## Outline

- Concept learning
- General-to-specific ordering over hypotheses
- Version spaces and candidate elimination algorithm
- Picking new examples
- The need for inductive bias



## Training Examples for EnjoySport

| Sky | Temp | Humid | Wind | Water | Forecst | EnjoySpt |
|---|---|---|---|---|---|---|
| Sunny | Warm | Normal | Strong | Warm | Same | Yes |
| Sunny | Warm | High | Strong | Warm | Same | Yes |
| Rainy | Cold | High | Strong | Warm | Change | No |
| Sunny | Warm | High | Strong | Cool | Change | Yes |

What is the general concept?



## Representing Hypotheses

Many possible representations

Here, $h$ is conjunction of constraints on attributes

Each constraint can be

- a specfic value (e.g., $Water = Warm$)
- don't care (e.g., "$Water =?$")
- no value allowed (e.g.,"Water=$\emptyset$")

For example,

| Sky | AirTemp | Humid | Wind | Water | Forecst |
|---|---|---|---|---|---|
| $\langle Sunny$ | ? | ? | $Strong$ | ? | $Same\rangle$ |



## Prototypical Concept Learning Task

- **Given:**
  - Instances $X$: Possible $Days$, each described by the attributes *Sky, AirTemp, Humidity, Wind, Water, Forecast*
  - Target function $c$: $EnjoySport : Day \rightarrow \{yes, no\}$
  - Hypotheses $H$: Conjunctions of literals. E.g.

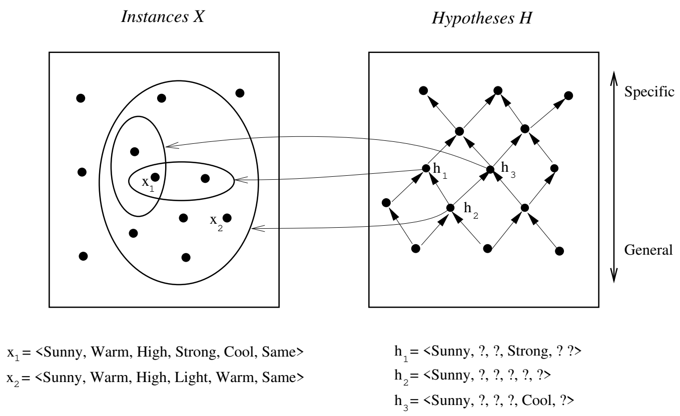
    $$\langle ?, Cold, High, ?, ?, ?\rangle.$$

  - Training examples $D$: Positive and negative examples of the target function
- **Determine:**

**The inductive learning hypothesis:** Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.



## Instance, Hypotheses, and More-General-Than



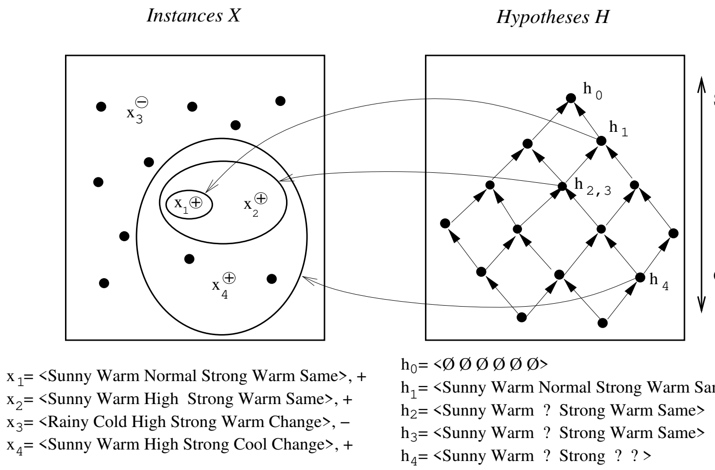



## Find-S **Algorithm**

1. Initialize $h$ to the most specific hypothesis in $H$
2. For each positive training instance $x$
   - For each attribute constraint $a_i$ in $h$
     If the constraint $a_i$ in $h$ is satisfied by $x$
     Then do nothing
     Else replace $a_i$ in $h$ by the next more general constraint that is satisfied by $x$
3. output hypothesis $h$



## Hypothesis Space Search by Find-S



## Complaints about Find-S

- Can't tell whether it has learned concept
- Can't tell when training data inconsistent
- Picks a maximally specific $h$ (why?)
- Depending on $H$, there might be several!



