Laboratory Exercise 6
Adders, Subtractors, and Multipliers

The purpose of this exercise is to examine arithmetic circuits that add, subtract, and multiply numbers. Each type of circuit will be implemented in two ways: first by writing Verilog code that describes the required functionality, and second by making use of predefined subcircuits from Altera’s library of parameterized modules (LPMs). The results produced for various implementations will be compared, both in terms of the circuit structure and its speed of operation.

Part I

Consider again the four-bit ripple-carry adder circuit that was used in lab exercise 2; a diagram of this circuit is reproduced in Figure 1a. You are to create an 8-bit version of the adder and include it in the circuit shown in Figure 1b. Your circuit should be designed to support signed numbers in 2’s-complement form, and the Overflow output should be set to 1 whenever the sum produced by the adder does not provide the correct signed value. Perform the steps shown below.

![Four-bit ripple-carry adder circuit](image)

![Eight-bit registered adder circuit](image)

Figure 1. An 8-bit signed adder with registered inputs and outputs.
1. Make a new Quartus II project and write Verilog code that describes the circuit in Figure 1b. Use the circuit structure in Figure 1a to describe your adder.

2. Include the required input and output ports in your project to implement the adder circuit on the DE1 board. Connect the inputs $A$ and $B$ to two registers and connect both registers to switches $SW_{7-0}$. Use $KEY_0$ as an active-low asynchronous reset input, $KEY_1$ as a manual clock input, $KEY_2$ and $KEY_3$ as the enable signal for input registers. Display $A$ and $B$ on the red $\text{LED}_{R7-0}$ lights and the green $\text{LED}_{G7-0}$ lights respectively. The overflow output should be displayed on the red $\text{LED}_{R8}$ light. The hexadecimal value of $S$ should appear on $\text{HEX}_{1-0}$.

3. Compile your code and use timing simulation to verify the correct operation of the circuit. Once the simulation works properly, download the circuit onto the DE1 board and test it by using different values of $A$ and $B$. Be sure to check for proper functionality of the Overflow output.

4. Open the Quartus II Compilation Report and examine the results reported by the Timing Analyzer. What is the maximum operating frequency, $f_{max}$, of your circuit? What is the longest path in the circuit in terms of delay?

Part II

Modify your circuit from Part I so that it can perform both addition and subtraction of eight-bit numbers. Use switch $SW_9$ to specify whether addition or subtraction should be performed. Connect the other switches, lights, and displays as described for Part I.

1. Simulate your adder/subtractor circuit to show that it functions properly, and then download it onto the DE1 board and test it by using different switch settings.

2. Open the Quartus II Compilation Report and examine the results reported by the Timing Analyzer. What is the $f_{max}$ of your circuit? What is the longest path in the circuit in terms of delay?

Part III

Repeat Part I using the predefined adder circuit called $\text{lpm\_add\_sub}$, instead of your ripple-carry adder structure from Figure 1. The $\text{lpm\_add\_sub}$ module can be found in Altera’s library of parameterized modules (LPMs), which is provided as part of the Quartus II system. The procedure for using these predefined modules in Quartus II projects is described in the tutorial Using Library Modules in Verilog Designs, which is available on the DE1 System CD and in the University Program section of Altera’s web site.

1. Configure the $\text{lpm\_add\_sub}$ module so that it performs only addition, to make the functionality comparable to Part I. Store your configuration of the $\text{lpm\_add\_sub}$ module in the file $\text{lpm\_add8.v}$. After instantiating this module in your Verilog code, compile the project and use the Quartus II Chip Planner tool to examine some of the details of the implemented circuit.

