Introduction to Artificial Intelligence

Rational decisions

Chapter 16

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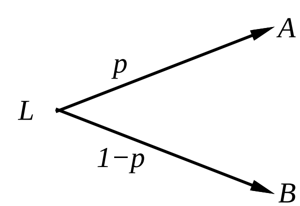
Outline

- ◊ Rational preferences
- ◊ Utilities
- ◊ Money
- ◊ Multiattribute utilities
- ◊ Decision networks
- ◊ Value of information

Preferences

An agent chooses among **prizes** (A, B, etc.) and **lotteries**, i.e., situations with uncertain prizes

Lottery $L = [p, A;\ (1-p), B]$



Notation:

$A \succ B$	A preferred to B
$A \sim B$	indifference between A and B
$A \succsim B$	B not preferred to A

Rational preferences

Idea: preferences of a rational agent must obey constraints.
Rational preferences $\Rightarrow$
behavior describable as maximization of expected utility

Constraints:

Orderability
$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$
Transitivity
$$(A \succ B) \wedge (B \succ C) \ \Rightarrow\ (A \succ C)$$
Continuity
$$A \succ B \succ C \ \Rightarrow\ \exists p\ [p, A;\ 1-p, C] \sim B$$
Substitutability
$$A \sim B \ \Rightarrow\ [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$
Monotonicity
$$A \succ B \ \Rightarrow\ (p \geq q \ \Leftrightarrow\ [p, A;\ 1-p, B] \succsim [q, A;\ 1-q, B])$$

Rational preferences contd.

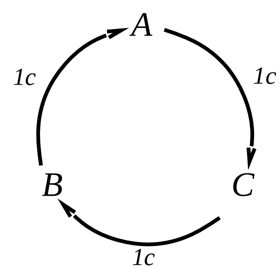
Violating the constraints leads to self-evident irrationality

For example: an agent with intransitive preferences can be induced to give away all its money

If $B \succ C$, then an agent who has C would pay (say) 1 cent to get B

If $A \succ B$, then an agent who has B would pay (say) 1 cent to get A

If $C \succ A$, then an agent who has A would pay (say) 1 cent to get C



Maximizing expected utility

Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):
Given preferences satisfying the constraints
there exists a real-valued function U such that

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succsim B$$

$$U([p_1, S_1; \ldots ; p_n, S_n]) = \sum_i p_i U(S_i)$$

MEU principle:
Choose the action that maximizes expected utility

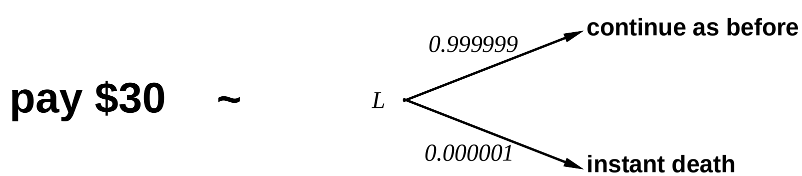
Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities

E.g., a lookup table for perfect tictactoe

Utilities

Utilities map states to real numbers. Which numbers?

Standard approach to assessment of human utilities:
compare a given state A to a **standard lottery** L_p that has
"best possible prize" $u_\top$ with probability p
"worst possible catastrophe" $u_\perp$ with probability $(1-p)$
adjust lottery probability p until $A \sim L_p$



Utility scales

Normalized utilities: $u_\top = 1.0$, $u_\perp = 0.0$

Micromorts: one-millionth chance of death
useful for Russian roulette, paying to reduce product risks, etc.

QALYs: quality-adjusted life years
useful for medical decisions involving substantial risk

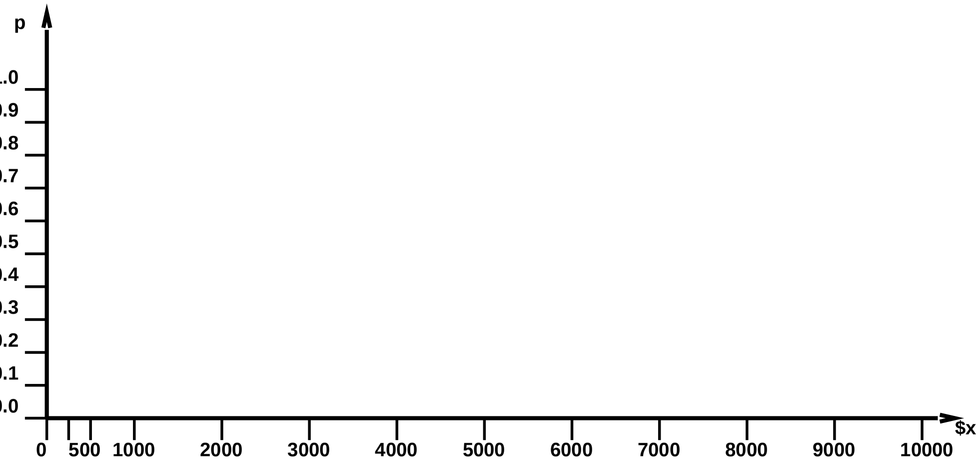
Note: behavior is **invariant** w.r.t. linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

With deterministic prizes only (no lottery choices), only **ordinal utility** can be determined, i.e., total order on prizes

Student group utility

For each x, adjust p until half the class votes for lottery (M=10,000)



