

Inference Rules in FOL

Premises:

1. If x is a parent of y, then y is older than x
2. If x is the mother of y, then x is a parent of y
3. Lulu is the mother of Fifi

Conclusion:

Lulu is older than Fifi

Premises in first-order logic:

1. $\forall x \forall y. \text{Parent}(x, y) \Rightarrow \text{Older}(x, y)$
2. $\forall x \forall y. \text{Mother}(x, y) \Rightarrow \text{Parent}(x, y)$
3. $\text{Mother}(\text{Lulu}, \text{Fifi})$

Inference Rules in FOL (cont.)

Previous inference easier with strengthened rule:

Generalized Modus Ponens

$$\frac{\alpha_1[\underline{x}/\underline{k}], \dots, \alpha_n[\underline{x}/\underline{k}], \forall x_1 \dots \forall x_m. (\alpha_1 \wedge \dots \wedge \alpha_n) \Rightarrow \beta}{\beta[\underline{x}/\underline{k}]}$$

$$\underline{x}/\underline{k} : \{x_1/k_1, \dots, x_m/k_m\}$$

A proof of **Older(Lulu, Fifi)** using GMP:

- | | |
|---|--------------|
| 1. Mother(Lulu, Fifi) | given |
| 2. Alive(Lulu) | given |
| 3. $\forall x \forall y. \text{Mother}(x, y) \Rightarrow \text{Parent}(x, y)$ | given |
| 4. $\forall x \forall y. (\text{Parent}(x, y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x, y)$ | given |
| 5. Parent(Lulu, Fifi) | 1, 3, GMP |
| 6. Older(Lulu, Fifi) | 5, 2, 4, GMP |

This use of GMP is called **forward-chaining**

Inference Rules in FOL (cont.)

Another style of proof is to “reason backward” (**backward-chaining**):

Start with a goal (to be proved) & then derive new sub-goals until we have sub-goals known to be true

→ similar to problem reduction

Backward-chaining proof of **Older(Lulu, Fifi)** from premisses:

1. **Mother(Lulu, Fifi)**
2. **Alive(Lulu)**
3. $\forall x \forall y. \text{Mother}(x, y) \Rightarrow \text{Parent}(x, y)$
4. $\forall x \forall y. (\text{Parent}(x, y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x, y)$

Goal: (i) **Older(Lulu, Fifi)**

Match (i) against RHS of (4)

Subgoals: (ii) **Parent(Lulu, Fifi)** (iii) **Alive(Lulu)**

Match (iii) against (2) True

Match (ii) against RHS of (3)

Subgoal: (iv) **Mother(Lulu, Fifi)**

Match (iv) against (1)