   One way to examine the adder subcircuit using the Chip Planner tool is illustrated in Figure 2. In the Quartus II Project Navigator window right-click on the part of your circuit hierarchy that represents the $\text{lpm\_add8}$ subcircuit, and select the command $\text{Locate} > \text{Locate in Chip Planner}$. This opens the Chip Planner window shown in Figure 3. The logic elements in the Cyclone II FPGA that are used to implement the adder are highlighted in blue in the Chip Planner tool. Position your mouse pointer over any of these logic elements and double-click to open the Resource Property Editor window displayed in Figure 4. In the box labeled $\text{Node name}$ you can select any of the nine logic elements that implement the adder module. The Resource Property Editor allows you to examine the contents of a logic element and to see how one logic element is connected others.
Using the tools described above, and referencing the Data Sheet information for the Cyclone II FPGA, describe the eight-bit adder circuit implemented with the lpm_add_sub module.
2. Open the Quartus II Compilation Report and compare the fmax of your adder circuit with the one designed in Part I. Discuss any differences in performance that are observed.

Part IV

Repeat Part II using the predefined adder circuit called lpm_add_sub, instead of your adder-subtractor circuit based on Figure 1.

Comment briefly on the circuit structure obtained using the LPM module, and compare the fmax of this circuit to the one from Part II. Describe how the lpm_add_sub module implements the Overflow signal.
Part V

Figure 5a gives an example of the traditional paper-and-pencil multiplication \( P = A \times B \), where \( A = 12 \) and \( B = 11 \). We need to add two summands that are shifted versions of \( A \) to form the product \( P = 132 \). Part b of the figure shows the same example using four-bit binary numbers. Since each digit in \( B \) is either 1 or 0, the summands are either shifted versions of \( A \) or 0000. Figure 5c shows how each summand can be formed by using the Boolean AND operation of \( A \) with the appropriate bit in \( B \).

\[
\begin{array}{c}
12 \\
\times 11 \\
\hline
12 \\
12 \\
\hline
132
\end{array}
\]  
\[
\begin{array}{c}
1100 \\
\times 1011 \\
\hline
1100 \\
1100 \\
0000 \\
\hline
10000010
\end{array}
\]

\[a) \text{ Decimal} \quad b) \text{ Binary}\]

\[
\begin{array}{cccccc}
\times & a_3 & a_2 & a_1 & a_0 \\
& b_3 & b_2 & b_1 & b_0 \\
\hline
a_3b_0 & a_2b_0 & a_1b_0 & a_0b_0 \\
a_3b_1 & a_2b_1 & a_1b_1 & a_0b_1 \\
a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 \\
a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 \\
\hline
p_7 & p_6 & p_5 & p_4 & p_3 & p_2 & p_1 & p_0
\end{array}
\]

c) Implementation

Figure 5. Multiplication of binary numbers.

A four-bit circuit that implements \( P = A \times B \) is illustrated in Figure 6. Because of its regular structure, this type of multiplier circuit is usually called an array multiplier. The shaded areas in the circuit correspond to the shaded columns in Figure 5c. In each row of the multiplier AND gates are used to produce the summands, and full adder modules are used to generate the required sums.
Use the following steps to implement the array multiplier circuit:

1. Create a new Quartus II project which will be used to implement the desired circuit on the Altera DE1 board.

2. Generate the required Verilog file, include it in your project, and compile the circuit.

3. Use functional simulation to verify that your code is correct.

4. Augment your design to use switches $SW_{7-4}$ to represent the number $A$ and switches $SW_{3-0}$ to represent $B$. The hexadecimal values of $A$ and $B$ are to be displayed on the 7-segment displays $HEX3$ and $HEX2$, respectively. The result $P = A \times B$ is to be displayed on $HEX1$ and $HEX0$.

5. Assign the pins on the FPGA to connect to the switches and 7-segment displays, as indicated in the User Manual for the DE1 board.

6. Recompile the circuit and download it into the FPGA chip.

7. Test the functionality of your design by toggling the switches and observing the 7-segment displays.
Part VI

Extend your multiplier to multiply 8-bit numbers and produce a 16-bit product. The result \( P = A \times B \) is to be displayed on HEX3–0. Add registers to your circuit to store the values of \( A, B \), and the product \( P \), using a similar structure as shown for the registered adder in Figure 1. Connect switches, lights, and pushbuttons as described for Part I.