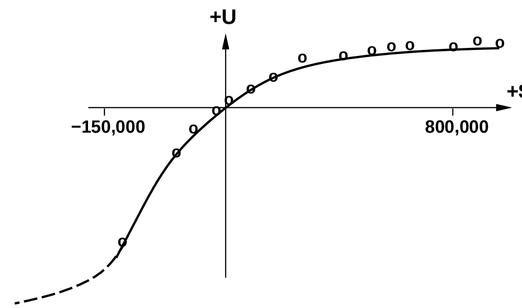
Money

Money does **not** behave as a utility function

Given a lottery L with expected monetary value $EMV(L)$, usually $U(L) < U(EMV(L))$, i.e., people are **risk-averse**

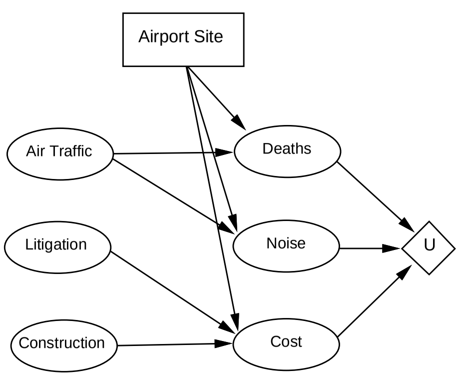
Utility curve: for what probability p am I indifferent between a fixed prize x and a lottery $[p, \$M;\ (1-p), \$0]$ for large M?

Typical empirical data, extrapolated with **risk-prone** behavior:



Decision networks

Add **action nodes** and **utility** nodes to belief networks to enable rational decision making



Algorithm:
- For each value of action node
  - compute expected value of utility node given action, evidence
- Return MEU action

Multiattribute utility

How can we handle utility functions of many variables $X_1 \ldots X_n$?
E.g., what is $U(Deaths, Noise, Cost)$?

How can complex utility functions be assessed from preference behaviour?

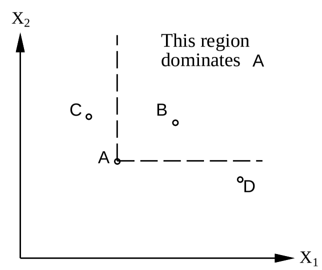
Idea 1: identify conditions under which decisions can be made without complete identification of $U(x_1, \ldots, x_n)$

Idea 2: identify various types of **independence** in preferences and derive consequent canonical forms for $U(x_1, \ldots, x_n)$

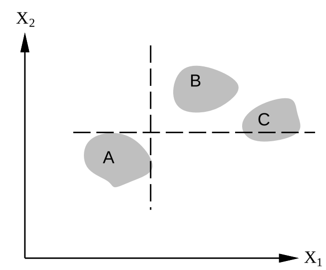
Strict dominance

Typically define attributes such that U is **monotonic** in each

Strict dominance: choice B strictly dominates choice A iff
$\forall\, i \;\; X_i(B) \geq X_i(A)$ (and hence $U(B) \geq U(A)$)



Deterministic attributes



Uncertain attributes

Strict dominance seldom holds in practice

Value of information

Idea: compute value of acquiring each possible piece of evidence
Can be done **directly from decision network**

Example: buying oil drilling rights
Two blocks A and B, exactly one has oil, worth k
Prior probabilities 0.5 each, mutually exclusive
Current price of each block is $k/2$
Consultant offers accurate survey of A. Fair price?

Solution: compute expected value of information
= expected value of best action given the information
minus expected value of best action without information
Survey may say "oil in A" or "no oil in A", prob. 0.5 each
= [$0.5 \times$ value of "buy A" given "oil in A"
+ $0.5 \times$ value of "buy B" given "no oil in A"]
– 0
= $(0.5 \times k/2) + (0.5 \times k/2) - 0 = k/2$

General formula

Current evidence E, current best action α
Possible action outcomes S_i, potential new evidence E_j

$$EU(\alpha|E) = \max_a \sum_i U(S_i)\; P(S_i|E,a)$$

Suppose we knew $E_j = e_{jk}$, then we would choose $\alpha_{e_{jk}}$ s.t.

$$EU(\alpha_{e_{jk}}|E, E_j = e_{jk}) = \max_a \sum_i U(S_i)\; P(S_i|E,a,E_j = e_{jk})$$

E_j is a random variable whose value is *currently* unknown
$\Rightarrow$ must compute expected gain over all possible values:

$$VPI_E(E_j) = \left(\sum_k P(E_j = e_{jk}|E) EU(\alpha_{e_{jk}}|E, E_j = e_{jk})\right) - EU(\alpha|E)$$

(VPI = value of perfect information)

Properties of VPI

Nonnegative—in *expectation*, not *post hoc*

$$\forall\, j, E \;\; VPI_E(E_j) \geq 0$$

Nonadditive—consider, e.g., obtaining E_j twice

$$VPI_E(E_j, E_k) \neq VPI_E(E_j) + VPI_E(E_k)$$

Order-independent

$$VPI_E(E_j, E_k) = VPI_E(E_j) + VPI_{E,E_j}(E_k) = VPI_E(E_k) + VPI_{E,E_k}(E_j)$$

Note: when more than one piece of evidence can be gathered,
maximizing VPI for each to select one is not always optimal
$\Rightarrow$ evidence-gathering becomes a **sequential** decision problem

Qualitative behaviors

a) Choice is obvious, information worth little
b) Choice is nonobvious, information worth a lot
c) Choice is nonobvious, information worth little