## Version Spaces

A hypothesis $h$ is **consistent** with a set of training examples $D$ if and only if $h(x) = c(x)$ for each example $\langle x, c(x) \rangle$ in $D$.

$$Consistent(h, D) \equiv (\forall \langle x, c(x) \rangle \in D)\; h(x) = c(x)$$

The **version space**, $VS_{H,D}$, with respect to hypothesis space $H$ and training examples $D$, is the subset of hypotheses from $H$ consistent with the training examples in $D$.

$$VS_{H,D} \equiv \{h \in H | Consistent(h, D)\}$$



## Representing Version Spaces

The **General boundary**, G, of version space $VS_{H,D}$ is the set of its maximally general members

The **Specific boundary**, S, of version space $VS_{H,D}$ is the set of its maximally specific members

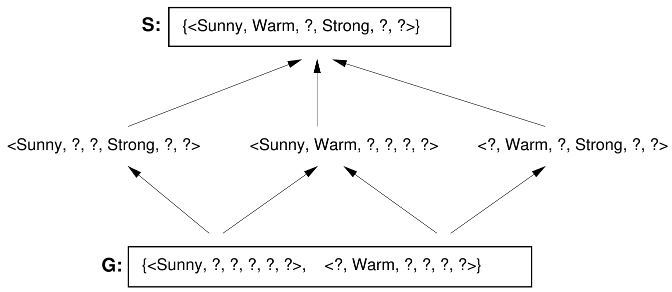
Every member of the version space lies between these boundaries

$$VS_{H,D} = \{h \in H | (\exists s \in S)(\exists g \in G)(g \geq h \geq s)\}$$

where $x \geq y$ means $x$ is more general or equal to $y$



## Example Version Space



## Version Space Candidate Elimination Algorithm

Initialize $G$ to the set of maximally general hypotheses in $H$
Initialize $S$ to the set of maximally specific hypotheses in $H$
For each training example $d$, do

- If $d$ is a positive example
  - Remove from $G$ any hypothesis inconsistent with $d$
  - For each hypothesis $s$ in $S$ that is not consistent with $d$
    - Remove $s$ from $S$
    - Add to $S$ all minimal generalizations $h$ of $s$ such that
      1. $h$ is consistent with $d$, and
      2. some member of $G$ is more general than $h$



    - Remove from $S$ any hypothesis that is more general than another hypothesis in $S$
- If $d$ is a negative example
  - Remove from $S$ any hypothesis inconsistent with $d$
  - For each hypothesis $g$ in $G$ that is not consistent with $d$
    - Remove $g$ from $G$
    - Add to $G$ all minimal specializations $h$ of $g$ such that
      1. $h$ is consistent with $d$, and
      2. some member of $S$ is more specific than $h$
    - Remove from $G$ any hypothesis that is less general than another hypothesis in $G$