After successfully compiling and testing your multiplier circuit, examine the results produced by the Quartus II Timing Analyzer to determine the fmax of your circuit. What is the longest path in terms of delay between registers?

Part VII

Change your Verilog code to implement the 8 × 8 multiplier by using the \textit{lpm} mult module from the library of parameterized modules in the Quartus II system. Complete the design steps above. Compare the results in terms of the number of logic elements (LEs) needed and the circuit fmax.

Part VIII

It many applications of digital circuits it is useful to be able to perform some number of multiplications and then produce a summation of the results. For this part of the exercise you are to design a circuit that performs the calculation

\[ S = (A \times B) + (C \times D) \]

The inputs \( A, B, C, \) and \( D \) are eight-bit unsigned numbers, and \( S \) provides a 16-bit result. Your circuit should also provide a carry-out signal, \( C_{out} \). All of the inputs and outputs of the circuit should be registered, similar to the structure shown in Figure 1b.

1. Create a new Quartus II project which will be used to implement the desired circuit on the Altera DE1 board. Use the \textit{lpm} mult and \textit{lpm} add sub modules to realize the multipliers and adders in your design.

2. Connect the inputs \( A, B, C, \) and \( D \) to switches SW7–0. Use switch SW8 to select between these two sets of inputs: \( A, B \) or \( C, D \). Also, use the pushbuttons KEY2 and KEY3 as write enable (WE) inputs for \( A, C \) and \( B, D \) respectively. Push the buttons to set WE signals to 0, which should allow data to be loaded into the input registers when an active clock edge occurs, while setting WE signals to 1 should prevent loading of these registers.

3. Use KEY0 as an active-low asynchronous reset input, and use KEY1 as a manual clock input.

4. Display the binary value of either \( A \) or \( C \), as selected by SW8, on red LEDR7–0 and display either \( B \) or \( D \) on LEDG7–0. The sum \( S \) should be shown on HEX3-0, and the \( C_{out} \) signal should appear on LEDR8.

5. Compile your code and use either functional or timing simulation to verify that your circuit works properly. Then download the circuit onto the DE1 board and test its operation.

6. It is often necessary to ensure that a digital circuit is able to meet certain speed requirements, such as a particular frequency of a signal applied to a clock input. Such requirements are provided to a CAD system in the form of timing constraints. The procedure for using timing constraints in the Quartus II CAD system is described in the tutorial Timing Considerations with Verilog-Based Designs, which is available on the DE1 System CD and in the University Program section of Altera’s web site.

For this exercise we are using a manual clock that is applied by a pushbutton switch, so no realistic timing requirements exist. But to demonstrate the design issues involved, assume that your circuit is required to operate with a clock frequency of 220 MHz. Enter this frequency as a timing constraint in the Quartus II software, and recompile your project. The Timing Analyzer should report that it is unable to meet the timing requirements due to the lengths of various register-to-register paths in the circuit. Examine the timing analysis report and describe briefly the timing violations observed.
7. One way to increase the speed of operation of a given circuit is to insert registers into the circuit in a way that shortens the lengths of its longest paths. This technique is referred to as pipelining a circuit, and the inserted registers are often called pipeline registers. Insert pipeline registers into your design between the multipliers and the adder. Recompile your project and discuss the results obtained.

Part IX

The Quartus II software includes a predesigned module called altmult.add that can perform calculations of the form $S = (A \times B) + (C \times D)$. Repeat Part VIII using this module instead of the lpm_mult and lpm_add_sub modules. Test your circuit using both simulation and by downloading the circuit onto the DE1 board.

Briefly describe how the implementation of your circuit differs when using the altmult.add module. Examine its performance both with and without the pipeline registers discussed in Part VIII.

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