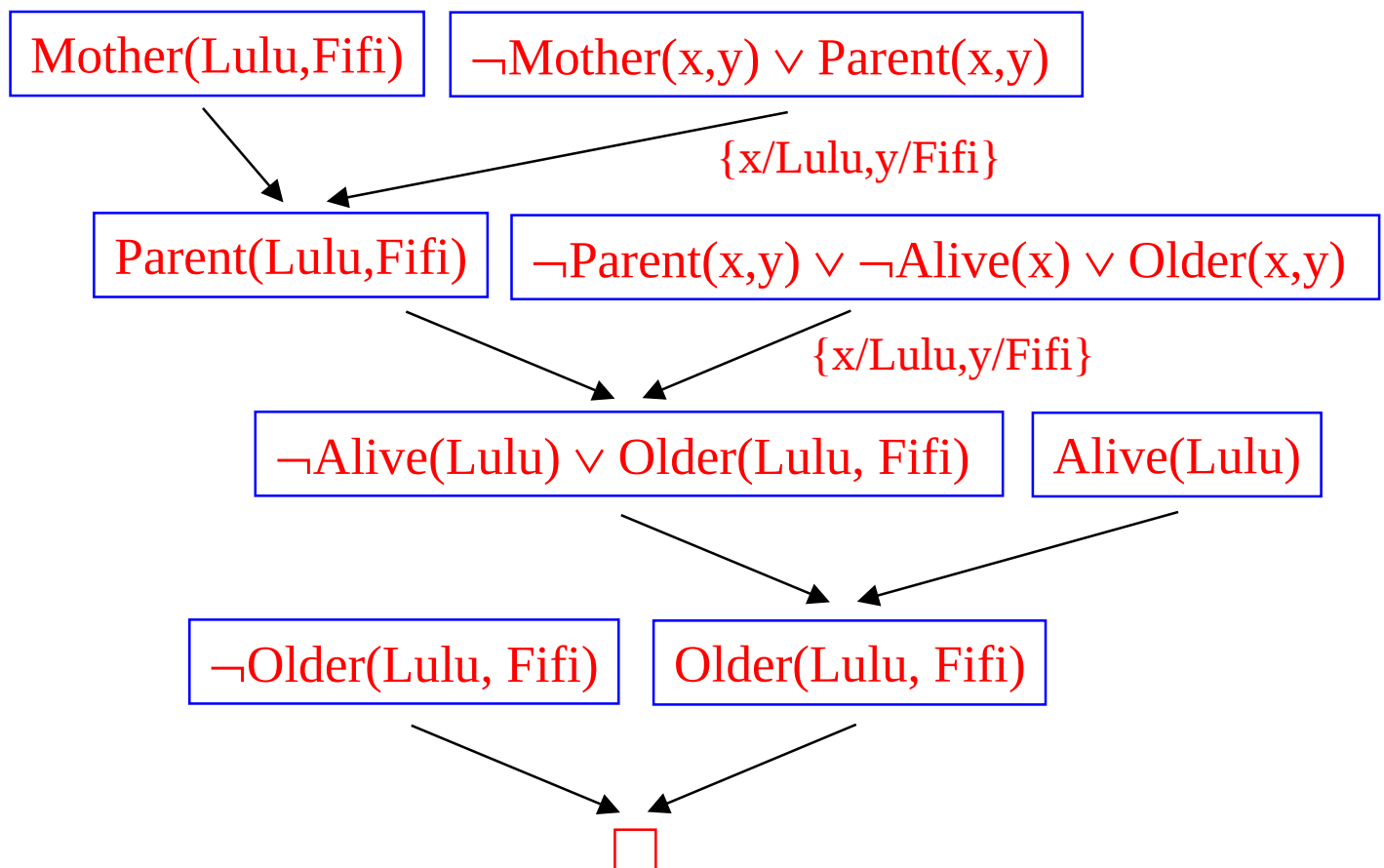
$\forall x \forall y. (\text{Parent}(x, y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x, y)$

Prove:

$\text{Older}(\text{Lulu}, \text{Fifi})$

Denial:

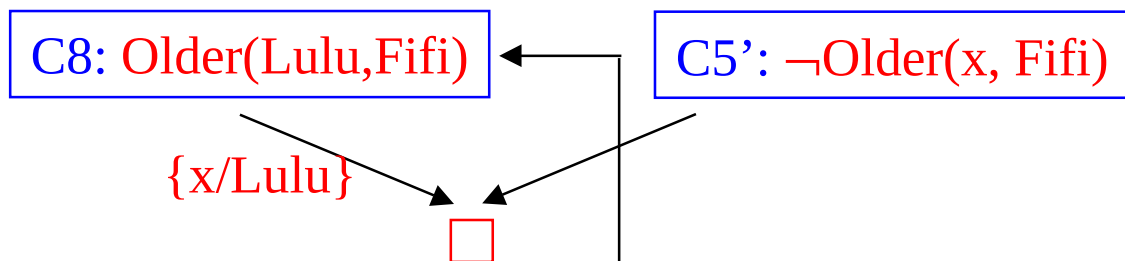
$\neg \text{Older}(\text{Lulu}, \text{Fifi})$



Resolution Examples (cont.)

Example 3:

- Suppose the desired conclusion had been
“Something is older than Fifi”
 $\exists x. \text{Older}(x, \text{Fifi})$
- Denial:
 $\neg \exists x. \text{Older}(x, \text{Fifi})$
also written as: $\forall x. \neg \text{Older}(x, \text{Fifi})$
in clause form: $\neg \text{Older}(x, \text{Fifi})$
- Last proof step would have been



Don't make mistake of first forming clause from conclusion & then denying it:

- Conclusion:

$\exists x. \text{Older}(x, \text{Fifi})$

clause form: $\text{Older}(C, \text{Fifi})$

denial: $\neg \text{Older}(C, \text{Fifi})$

Cannot unify
Lulu, C!!

Question-Answering

Example 1:

“Who is Lulu older than?”

- Prove that
“there is an x such that Lulu is older than x”
- In FOL form:
 $\exists x. \text{Older}(\text{Lulu}, x)$
- Denial:
 $\neg \exists x. \text{Older}(\text{Lulu}, x)$
 $\forall x. \neg \text{Older}(\text{Lulu}, x)$
in clause form: $\neg \text{Older}(\text{Lulu}, x)$
- Successful proof gives
 $\{x/\text{Fifi}\}$ [Verify!!]

Example 2:

“What is older than what?”

- In FOL form:
 $\exists x \exists y. \text{Older}(x, y)$
- Denial:
 $\neg \exists x \exists y. \text{Older}(x, y)$
in clause form: $\neg \text{Older}(x, y)$
- Successful proof gives
 $\{x/\text{Lulu}, y/\text{Fifi}\}$ [Verify!!]