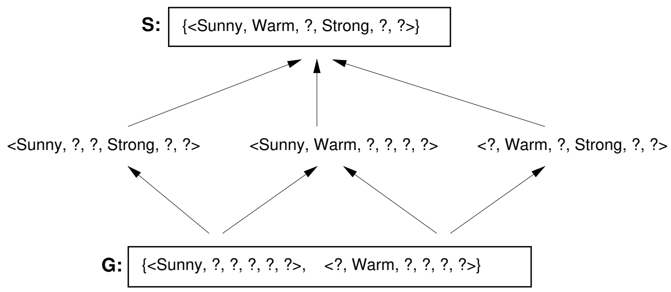
## Example Trace

$S_0$: {<Ø, Ø, Ø, Ø, Ø, Ø>}

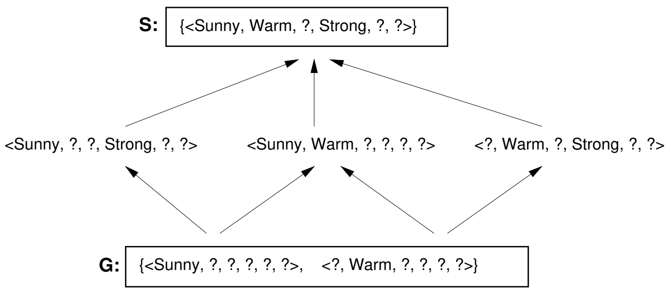
$G_0$: {<?, ?, ?, ?, ?, ?>}



## What Next Training Example?



## How Should These Be Classified?




$$\langle Sunny\ Warm\ Normal\ Strong\ Cool\ Change \rangle$$

$$\langle Rainy\ Cool\ Normal\ Light\ Warm\ Same \rangle$$

$$\langle Sunny\ Warm\ Normal\ Light\ Warm\ Same \rangle$$



## What Justifies this Inductive Leap?

$+\ \langle Sunny\ Warm\ Normal\ Strong\ Cool\ Change \rangle$

$+\ \langle Sunny\ Warm\ Normal\ Light\ Warm\ Same \rangle$

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$S: \quad \langle Sunny\ Warm\ Normal\ ?\ ?\ ? \rangle$

Why believe we can classify the unseen

$\langle Sunny\ Warm\ Normal\ Strong\ Warm\ Same \rangle$



## An UNBiased Learner

Idea: Choose $H$ that expresses every teachable concept (i.e., $H$ is power set of $X$)

Consider $H'$ = disjunctions, conjunctions, negations over previous $H$

What are $S$, $G$ in this case?

S $\leftarrow$

G $\leftarrow$



## Inductive Bias

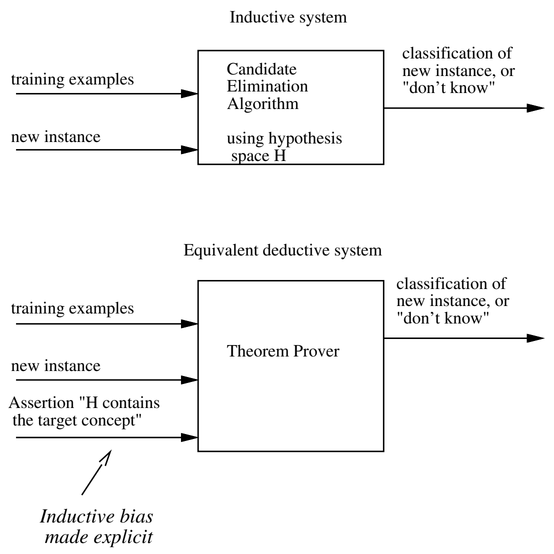
**Definition**: The **inductive bias** of a learning system is the set of assumptions that, combined with the observed training examples, deductively entail subsequent instance classifications $\hat{c}(x)$ made by the learner.

$$(\forall x \in X)(D + Bias + x \vdash \hat{c}(x))$$

Here

- Training data $D = \{\langle x, c(x) \rangle\}$, where $x$ is the instance, $c(x)$ is its classification
- $X$ is the set of all possible instances
- $\hat{c}(x)$ is prediction made by learner regarding $x$

## Inductive Systems and Equivalent Deductive Systems






## Three Learners with Different Biases

1. *Rote learner:* Store examples, Classify $x$ iff it matches previously observed example.
2. *Version space candidate elimination algorithm*
3. *Find-S*



## Summary Points

1. Concept learning as search through $H$
2. General-to-specific ordering over $H$
3. Version space candidate elimination algorithm
4. $S$ and $G$ boundaries characterize learner's uncertainty
5. Learner can generate useful queries
6. Inductive leaps possible only if learner is biased
7. Inductive learners can be modelled by equivalent deductive systems