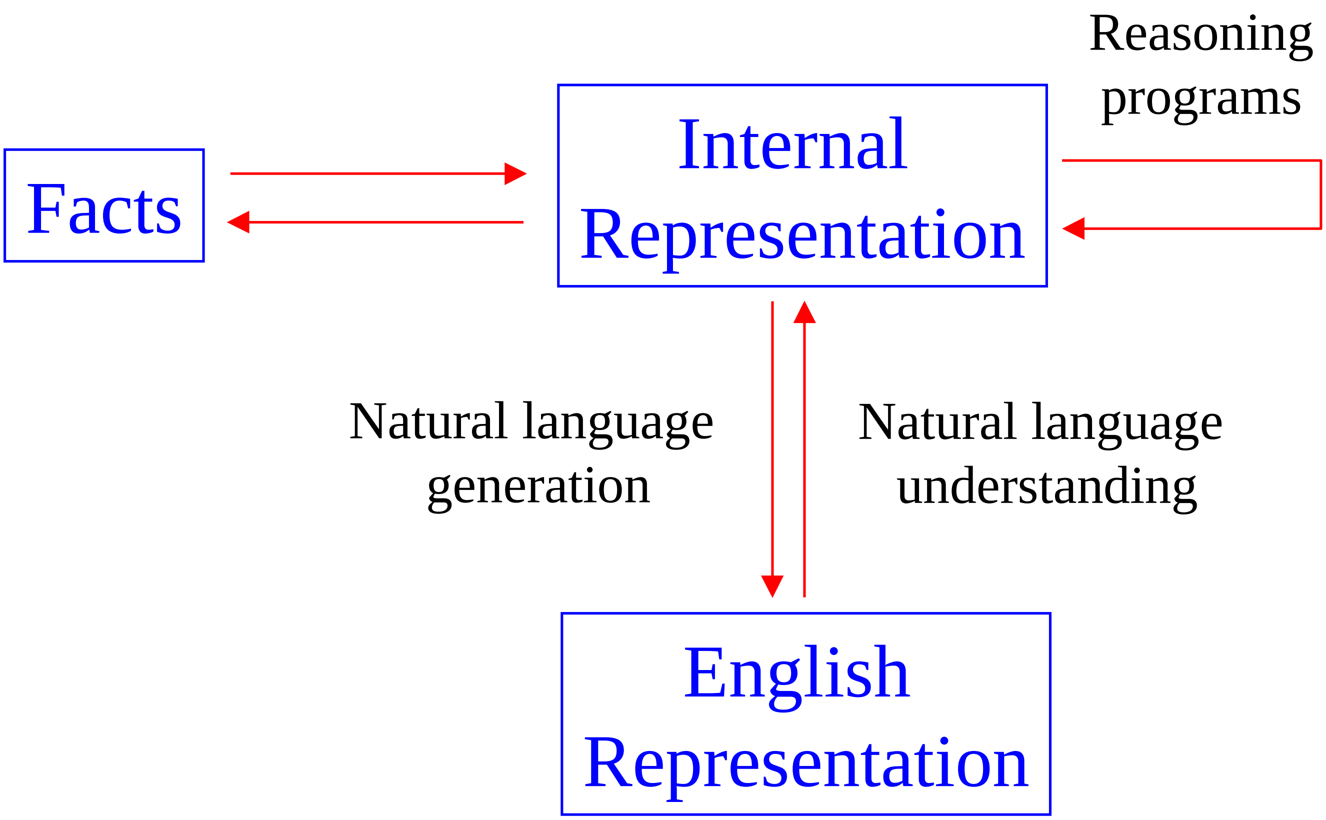
Facts & Representations



Examples:

"Socrates is a man" → MANSOCRATES

"Spot is a dog" → Dog(Spot)

Validity & Soundness

- Validity:

  Some sentences are always true (valid) & so can always be assumed

  Truth table for $\neg\neg\alpha \Leftrightarrow \alpha$:

α	$\neg\neg\alpha$	$\neg\neg\alpha \Leftrightarrow \alpha$
$\perp$	$\perp$	T
T	T	T

  Verify for $(\alpha \Rightarrow \beta) \Leftrightarrow (\neg\alpha \vee \beta)$

- Soundness:

  A rule of inference is sound if it always leads from the premises to a true conclusion

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

MP:

α	β	$\alpha \Rightarrow \beta$
$\perp$	$\perp$	T
$\perp$	T	T
T	$\perp$	$\perp$
T	T	T

Entailment & Completeness

- Entailment:

  "Whenever premises $\alpha_1,\alpha_2,\ldots\alpha_n$ are true, conclusion β is true"

  We write: $\alpha_1,\alpha_2,\ldots\alpha_n \models \beta$

- A "good" proof system should also allow us to prove β from $\alpha_1,\alpha_2,\ldots\alpha_n$

Completeness

A proof system is complete if, whenever

$$\alpha_1,\alpha_2,\ldots\alpha_n \models \beta \qquad \text{(semantic entailment)}$$

we can prove β from $\alpha_1,\alpha_2,\ldots\alpha_n$ using the inference axioms & inference rules of the proof system

$$\alpha_1,\alpha_2,\ldots\alpha_n \vdash \beta \qquad \text{(syntactic proof)}$$

There are many complete proof systems for propositional logic

Writing out the truth table for $\alpha_1,\alpha_2,\ldots\alpha_n,\beta$ is one method that is complete!

Another complete proof system:

1. $(\alpha \wedge \beta) \Leftrightarrow \neg(\neg\alpha \vee \neg\beta)$
2. $(\alpha \Rightarrow \beta) \Leftrightarrow (\neg\alpha \vee \beta)$
3. $(\alpha \Leftrightarrow \beta) \Leftrightarrow ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$
4. $(\alpha \wedge \alpha) \Leftrightarrow \alpha$
5. $\beta \Leftrightarrow (\alpha \vee \beta)$
6. $(\alpha \vee \beta) \Leftrightarrow (\beta \vee \alpha)$
7. $(\beta \Rightarrow \gamma) \Rightarrow ((\alpha \vee \beta) \Rightarrow (\alpha \vee \gamma))$

Satisfiability

- Suppose S is a sentence composed of atomic sentences (letters) $\alpha_1,\alpha_2,\ldots\alpha_n$

- An assignment of truth values to $\alpha_1,\alpha_2,\ldots\alpha_n$ that make S true is called a <u>satisfying assignment</u>

- In general, we may need to go through all 2^n possible interpretations of $\alpha_1,\alpha_2,\ldots\alpha_n$ to find a satisfying assignment

- nSAT: The problem of finding a satisfying assignment for sentences with n distinct letters

- nSAT is NP-complete